Towards Combinatorial Engineering of Decomposable Systems

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Abstract

The paper describes combinatorial modeling of decomposable systems: hierarchical system models, system components, design alternatives (DAs) for system components and their interconnection (Is), estimates of DAs and Is, changes of the systems. We point out some basic combinatorial operations: combinatorial description and presentation; analysis and evaluation; revealing of bottlenecks by elements; comparison of system versions; synthesis of composite DAs; modification (e.g., improvement, adaptation). The investigation is based on *hierarchical morphological multicriteria design* (HMMD) which involves an examination of the following: the design of hierarchical system model, the generation and assessment DAs and Is, composing of composite DAs, and improvements on the base of bottlenecks. Our list of support combinatorial problems is the following: design of hierarchical system models; multicriteria ranking (ordinal assessment), morphological clique (synthesis), etc.

1 Introduction

This paper is our attempt to describe combinatorial engineering of decomposable systems. We consider the following issues: (a) a hierarchical combinatorial description of decomposable systems; (b) functional operations of combinatorial engineering (e.g., analysis, design, transformation); (c) basic combinatorial elements (e.g., chains, trees) and their proximity; (d) approaches to structural modeling; (e) compatibility of system components; (f) basic combinatorial problems (multicriteria selection, multiple-choice problem, morphological analysis, clique, morphological clique, etc.).

Combinatorial problems have been used in many applications, for example, as follows:

(1) planning and scheduling in computer systems, in manufacturing, in project management;

(2) design and management of networks;

(3) VLSI and IC design;

(4) engineering design (e.g., in mechanics, electronics, architecture, software engineering, etc.);

(5) information design;

(6) analysis and transformation of genome information, etc.

Roberts has pointed out that combinatorics is concerned with the study of arrangement, patterns, designs, assignment, schedules, connections, and configurations [Roberts, 1984]. Here we examine deterministic combinatorial problems to analyze and compose a decomposable system from components, for each of them there exists a set of design alternatives (DAs). In addition, we take into account ordinal pairwise interconnection or compatibility (Is) among DAs. Our approach (*hierarchical morphological multicriteria design* or HMMD) is described in [Levin, 1995c]. A decomposable system is depicted in Figure 1. Here we consider the following situation: (a) initial system S consists of three components A, B, C with corresponding DAs: A_1, A_2, B_1, B_2, B_3, C_1, C_2. And we can consider system changes as follows:

(1) to add DAs (A_3, B_4, C_3);

(2) to delete DAs (B_2, C_1);

(3) to add a component D with DAs (D_1, D_2, D_3, D_4).

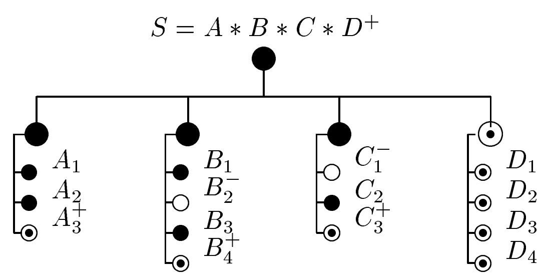

Figure 1. Decomposable system

The list of HMMD stages is the following:

(1) specification of requirements (objectives or criteria, constraints) to a designed system, and its components;

(2) designing the structure of the system;

(3) generation of DAs;

(4) evaluation of DAs, and Is between DAs;

(5) composing of composite DAs from the initial DAs with taking into account their quality and compatibility;

(6) analysis of composite DAs, and refinement;

(7) comparison of composite DAs.

Note that the following applications of HMMD are published: design of user interfaces [Levin, 1994]; information design in hypertexts [Levin, 1995b]; planning a parallel-series solving process [Levin, 1995a]; planning an information center [Levin, 1995d].

HMMD corresponds to hierarchic decomposition approaches: development of a hierarchic structure of the designed system or initial problem on the base of hierarchical levels and decomposition (partitioning); solving local subproblems; and composition of a global solution from local solutions [Newell and Simon, 1972; etc.]. In recent years, hierarchical approaches to the design of complex systems has been actively applied in various methodologies.

2 Functional operations

Let us consider a set of engineering functional operations on the base of Altshuller's levels of creative problems as follows: selection of objects, modification of an object, design of a new object, synthesis of a new system from a set of initial objects) [Altshuller, 1984]. Thus we may examine the following two-dimensional cartesian space:

(i) kind of a system (the whole object or system, the decomposable system consisting of a set of simple objects);

(ii) functional engineering operations: description and/or presentation; analysis as evaluation, assessment; analysis as revealing bottlenecks; comparison; selection; synthesis; modification (correction, improvement, adaptation, reconstruction, reengineering).

As a results we obtain the following basic objects of our examination (e.g., system descriptions) and problems for the whole system (a), and decomposable system (b):

1. *Description and/or presentation:* (1a) functional description, multidimensional representation; (1b) tree-like system model, external requirements: criteria, constraints for the system and its elements, design alternatives (DAs) for the elements (nodes of the system model), interconnection (Is) among DAs, estimates of DAs and Is.

2. *Analysis and evaluation:* (2a) assessment in multiparameter space; (2b) multi-level assessment of the system and its elements, including assessment of composite DAs in a complex space of system excellence.

3. *Analysis as revealing of bottlenecks:* (3a) revealing of critical parameters; (3b) revealing of bottlenecks (by system parts, by Is, by system structure).

4. *Comparison:* (4a) multiparameter comparison of the objects; (4b) comparison of system versions (by components and DAs, by Is, by structure).

5. *Selection:* (5a) multicriteria selection; (5b) multilevel system selection.

6. *Synthesis:* (6a) optimal design; (6b) two problems: (i) system selection; (ii) hierarchical synthesis (design of system model, specification of requirements, generation of DAs, assessment of DAs and Is, composing of composite DAs).

7. *Modification (e.g., improvement, adaptation):* (7a) parameter optimization; (7b) generation of improvement actions (improvement of DAs and/or Is, modification of the system model), and scheduling.

3 Structural modeling

Here we point out the following basic problems of structural modeling:

1. Design of structural (combinatorial) objects.

2. Presentation of the objects.

3. Analysis as the evaluation, comparison of structural objects.

4. Integration (aggregation, computation of consensus for a set of structural objects, synthesis).

5. Transformation (correction, approximation, reconstruction) of a structural object.

We assume that a combinatorial (structural) model is a composition (combination) of basic structural elements, which are the following: sets; chains (or linear orders, strings); clusters (partitions); layered structures (e.g., k-partite graphs); trees; parallel-series graphs; hierarchies; acyclic digraphs (or posets); digraphs; networks; composite structures (e.g., a tree with additional element chains for leaf nodes, a tree with the chains for all nodes, etc.). An example is presented in Figure 1. In this case, posets on the base of multicriteria estimates of DAs have to be examined too.

Also it is reasonable to point out the following kinds of important constructions: (a) metrics and/or proximity of the structural elements (e.g., for the comparison, integration); (b) ways of the transformation (improvement, adaptation) of the structural elements.

In recent years a significance of designing the structural hierarchical models of complex systems has been increased. The following approaches are used for designing of structural models:

(1) design of hierarchical multilevel models of complex systems [Waller, 1976; etc.];

(2) structural modeling [Geoffrion, 1987; etc.];

(3) usage of knowledge based systems for a configuration design [McDermott, 1982; Sykes and White, 1989; etc.];

(4) selection, modification, and aggregation of some standard frames (e.g., technological frames) [Pospelov, 1986];

(5) hierarchical approximation of an initial system structure on the base of spanning trees, Steiner trees, hierarchies, etc. [Garey and Johnson, 1979; Botafogo et al., 1992; etc.].

In this paper, we examine the above-mentioned description of decomposable systems. It is reasonable to use the following notations of basic structural elements: (1) points in a parameter space (e_1); (2) a set (e_2); (3) clusters (e_3); (4) layered structure or k-partite graph (e_4); (5) trees (e_5); (6) posets (e_6).

Finally a relationship between our system description and basic structural elements is as follows:

(a) tree-like system model: e_5;

(b) leaf nodes of the system model: e_2;

(c) a set of DAs: $e_1 \cup e_2 \cup e_4$;

(d) a set of Is: $e_1 \cup e_2 \cup e_4$;

(e) ordinal estimates of DAs/Is: $e_1 \cup e_4$;

(f) multicriteria estimates of DAs/Is: $e_1 \cup e_6$;

(g) criteria for DAs/Is: $e_2 \cup e_3 \cup e_4 \cup e_5$;

(h) constraints for DAs: $e_2 \cup e_3 \cup e_4 \cup e_5$.

Note there exists a dependence between our description above and Kolmogorov's complexes [Uspensky and Semenov, 1987].

4 Interconnection

Complex systems include different types of components and their interrelations.Interconnection (compatibility) of system components is a well-known factor in the design of large-scale systems (e.g., software systems, multiple processor systems, network systems, organization structure, technical systems) [Hubka and Eder, 1988; Morse and Hendrickson, 1990; Selby, 1993; etc.]. Hubka and Eder have pointed out couplings between technical system elements of different kinds, e.g., as follows [Hubka and Eder, 1988]: mechanical; thermal; electrical; chemical; magnetic. Griffin and Hauser have considered communication among product life cycle stages (marketing, engineering, manufacturing) and show that greater communication provides an enhancement of product development [Griffin and Hauser, 1992]. We assume two way of ordinal assessment of Is: (a) direct assessment; (b) a preliminary multicriteria evaluation of compatibility, and mapping of the obtained estimates into an ordinal scale.

5 Integration

Here we examine several levels of the integration or synthesis as follows:

(1) computation of a consensus for a set of structural objects (e.g., consensus of linear orders [Cook and Kress, 1992; etc.]);

(2) aggregation as a selection of a structure from a specified set of structures of a concrete kind, e.g., aggregation of layered structures on the base of searching for a "fuzzy" layered structure with the use of multiple-choice problem [Belkin and Levin, 1990];

(3) synthesis as composing a system from elements, e.g., selection of DAs and their composing on the base of the following problems: multiple-choice problem [Martello and Toth, 1990], problem of representatives [Knuth and Raghunathan, 1992], quadratic assignment problem [Carraway and Schmidt, 1991], nonlinear integer programming [Berman and Ashrafi, 1993], morphological analysis [Zwicky, 1969; Ayres, 1969], morphological clique [Levin, 1995c]; mixed integer programming [Grossmann, 1990].

Now let us consider the morphological clique problem [Levin, 1995c; Levin, 1996]. We assume that a system consists of m components, and for each component $i = 1, ..., m$ there exists a set of DAs (morphological class i). The problem is: Find a composition (morphological scheme) $S = S(1) * ... * S(i) * ... * S(m)$, where $S(i)$ is a selected DA for component i (one representative from each morphological class) with non-zero Is. And we apply a vector-like measure of system excellence: $N(S) = (w(S); n(S))$, where $w(S)$ is the minimum of pairwise compatibility in S, $n(S) = (n(1), ..., n(r), ..., n(k))$ describes selected DAs ($n(r)$ is the number of DAs with priority r in S, where $r = 1$ is the best one, and $w = 0, ..., l$, l corresponds to the best compatibility). Figure 2 presents a system excellence lattice, that corresponds to vector N.

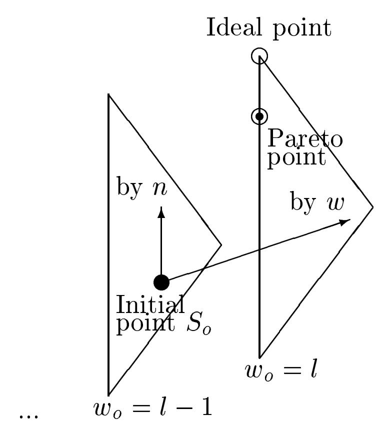

Figure 2. Excellence lattice, improvements (→)

Thus we search for S which is nondominated by $N(S)$. In the same way we can examine a problem, when several representatives maybe selected from each morphological class. Our basic algorithm consists in the construction of feasible compositions, and selection of Pareto-effective ones (by N).

6 Presentation

Basic approaches to the presentation of structural objects are based on traditional mathematical descriptions. On the other hand, it is reasonable to investigate new visual techniques for the presentation and manipulation of the objects. An example of similar software tool is proposed in [Jourdan et al., 1995].

In HMMD, we apply hierarchical (Figure 1) and concentric (Figure 3) presentations of decomposable systems, matrix and graph presentations of compatibility, vector-like and lattice (Figure 2) presentations of the system excellence [Levin, 1995c; Levin, 1996]. Figure 3 demonstrates two compositions: $S_1 = A_2 * B_1 * C_1$, and $S_2 = A_2 * B_3 * C_2$.

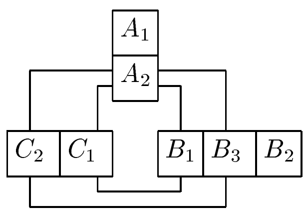

Figure 3. Concentric presentation

7 Analysis

The following types of an analysis are the basic ones:

(a) recognition and analysis of some features for structural objects, e.g., a kind, properties (recognition of tree-like graphs, recognition of parallel-series graphs, etc.);

(b) assessment (e.g., on the base of features);

(c) revealing of bottlenecks;

(c) comparison.

The feature recognition problems for structural objects are well-known [Garey and Johnson, 1979; etc.]. The assessment problem have to be examined from the following two viewpoints: (a) assessment of features (the problem is based on the recognition); (b) evaluation, including the multicriteria analysis, and assessment of a system excellence. Recently many techniques of multicriteria analysis have been developed [Dyer et al., 1992; Buede, 1992; etc.]. The assessment of the system excellence often is based on the traditional multiattribute comparison (e.g., a layer of Pareto-effective points).

Revealing of bottlenecks maybe based on a comparative analysis of system elements. In HMMD we examine the following kinds of elements (DAs, Is) with respect to solution S: *S-improving*, *S-neutral*, and *S-aggravating* ones by vector $N(S)$. Thus we can use *S-aggravating* elements as bottlenecks. And the following main types of the system comparison maybe considered: (1) comparison of DAs sets; (2) comparison of the system models (i.e., system structures); (3) combination of previous cases; (4) comparison of system requirements; An example of the system comparison for user interface versions is described in (Levin, 1994).

Issues of a metric/proximity for combinatorial objects are crucial ones. In our opinion, constructing a proximity of complex objects is a specific design, that maybe based on the selection, modification or composing of well-known metrics or proximity models. The following approaches have been used to compare combinatorial objects: (1) metrics; (2) attributes of an intersection or a covering structure; (3) attributes of a transformation path from an initial object into a target one.

Additional important issue consists in constructing or selecting a scale of the proximity (e.g., R^1, $[0, 1]$ or an ordinal one). In recent years, extensions of metric spaces have been proposed (Jawhari et al. 1986; Pouzet and Rosenberg, 1994; etc.) including the following: (a) graphs and posets; (b) conceptual lattices; (c) semilattices; (d) simplices. Thus it is reasonable to use the following approaches: (i) multidimensional scaling; (ii) usage of complex constructions for proximity spaces; (iii) integrating or composing a global proximity from distances or proximity models of system components.

8 Transformation

Main two types of structural problems (transformation and approximation) maybe used at different engineering levels as follows: (a) the local transformation; (b) the global modification, adaptation, improvement, reconstruction or re-engineering of the system.

Approximation of a combinatorial object corresponds to well-known combinatorial problems, e.g., spanning tree, Steiner tree problem, covering problem, etc. [Garey and Johnson, 1979]. Recently the problems above have been considered in important applications (e.g., network design on the base of tree-like approximation).

On the other hand, new practical approximation problems have been appeared, for example: (i) approximation of information structures with the use of hierarchices, clusters, cliques [Levin, 1989; Botafogo et al., 1992; Levin, 1995b; etc.] (ii) processing of preference relations in decision making on the base of approximation [Belkin and Levin, 1990].

Specific approximation combinatorial problems have been proposed for hypertext in [Levin, 1989; Levin, 1995b], including approximation and new kind of combinatorial problems with compatibility of selected items for knapsack, multiple-choice and clique problems.

In our opinion, the modification is now the most important kind of engineering activities (e.g., redesign, re-engineering). Some clear examples of the combinatorial transformation have been proposed in biological mathematics, when it is necessary to find the simplest path of a genome transformation [Hannenhalli and Pevzner, 1995; etc.]. Similar problems are often reduced to routing.

In HMMD, the improvement process consists of the following: revealing bottlenecks by DAs or/and Is, generating the set of improvement actions for the selected bottlenecks, and scheduling the improvement actions. Basic two kinds of improvement actions are depicted in Figure 3. This improvement approach has been used for an adaptation of an user interface in [Levin, 1995a]. Note that scheduling of improvement actions may be based on morphological clique problem too, and similar synthesis of parallel-series solving strategies is described in [Levin, 1995a; Levin, 1996].

Now let us point out basic changes of decomposable systems:

1. *Internal changes:* (a) local evolution (DAs and/or Is); (b) global evolution (subsystems); (c) reconstruction (system model).

2. *External changes:* requirements.

9 Education issues

In our opinion, the description, presentation, selection and composing of complex systems are critical ones for specialists in various disciplines. Basic combinatorial engineering problems may correspond to all standard stages of decision making at different levels (strategic planning, investment, design and marketing of products, selection and assignment of personnel, etc.) [Simon et al., 1987].

We have examined the analysis of decomposable systems, selection of DAs and composing of composite DAs, etc.) for teaching in [Levin, 1995e]. It is reasonable to use special introductions of combinatorial engineering in educational programs.

10 Conclusion

Finally let us emphasize the following directions of future investigations:

1. Modification of complex decomposable systems, including formal descriptions, issues of the users and designers behavior and required specific experience and knowledge (e.g., system thinking).

2. New combinatorial problems, including issues of algorithm complexity, and complexity of human activities in interactive procedures.

3. Case studies of combinatorial engineering for various application domains.

4. Issues of visualization, and manipulation of structutal objects and basic combinatorial functional operations, including the presentation of complex systems and their excellence.

5. Educational issues of combinatorial engineering.

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