

## **LECTURE 1. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Inst. of Physics and Technology (University)**

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### **L.1. Systems, structure, life cycle, examples.**

#### ***PLAN:***

1. Profile of specialist
2. About course
3. Illustrative example of system and life cycle
4. Role of mathematics (models, algorithms)
5. Life cycle and logistic curve
6. Russian engineering experience
7. Levels of system complexity
8. Simple examples of systems
9. Monitoring systems

Sept. 3, 2004

## 1. Profile of specialist

### STRUCTURE:

#### A. Basic scientific disciplines

1. Mathematics
2. Physics, physical-chemical processes , etc.

#### B. Special engineering disciplines

1. Radioengineering, etc.

#### C. Information technology

#### D. Management / economics



#### E. System thinking



#### F. Creativity



#### G. Experience in applied domains

## 2.About course

A.Systems, multi-disciplinary systems

(airplane, machine, radar, team, plan, manufacturing systems, etc.)

B.Life cycle (life cycle engineering)

C.Design schemes (frameworks), support of life cycle

D.Structure of the course:

(1)lecture blocks

(schemes, models/methods, technological problems, applied examples)

(2)Assignment (simple preliminary works)

(3)Projects (realistic applied systems)

E.Neighbor courses:

\*system engineering

\*system design (e.g., architecture, mechanical engineering)

\*technology management

\*multicriteria decision making

\*combinatorial optimization

\*knowledge engineering

\*applications (engineering, management, information technology)

### 3.Illustrative example of life cycle

#### A.Life cycle:

- \*preliminary research
- \*R&D
- \*manufacturing
- \*testing
- \*marketing
- \*utilization & maintenance
- \*recycling

#### B.A system (airplane):

- \*body
- \*engine
- \*electronics (control, communication, etc.)
- \*human environment

#### Additional support subsystem:

- \*maintenance
- \*training
- \*recycling subsystem
- \*etc.

## 4.Role of mathematics

### A.Models

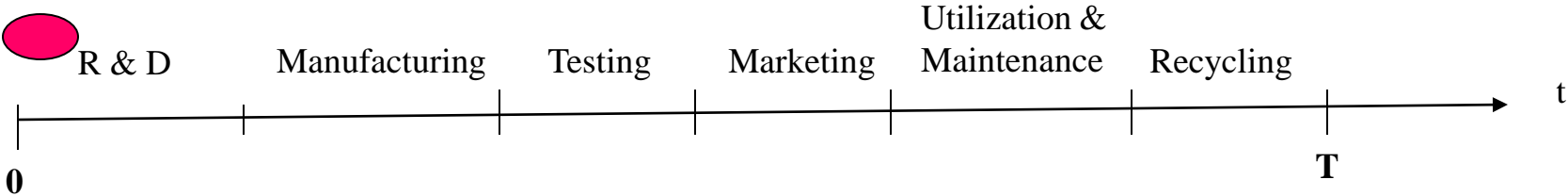
- \*structural models (e.g., graphs, networks)
- \*optimization models
- \*multicriteria decision making
- \*differential equations (dynamics)
- \*game theory
- \*uncertain models (probability, fuzzy sets)

### B.Algorithms

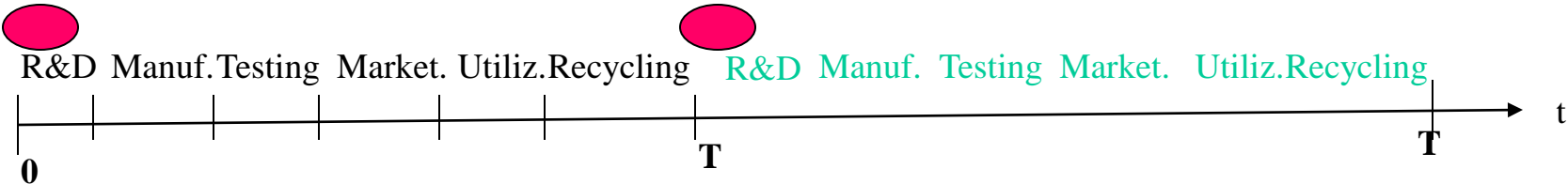
### C.Solving schemes

Real New Application  $\Rightarrow$   
new or modified models / algorithms

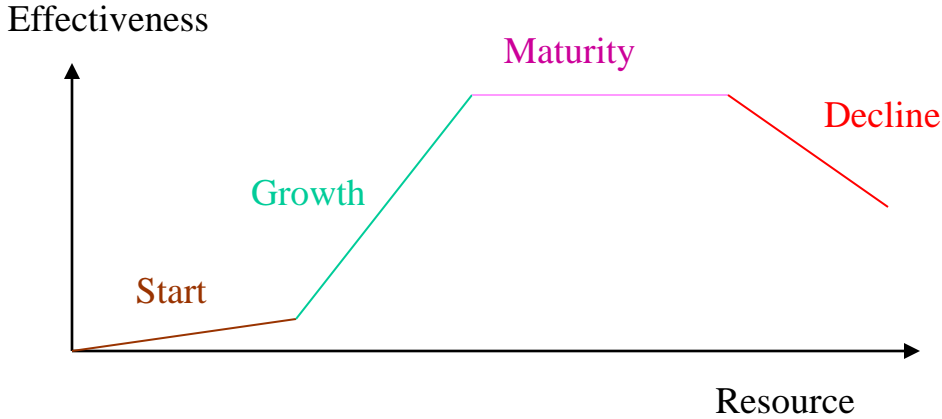
# 5. Life cycle and logistic curve



T: about 12 years (submarines, airplanes, nuclear technology, etc.) TENDENCY: increasing T (2 years, 6 months)



RESULT: need of specialists in system design & specialists in life cycle engineering



### **Complex systems:**

- 1.Airplanes**
- 2.Aerospace systems (stations, etc.)**
- 3.Communication systems**
- 4.Nuclear technology**
- 5.Defense systems (radars, etc.)**
- 6.etc.**

### **Factors:**

- 1.Creative people**
- 2.Educational system**
- 3.Engineering traditions (in design of complex systems)**
- 4.Complex problems**  
**(very large territory, various environments, etc.)**

## 7. Levels of system complexity (A. Shenhar)

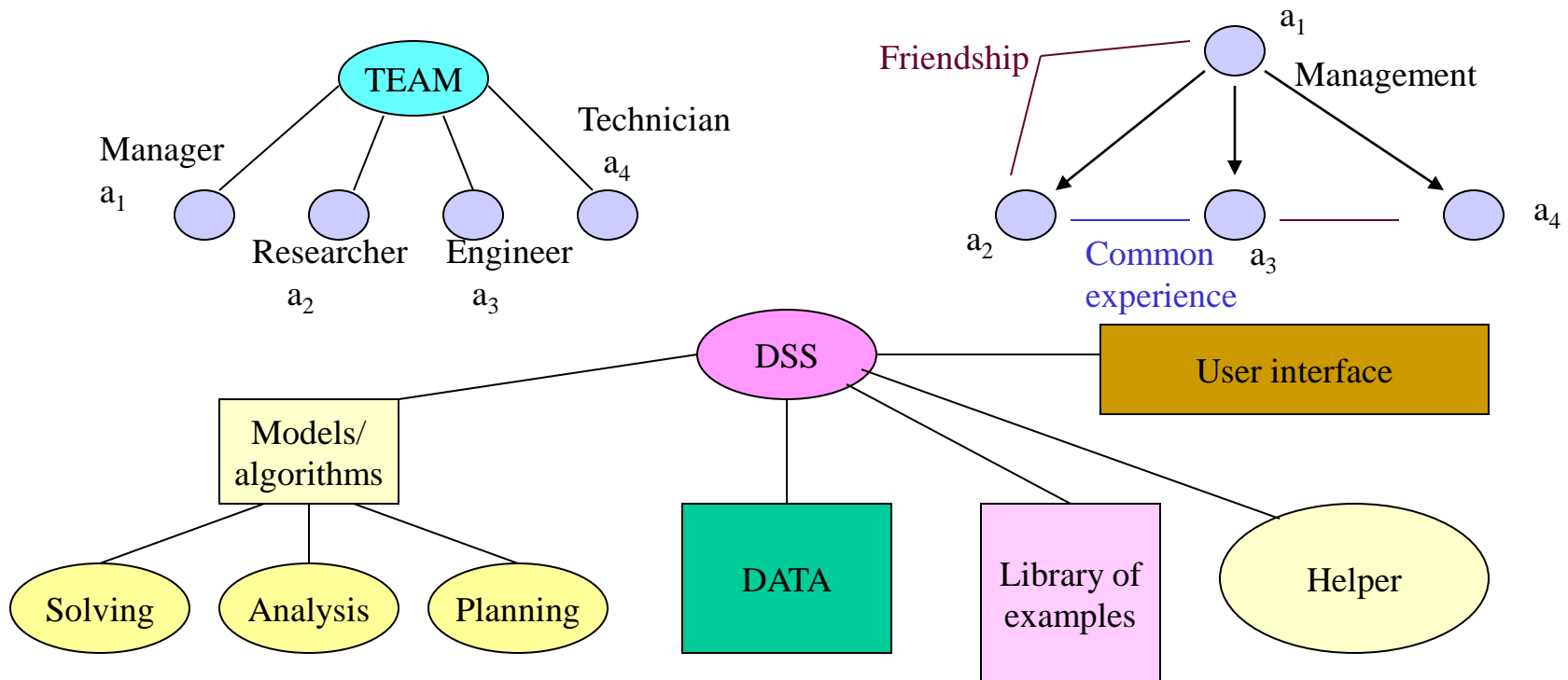
**Level 1. Arrays (network of systems, e.g., network of radar defense systems)**

**Level 2. System (multiple functions; radar, defense system)**

**Level 3. Assembly (one function: TV)**

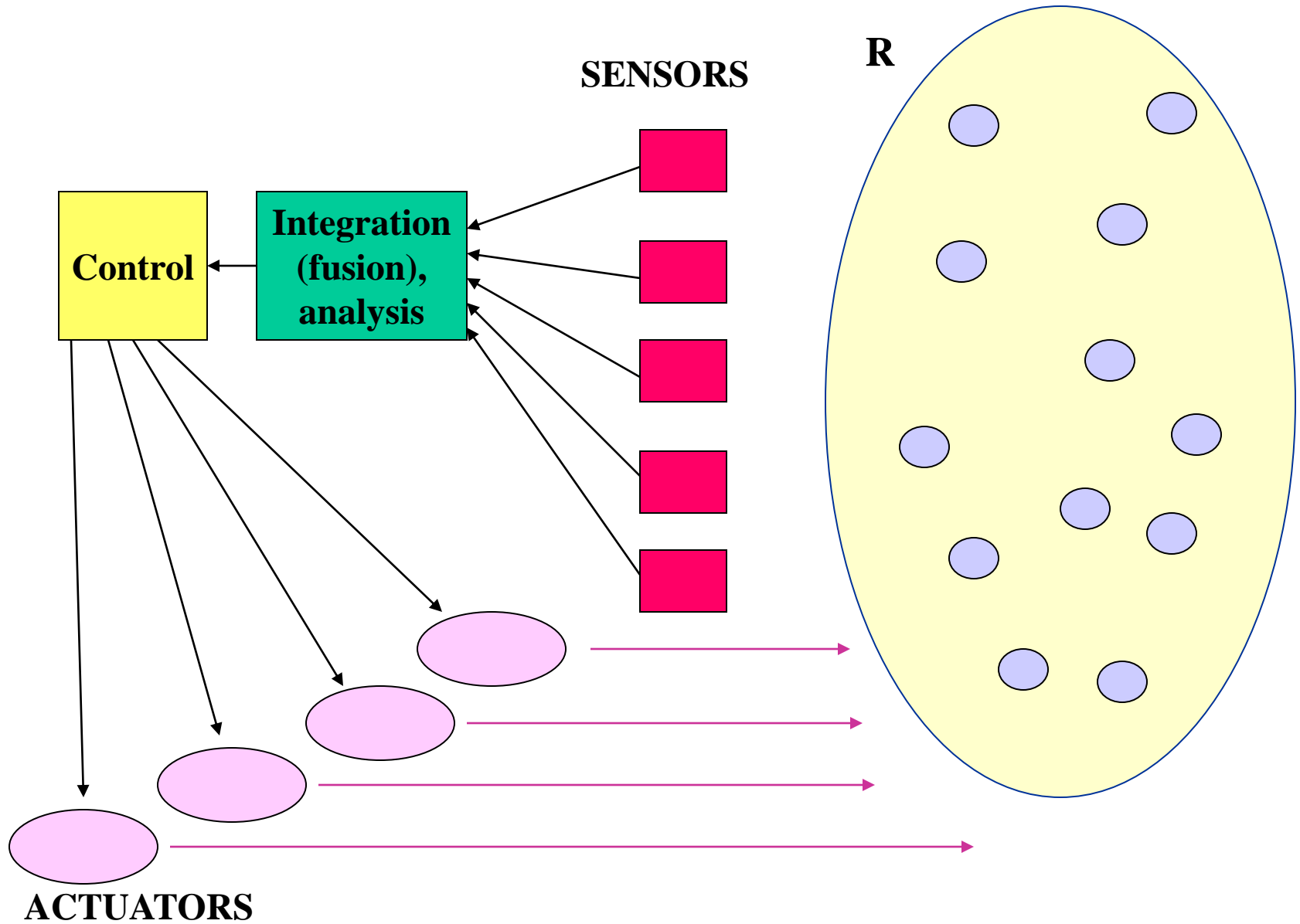
**Level 4. Component**

## 8. Simple examples of systems





# 9. Monitoring systems



**LECTURE 2-3. Course: “Design of Systems: Structural Approach”**

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**L.2. Modularity, system decomposition (partitioning), example.**

**L.3. Structural models (graphs, networks, binary relations), examples.**

***PLAN:***

1. Decomposition (partitioning) of systems \*decomposition – partitioning; \*illustrative examples; \*approaches

2. Issues of modularity: \*description and a basic linguistic analogue

\*applied examples (mechanical engineering, , aerospace engineering, etc.) \*goals and results

3. Structural models: \*graphs (graphs, digraphs, sign graphs)

\*simple structures (e.g., chains, trees, parallel-series graphs)

\*problems on graphs (metric/proximity, optimization, advance models)

Sept. 4, 2004

## 1. Decomposition / partitioning of systems

Decomposition: series process (e.g., dynamic programming)

Partitioning: parallel process / dividing (combinatorial synthesis)

Methods for partitioning:

- \*physical partitioning

- \*functional partitioning

Examples (for airplane, for human)

Examples for software:

1. Series information processing (input, solving, analysis, output)

2. Architecture:

- data subsystem, solving process, user interface,

- training subsystem, communication

3. Additional part: visualization (e.g., for data, for solving process)

4. Additional contemporary part (model management) as follows:

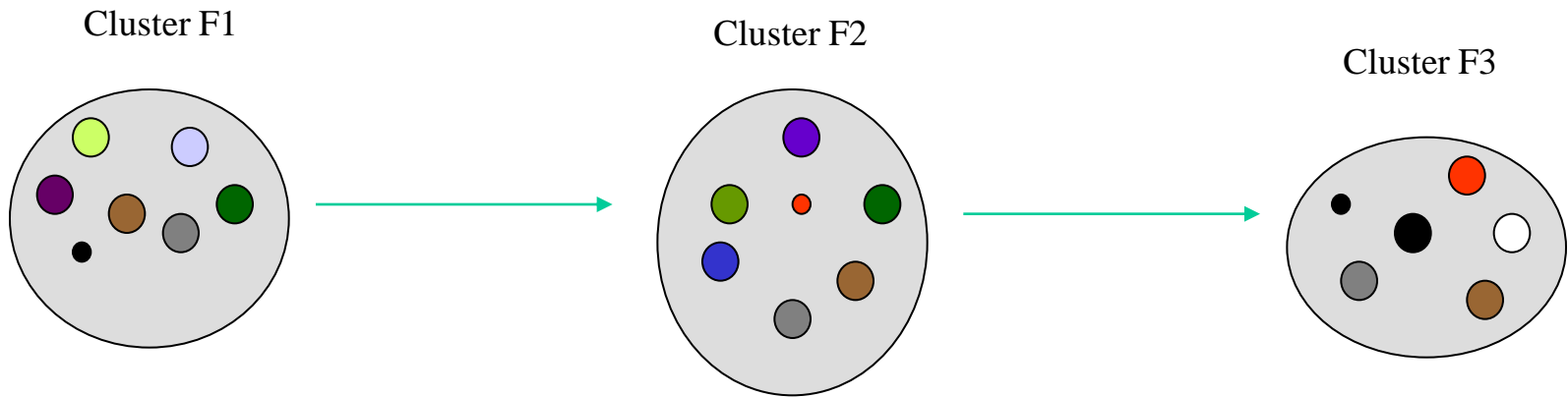
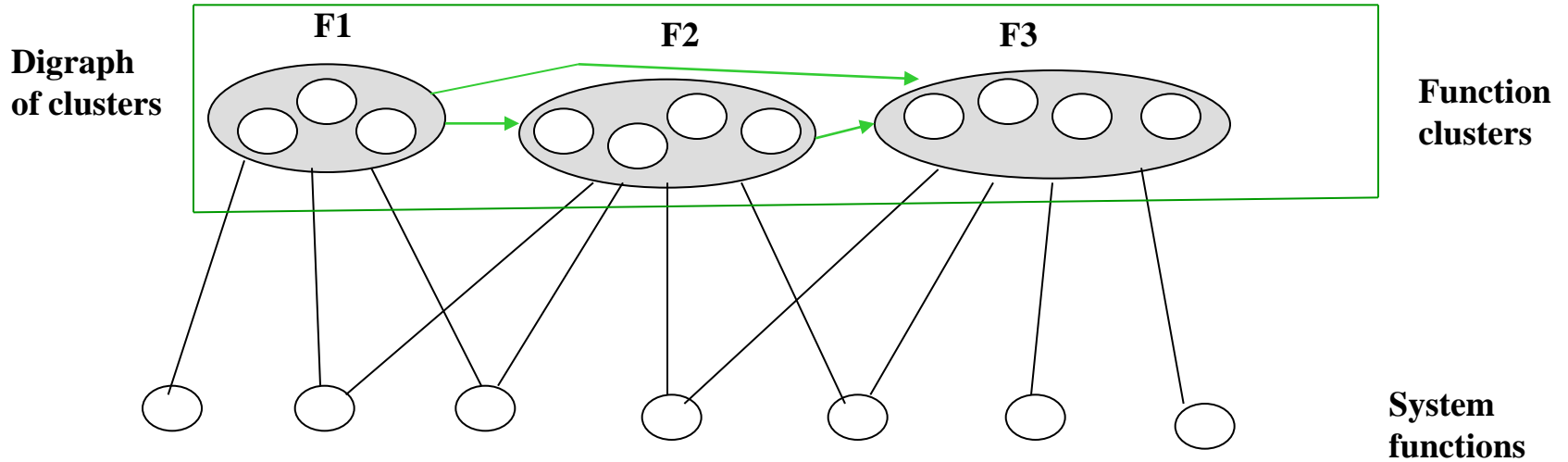
- \*analysis of an initial applied situation,

- \*library of models / methods,

- \*selection / design of models / methods,

- \*selection / design of multi-model solving strategy

# 1. Decomposition / partitioning of systems: Example for multifunction system testing



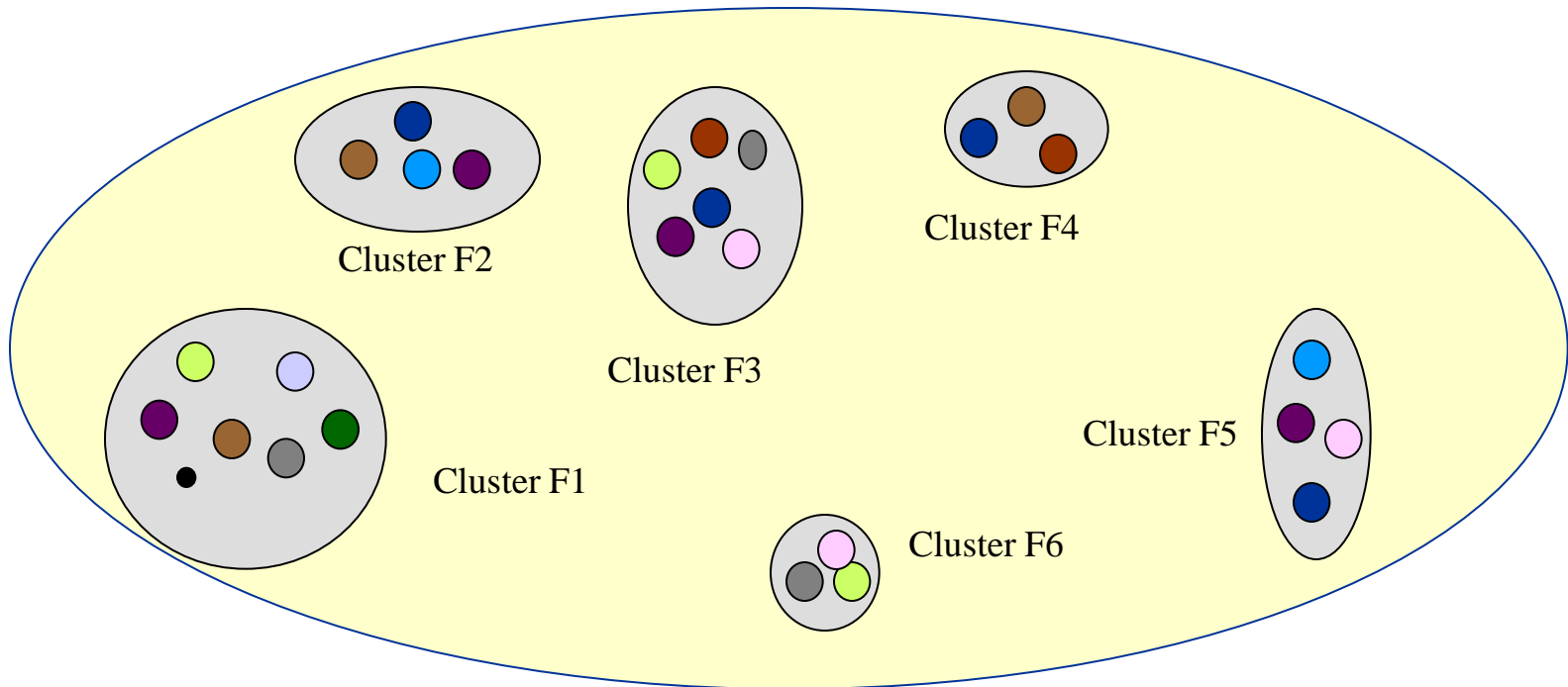
## Main approaches to partitioning of systems

A. Content analysis and experience:

\*by functions (basic functions, auxiliary functions)

\*by system parts (physical partitioning)

B. Cluster analysis (clustering)



## 2. Issues of modularity

### PRINCIPLES FOR MANAGEMENT OF COMPLEXITY :

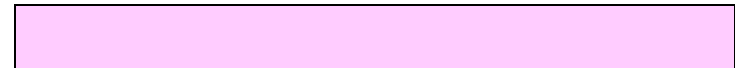
\*discrete pieces (modules)

\*standard interfaces for module communication

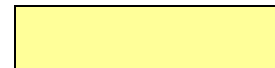
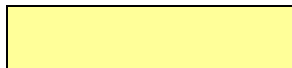
Applications: \*new technology design \* organizational design

### LINGUISTIC SYSTEM

TEXTS



PRASES



WORDS



ABC



## Applied examples for usage of modularity

- 1.Genetics
- 2.Reconfigurable manufacturing
- 3.Software libraries of standard modules
- 4.Combinatorial Chemistry:
  - \*molecular design in chemistry and biology
  - \*drug design
  - \*material engineering
  - \*etc.
- 5.Aerospace & mechanical engineering
- 6.Electronics
- 7.Civil engineering

## Main goals of modularity and resume

### Main goals:

1. Management of complexity
2. Parallel work
3. Accommodation of future uncertainty
4. Variety of resultant modular systems
5. Flexibility, adaptability, reconfigurability of resultant modular systems

### Resume:

1. Simple design process & simple all phases of life cycle
2. Short life cycle of product, long life cycle of product modules
3. Reconfigurable systems (e.g., manufacturing systems):  
long life cycle for system generation
4. Simple design and support of product families (airplanes, cars, etc.)
5. Simple design and support of different products  
(on the basis of module libraries as reuse)



## 3.Structural models

### A.GRAPHS

- 1.Graphs
- 2.Digraphs (directed graphs, oriented graphs - orgraphs)
- 3.Graphs / digraphs with weights (for vertices, for edges / arcs)
- 4.Simple graphs: chains, trees, parallel-series graphs, hierarchies
- 5.Sign graphs

### B.NETWORKS

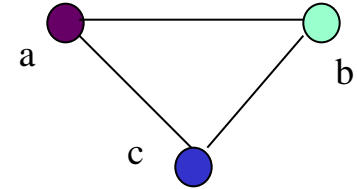
### C.AUTOMATA

### D.BINARY RELATIONS

## Illustration for graphs / digraphs

**Graph:**  $G = (A, E)$  where a set of nodes (vertices)  $A = \{1, \dots, n\}$   
and a set of edges  $E \subseteq A \times A$  (pairs of nodes)

Example:  $A = \{a, b, c\}$ ,  $E = \{(a, b), (b, c), (a, c)\}$

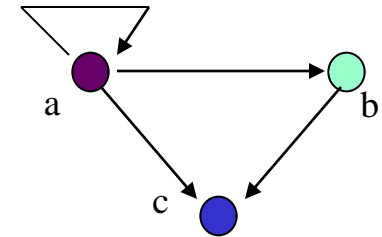


Matrix

	a	b	c
a		1	1
b	1		1
c	1	1	

**Digraph (orgraph):**  $G = (A, E)$  where a set of nodes (vertices)  $A = \{1, \dots, n\}$   
and a set of arcs  $E \subseteq A \times A$  (pairs of nodes)

Example:  $A = \{a, b, c\}$ ,  $E = \{(a, a), (a, b), (b, c), (a, c)\}$

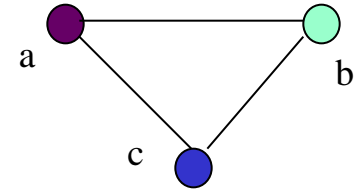


Matrix

	a	b	c
a	1	1	1
b			1
c			

## Illustration for graphs with weights

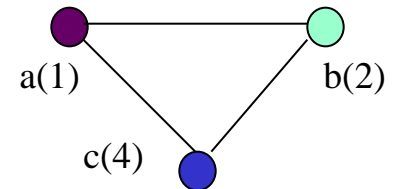
**Graph (weights of edges):**  $G = (A, E)$  where a set of nodes (vertices)  
 $A = \{1, \dots, n\}$  and a set of edges  $E \subseteq A \times A$  (pairs of nodes)  
 Example:  $A = \{a, b, c\}$ ,  $E = \{(a, b), (b, c), (a, c)\}$



Matrix

	a	b	c
a		2	5
b	2		3
c	5	3	

**Graph (weights of edges & nodes):**  $G = (A, E)$  where a set of nodes  
 (vertices)  $A = \{1, \dots, n\}$  and a set of edges  $E \subseteq A \times A$  (pairs of nodes)  
 Example:  $A = \{a, b, c\}$ ,  $E = \{(a, b), (b, c), (a, c)\}$   
 (weights of nodes are pointed out in brackets)

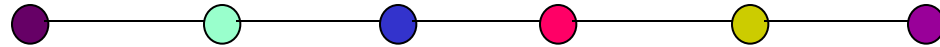


Matrix

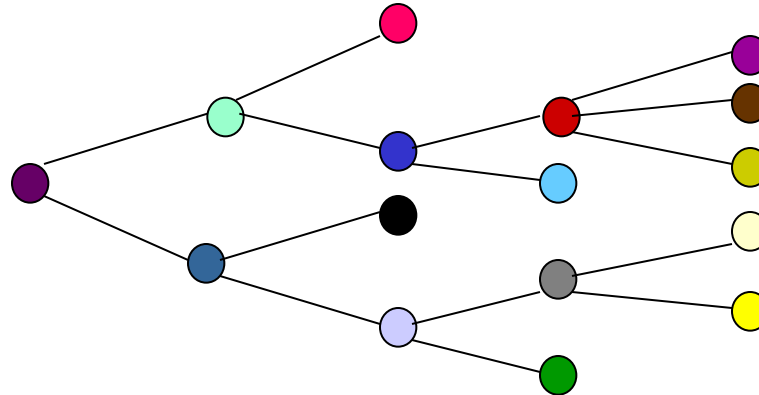
	a	b	c
a		2	5
b	2		3
c	5	3	

Simple structures (chains, trees, parallel-series graphs)

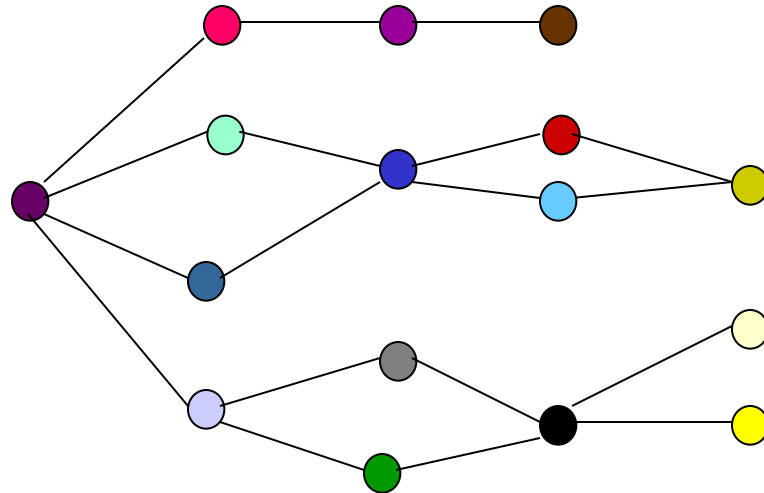
CHAIN



TREE



PARALLEL-SERIES  
GRAPH



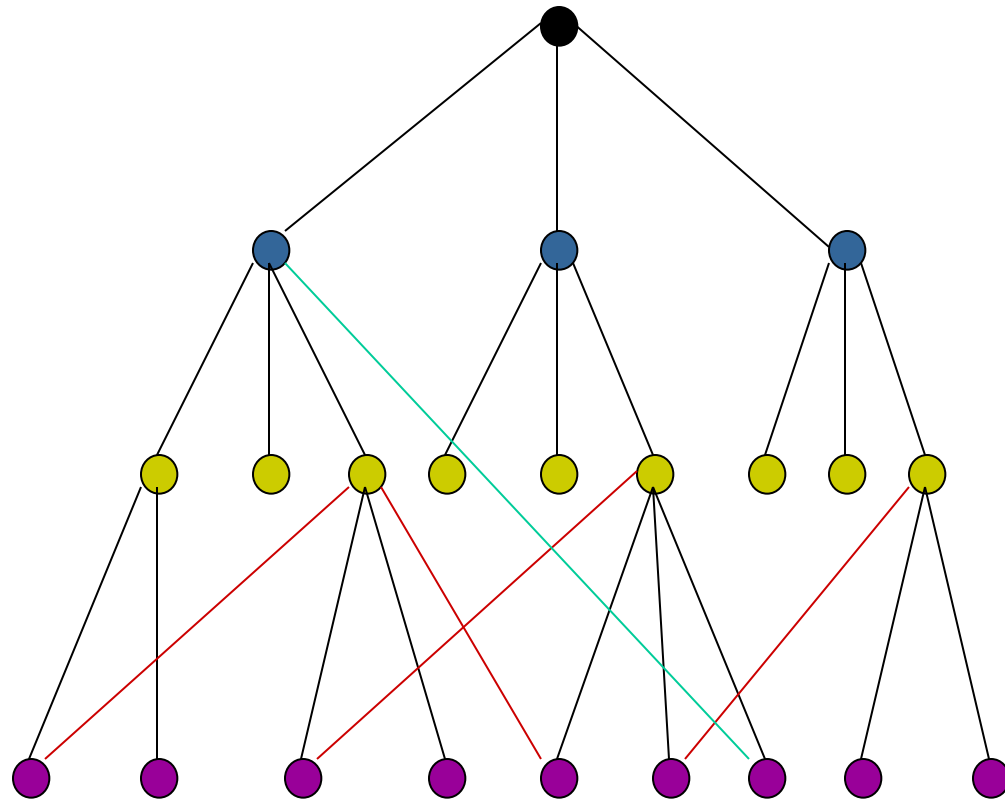
## Simple structures (hierarchy)

Level 4

Level 3

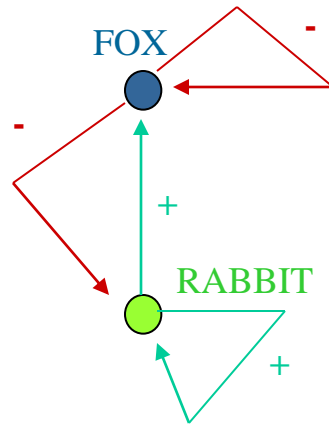
Level 2

Level 1



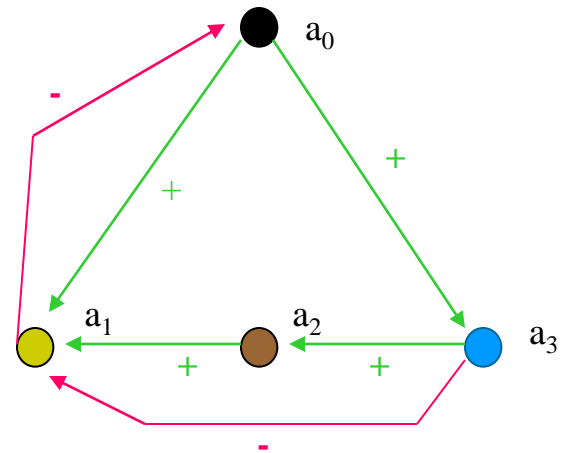
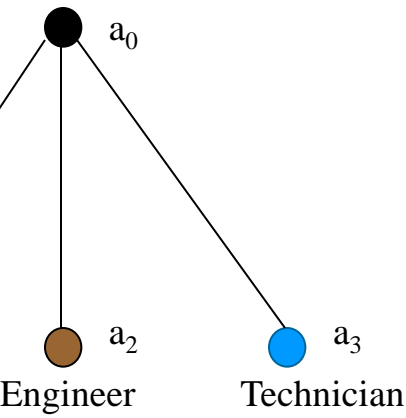
# Sign graph: illustrative examples

**Ecological system**



**Team**

Manager



## Some advanced structural models

1. Multigraphs

2. Graphs with versions for nodes (vertices)

3. Graphs with “vector weights”

4. Graphs with fuzzy weights

## Problems on graphs

### A.Metric / proximity (in graph between nodes, between graphs)

Proximity between graphs:

1.metrics, 2.edit distance (minimal “cost” transformation), 3.common part

### B.Optimization on graphs:

1.Shortest path

2.Spanning tree (& close approximation problems: spanning by other simple structures)

3.Traveling salesman problem

4.Minimal Steiner tree

5.Ordering of vertices

6.Alocation on graphs

7.Covering problems

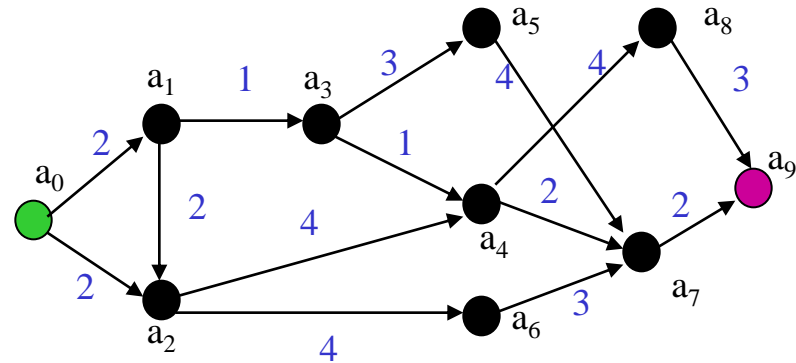
### C.Balance problem for sign graphs

### D.Clustering (dividing into interconnected groups)



## Optimization problems on graphs: illustrations

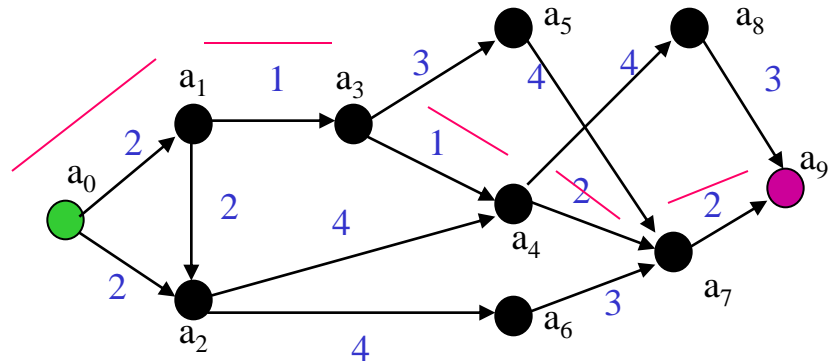
**BASIC GRAPH (DIGRAPH):**  
weights for arcs (or edges)



**Shortest Path for  $\langle a_0, a_9 \rangle$ :**

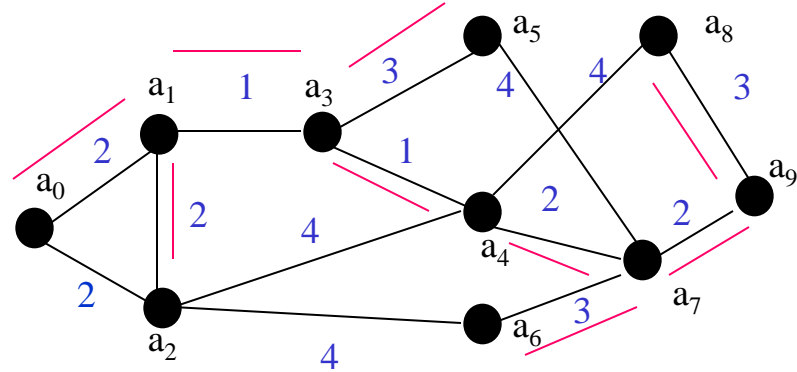
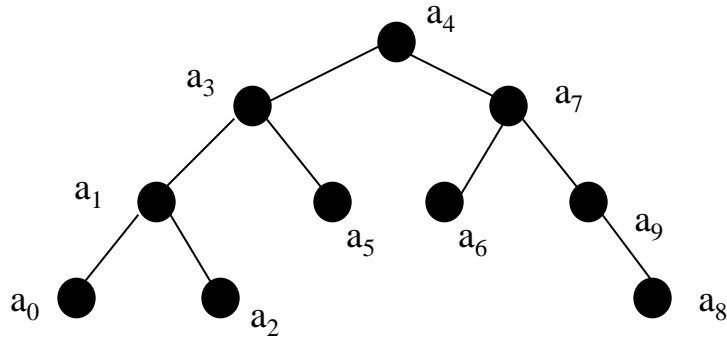
$L = \langle a_0, a_1, a_2, a_3, a_4, a_7, a_9 \rangle$

$2+1+1+2+2 = 8$



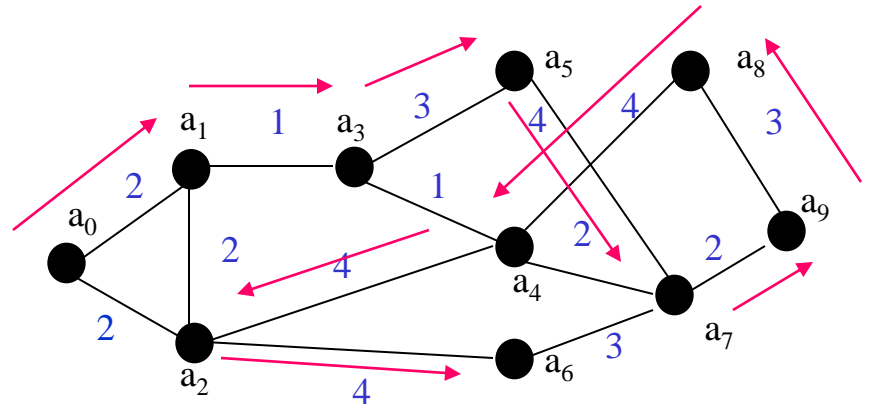
## Optimization problems on graphs: illustrations

**Spanning tree (length = 19):**



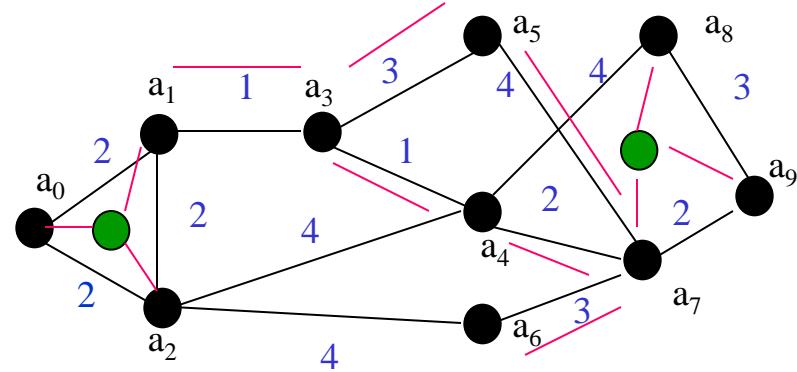
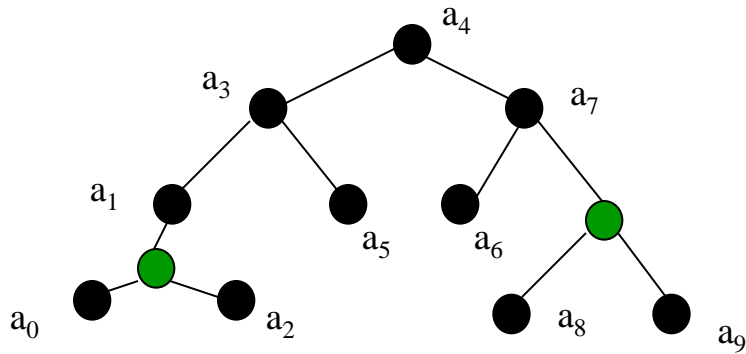
**Traveling Salesman Problem :**

$L = \langle a_0, a_1, a_3, a_5, a_7, a_9, a_8, a_4, a_2, a_6 \rangle$   
 $2+1+3+4+2+2+3+4+4+4$

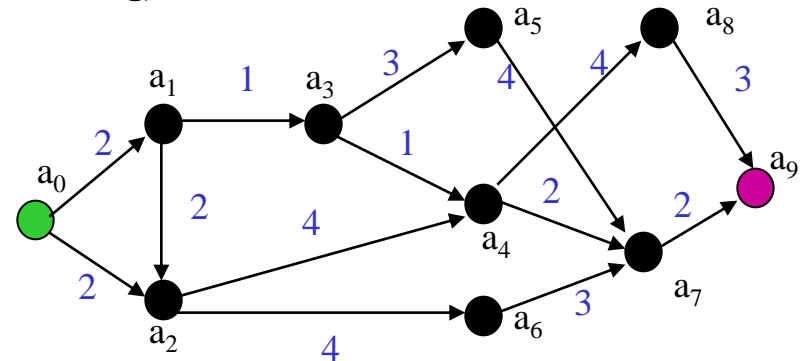
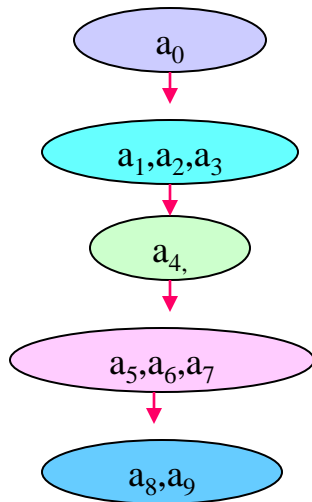


## Optimization problems on graphs: illustrations

### Steiner tree (example):



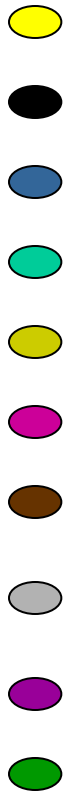
### “Ordering” Problem (close problems: sequencing, scheduling):



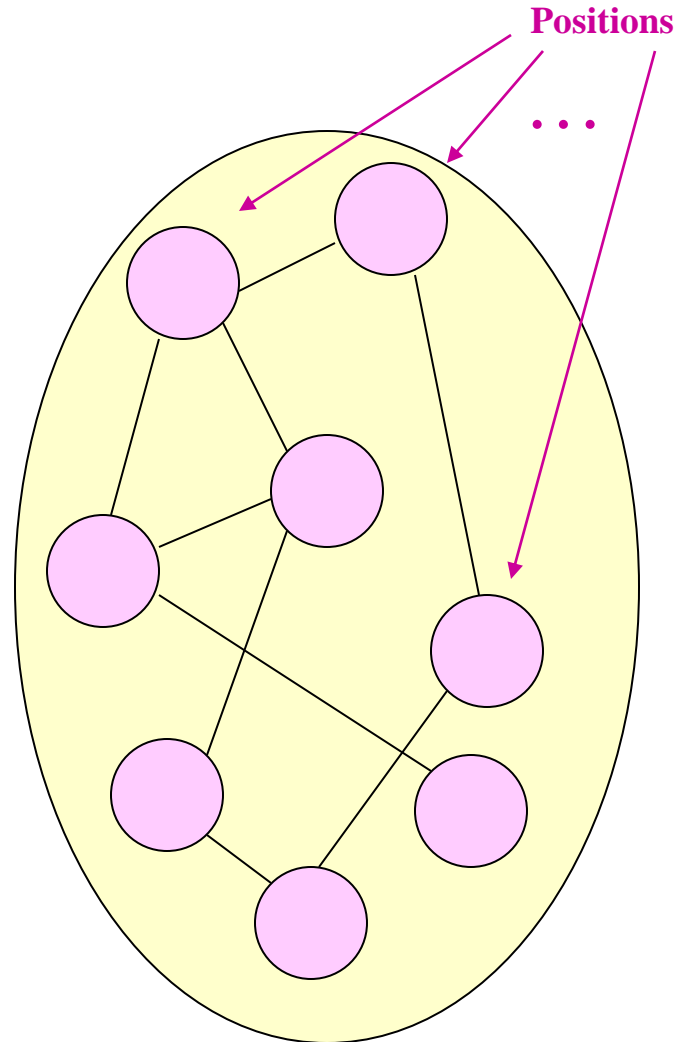
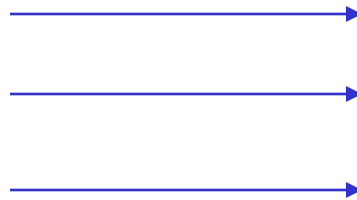
# Optimization problems on graphs: illustrations

**Allocation (assignment, mapping):**

**Set of  
elements**

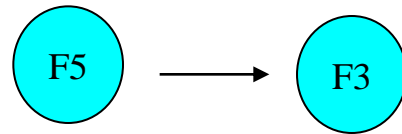
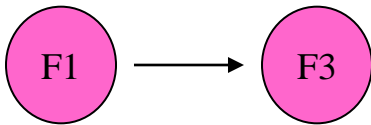
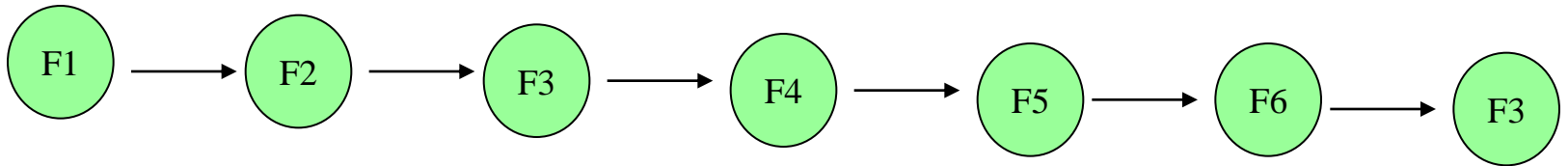
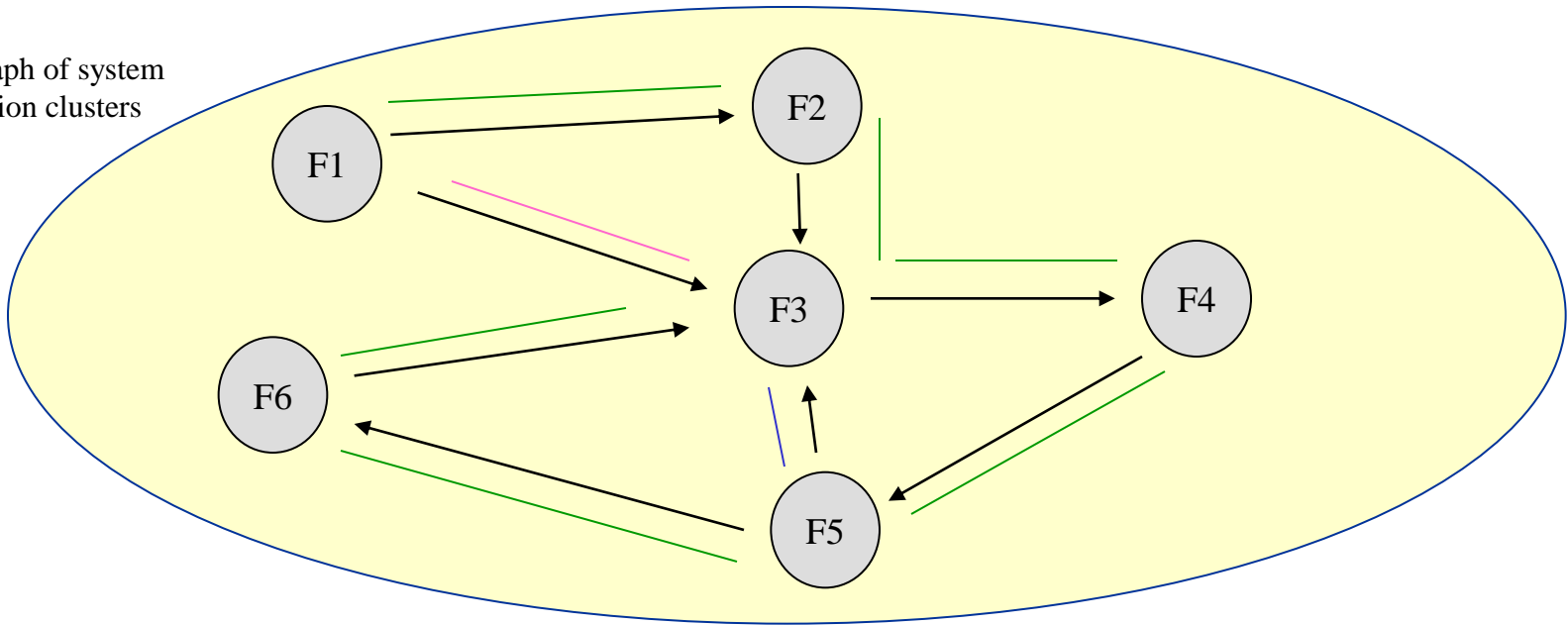


**ALLOCATION  
(mapping, assignment)**



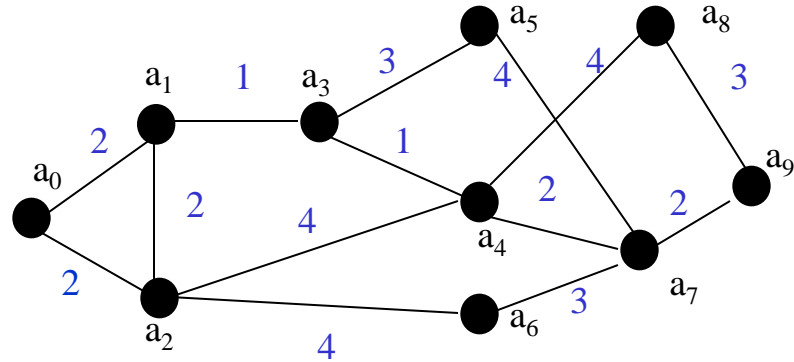
Example: system function clusters and covering by chains (covering of arcs)

Digraph of system function clusters



## Illustration for clustering

Basic graph



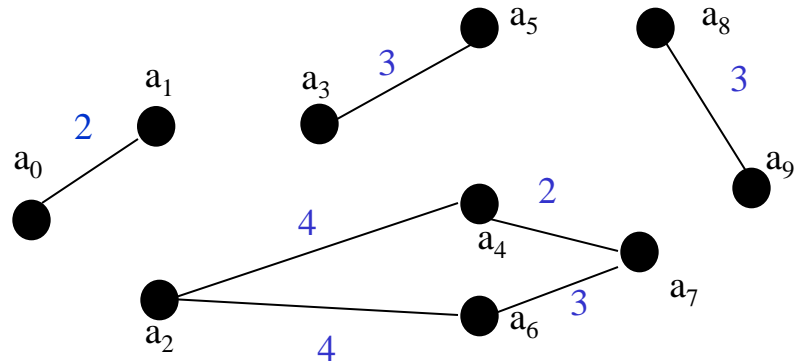
Clusters (a version):

$$C_1 = \{ a_0, a_1 \}$$

$$C_2 = \{ a_3, a_5 \}$$

$$C_3 = \{ a_8, a_9 \}$$

$$C_4 = \{ a_2, a_4, a_6, a_7 \}$$



## Binary relations

Initial set  $A = \{1, 2, \dots, n\}$ ,  $B = A \times A$  ( $\forall (x, y)$  such that  $x, y \in A$ )

**Definition. Binary relation  $R$  is a subset of  $B$**

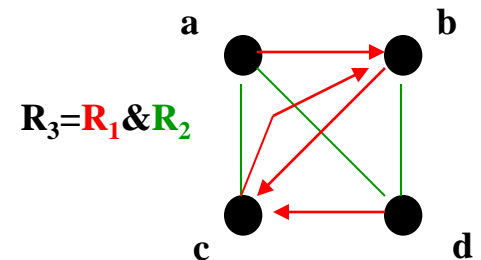
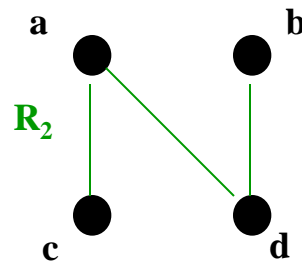
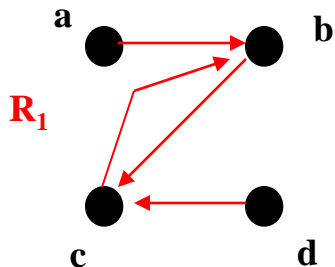
**EXAMPLE:**  $A = \{a, b, c, d\}$

$B = \{(a, a), (a, b), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, a), (c, b), (c, c), (c, d), (d, a), (d, b), (d, c), (d, d)\}$

$R_1 = \{(a, b), (b, c), (c, b), (d, c)\}$

$R_2 = \{(a, d), (b, d), (a, c)\}$

$R_3 = R_1 \& R_2$



## Binary relations

### Context Examples:

1. "Better" (dominance)
2. "Better & Equal" (dominance & equivalence)
3. "Equal" (equivalence)

### SOME PROPERTIES:

1. Symmetry:  $(x, y) \in R \Rightarrow (y, x) \in R \quad (\forall x \in R, \forall y \in R)$
2. Reflexivity:  $(x, x) \in R \quad \forall x \in R$
3. Transitivity:  $(x, y) \in R, (y, z) \in R \Rightarrow (x, z) \in R \quad (\forall x \in R, \forall y \in R, \forall z \in R)$

**APPLICATIONS:** \*Friendship, \*Partnership, \*Similarity, \*Etc.

### Extended models:

1. Weighted binary relations (e.g., power of dominance)
2. K-relations

### Prospective usage:

Combinatorial optimization problems on graphs with additional binary relations  
(over node/vertices, over edges / arcs, over elements / positions)



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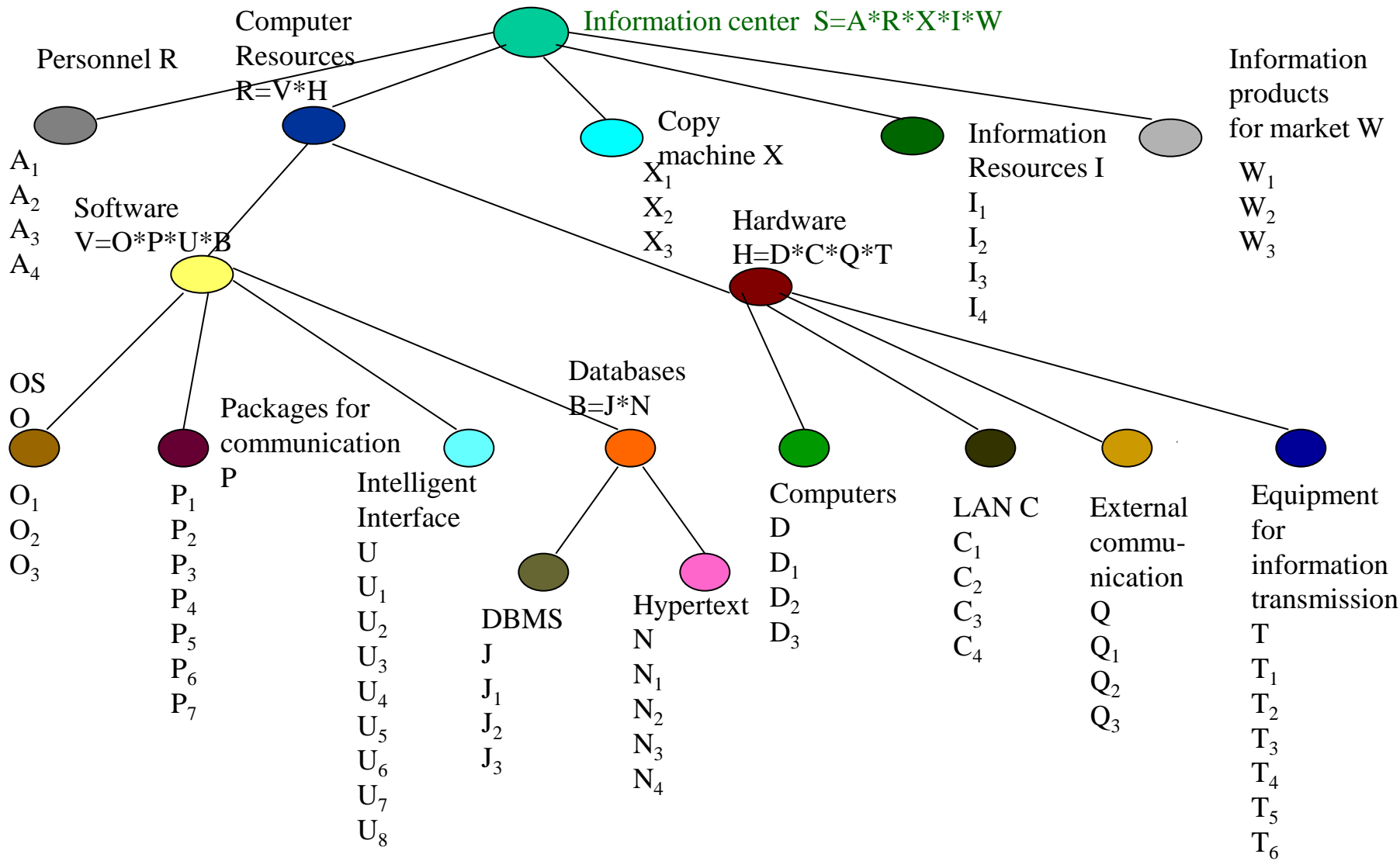
### **L.4. Example: joint design of hierarchical system. Communication systems.**

#### ***PLAN:***

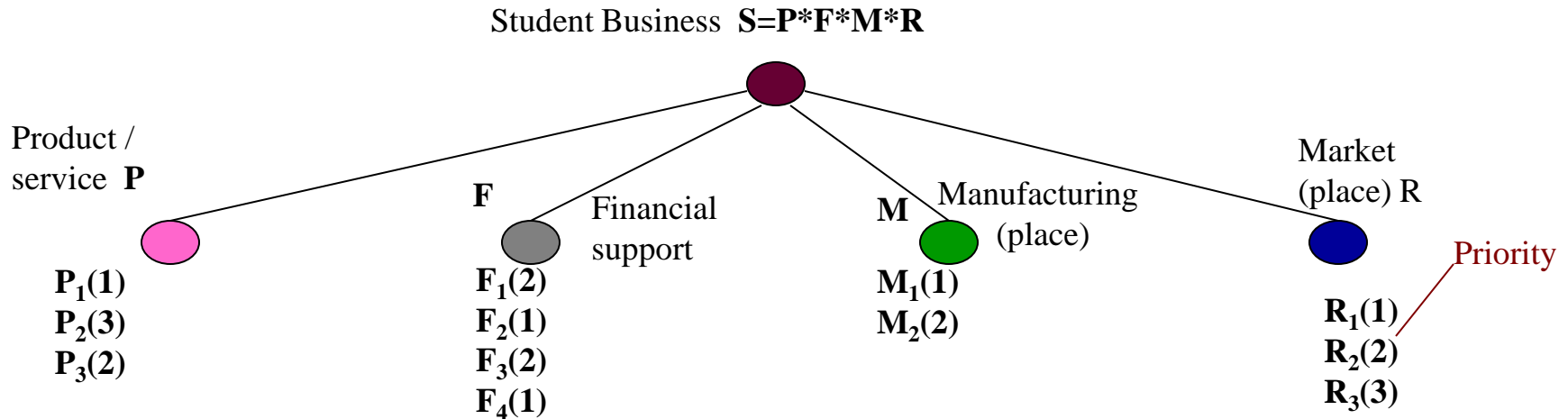
1. Joint design of a hierarchical system : \*structural scheme \*hierarchical model \*generation of alternatives for system parts  
\*generation of criteria for evaluation of alternatives for system parts \*multicriteria selection of alternatives  
\*synthesis of a composable system
2. Discussion of prospective research directions in communication networks

Sept. 10, 2004

Example of a hierarchical system: information center



## Joint design of a hierarchical system: student business



### ALTERNATIVES:

$P_1$  a simple food

$P_2$  support service (help) for usage of home PC

$P_3$  special consulting service for searching for personnel in HighTech (for companies, for specialists)

$F_1$  self-financing

$F_2$  financing by relatives & friends

$F_3$  financing by a bank

$F_4$  financing by a company

$M_1$  in university

$M_2$  special office in Dolgoprudny

$R_1$  Dolgoprudny

$R_2$  Moscow

$R_3$  New York

## Assessment of alternatives and total priority

Cost of manufacturing (-)    Volume of to-day's market (+)    Perspectives(+)    Resultant grade    Total priority

P <sub>1</sub>	1	5	3	7	1
P <sub>2</sub>	3	5	2	4	3
P <sub>3</sub>	4	4	5	5	2

Possible volume(+)    Responsibility(-)    Resultant grade    Total priority

F <sub>1</sub>	1	1	0	2
F <sub>2</sub>	2	1	1	1
F <sub>3</sub>	5	5	0	2
F <sub>4</sub>	4	3	1	1

Cost(-)    Usefulness(+)    Resultant grade    Total priority

M <sub>1</sub>	1	5	4	1
M <sub>2</sub>	3	5	2	2

Volume (+)    Possible competition (-)    Distance(-)    Resultant grade    Total priority

R <sub>1</sub>	1	0	0	1	1
R <sub>2</sub>	3	2	1	0	2
R <sub>3</sub>	3	3	5	-5	3

## Assessment of compatibility between alternatives

	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	M <sub>1</sub>	M <sub>2</sub>	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>
P <sub>1</sub>	5	4	0	0	5	3	5	2	0
P <sub>2</sub>	4	5	1	2	5	5	4	5	3
P <sub>3</sub>	0	1	4	3	0	5	2	5	5
F <sub>1</sub>					5	1	5	3	0
F <sub>2</sub>					5	1	5	3	0
F <sub>3</sub>					3	5	1	5	5
F <sub>4</sub>					3	5	1	5	5
M <sub>1</sub>							5	4	5
M <sub>2</sub>							3	5	4

NOTE: **5** corresponds to the best level of compatibility  
**0** corresponds to incompatibility

## Resultant composite decisions

$$S_1 = P_1 * F_2 * M_1 * R_1 ; \text{ vector of quality } N(S_1) = (4; 4,0,0);$$

**Description:** (a) all elements are at the top level (1); (b) grades of compatibility are (5,5,5,5,5,4)

**Another possible prospective decision is:**

$$S_2 = P_3 * F_4 * M_1 * R_2 ; \text{ vector of quality } N(S_1) = (4; 2,2,0);$$

**Description:** (a) levels of elements are (1,1,2,2); (b) grades of compatibility are (3,3,4,5,5,0)

**Note:** the decision is infeasible by compatibility

**Bottlenecks (possible problems for improving the initial situation):**

- 1.compatibility ( $P_3, M_1$ ) equals 0 ( $\Rightarrow$  to increase)
- 2.compatibility ( $F_4, M_1$ ) equals 3 ( $\Rightarrow$  to increase)
- 3.compatibility ( $M_1, R_2$ ) equals 3 ( $\Rightarrow$  to increase)
- 4.priority of  $P_3$  ( $\Rightarrow$  to improve)
- 5.priority of  $R_2$  ( $\Rightarrow$  to improve)

## Prospective research directions in communication systems (the list is under extension)

1. General design of communication network
2. GRID-like network environment (GRID-computing, GRID-communication)
3. Extension of communication network:
  - 3a. Improvement of an existing communication network
  - 3b. Extension of an existing communication network via additional territory
4. Allocation of resources in communication networks  
(applied situations, problem, models, and approaches)
5. Frequency assignment / allocation in communication networks  
(applied situation, problems, models, and approaches)
6. Reliability issues for communication networks
  - 6a. Evaluation of reliability
  - 6b. Design of reliable communication networks
7. Routing problems

## Prospective research directions in communication systems (the list is under extension)

8. System testing in communication networks:  
probing problem

9. Maintenance in communication networks

10. Mobile communication

11a. Movement of users

11b. Movement of all components of the system

11. System for information compression (algorithm part)

12. Communication networks & free scale networks

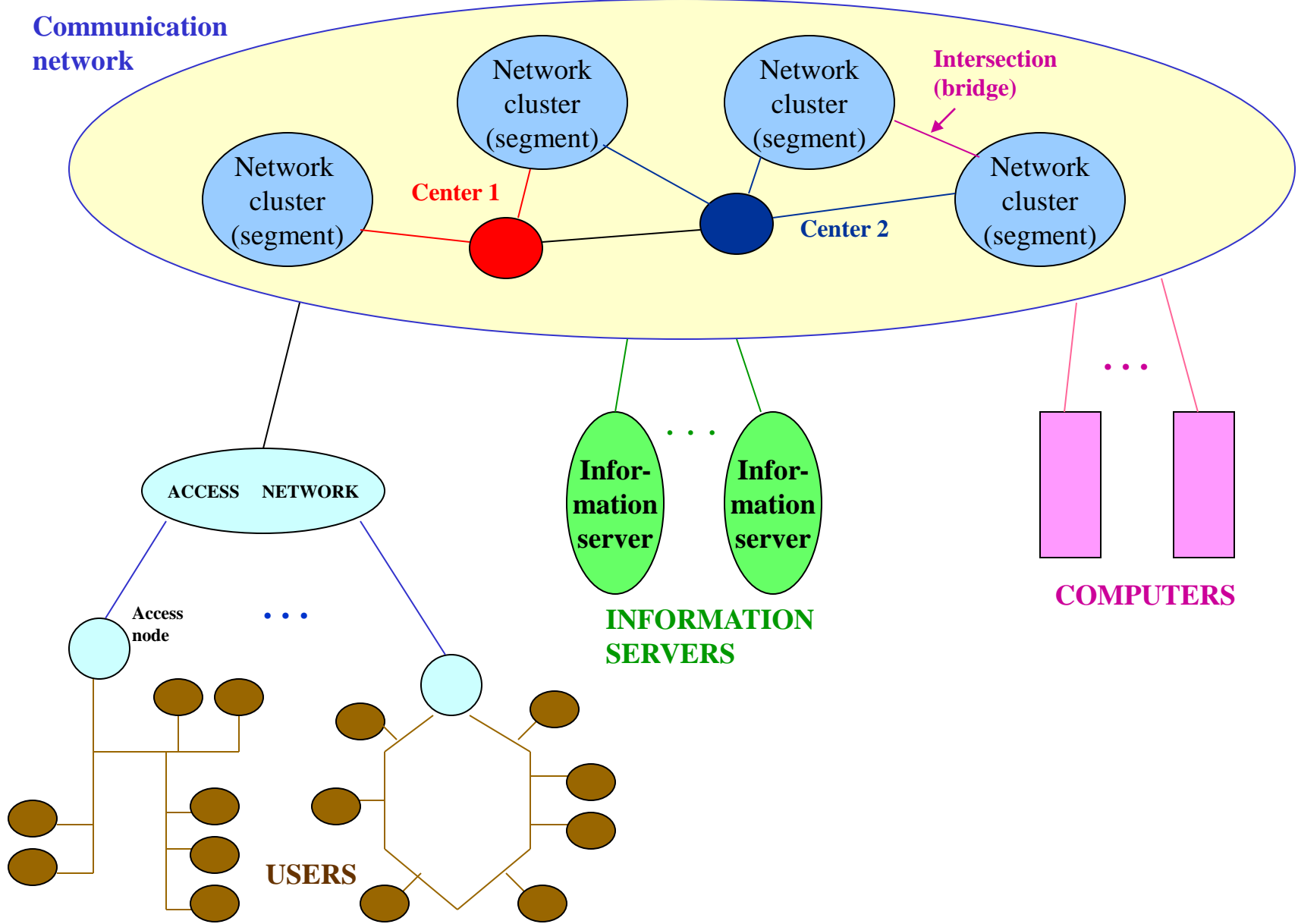
13. Planning of access to information/computer resources  
(information bases on servers or computers)  
in distributed information/computer environment  
(users, communication network, and  
a set of information servers/computer)

14. Design of topology for communication networks



Communication system: a scheme

Communication network



**LECTURE 5-6. Course: “Design of Systems: Structural Approach”**

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**L.5. Information technology. Human participation.**

**L.6. Schemes of design processes, Design problems.**

***PLAN:***

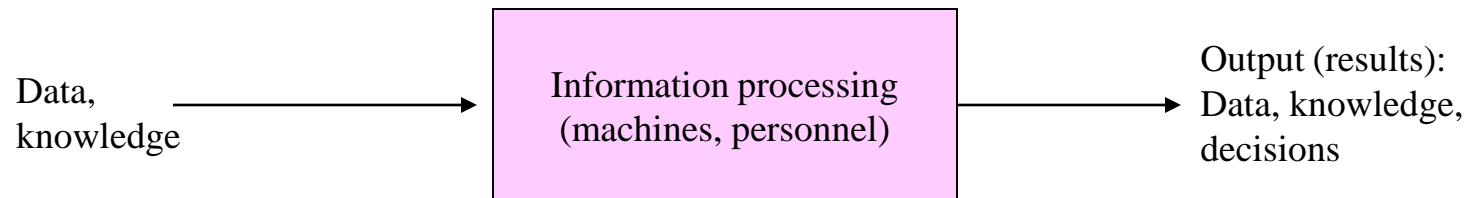
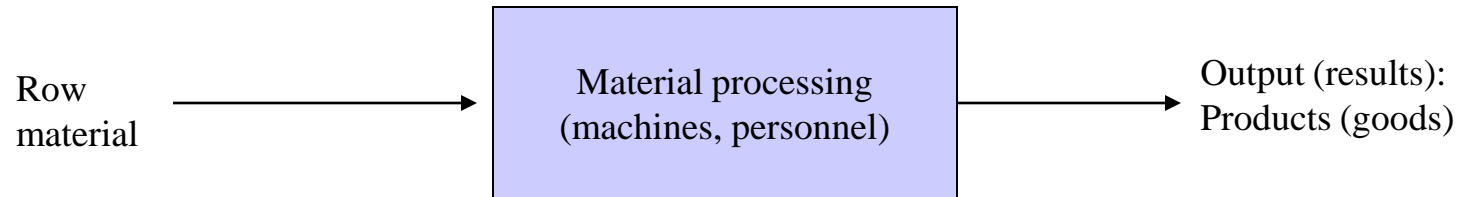
1. Information technology and its properties
2. Organizational-engineering systems. Human participation (in systems, in design)
3. Design frameworks (series process, cascade-like process). Close frameworks for information processing
4. Main design problems (design, redesign / upgrade process, multistage design, system evaluation, revelation of bottlenecks, system evolution / development)

Sept. 11, 2004

# Information technology: structure

	R&D	Manufacturing	Testing	Marketing	Utilization/Maintenance	Recycling/reusing
<b>HARDWARE PART</b> *VLSI *computers *communication			● ● ●			●
<b>SOFTWARE PART</b> *oper. systems *DBMSs *communication soft.						
<b>MATH./ALG. PART</b> *math. models *algorithms	● ●					
<b>INFORMATION PART</b> *data *knowledge	●	●			●	● ●
<b>ORG. PART</b> *specialists *users *HCI *group work						●
<b>APPL. SYSTEMS</b> *MISs *DSSs & ESs *etc.		●			●	

## “Processing” conveyor



## Comparison: material processing & information processing

STAGES	Technology for trees	Information technology
Source of raw materials	Forest	<ol style="list-style-type: none"> <li>1. Books, news papers</li> <li>2. Data &amp; knowledge bases</li> <li>3. People</li> </ol>
Raw materials	Woods	<ol style="list-style-type: none"> <li>1. Data</li> <li>2. Knowledge</li> </ol>
Transportation	Cars, trains	Communication systems
Manufacturing *machines *personnel	Machines Engineers, Workers	Computers, software, communication <ol style="list-style-type: none"> <li>1. Specialists</li> <li>2. Users</li> </ol>
Output	Boards, etc.	<ol style="list-style-type: none"> <li>1. Data</li> <li>2. Knowledge</li> <li>3. Decisions</li> </ol>
Keeping	Depository	<ol style="list-style-type: none"> <li>1. Data bases</li> <li>2. Knowledge bases</li> </ol>
Users	Firm for building, Private persons	<ol style="list-style-type: none"> <li>1. Government</li> <li>2. Firms</li> <li>3. University &amp; educational systems</li> <li>4. Research Institutes &amp; Universities</li> <li>5. Private persons</li> </ol>

## Properties of information technology

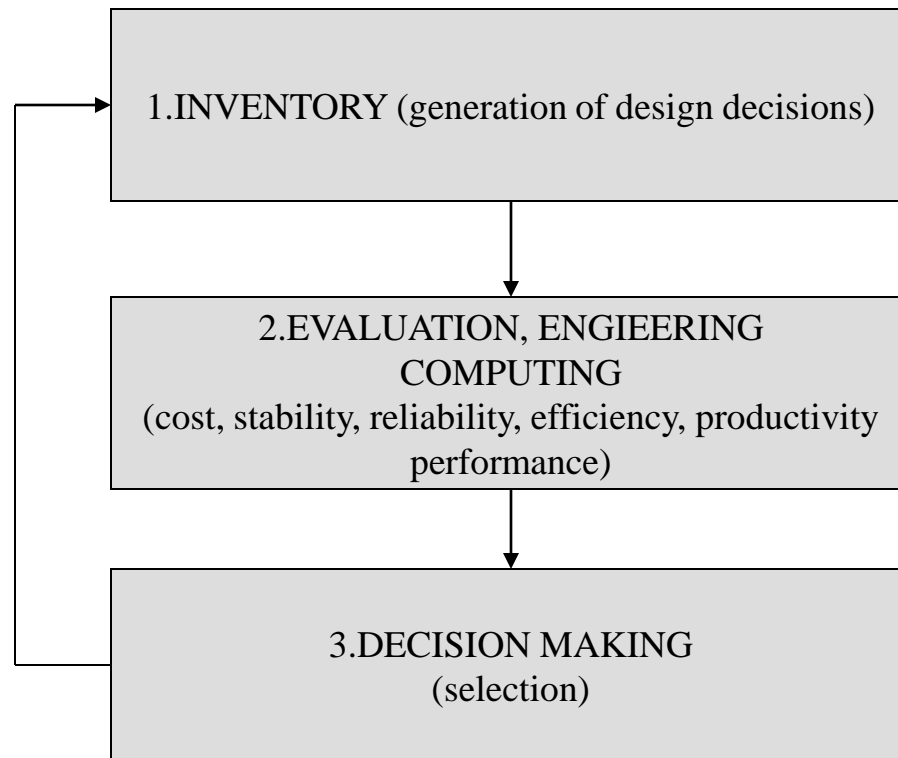
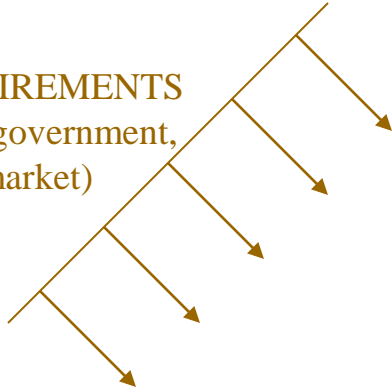
1. Various kinds of sources: \*statistics, books, data bases \*specialists, population
2. Preservation of initial information & possibility for re-processing
3. Possibility for parallel / concurrent processing
4. Possibility for usage of many different methods
5. Possibility to accumulate results (outputs)
6. High “ecologiability”
7. High requirements to professional skills
8. Unique role of human
9. High requirements to information presentation (e.g., visualization)
10. Integration:
  - \*exact science
  - \*engineering
  - \*psychology
  - \*education/training
  - \*art (e.g., TV, cinema)
11. Wide range of users:
  - \*science
  - \*industry
  - \*management, economics
  - \*education
  - \*art
  - \*private life

## Morphological description of specialist

System part/domain	Level of specialist (by J. Piaget)		
	A. Without object/operations (0..5)	B. Certain objects/operations(5...11)	C. Abstracted object/operations(11...)
Applied problem	● →	● ●	
Model		● ● ●	
Algorithm	● →	●	●
Software	● ● →		●
Hardware	●	● ●	
Etc.		● ● ●	

## Series design flow (J.R. Dixon)

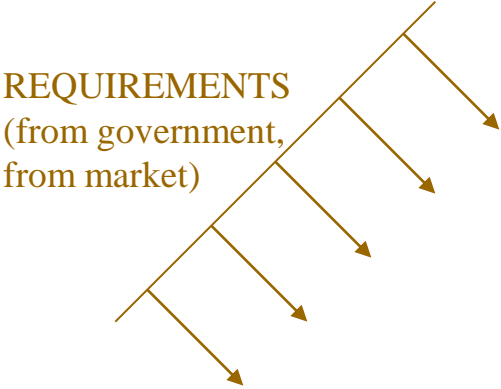
REQUIREMENTS  
(from government,  
from market)



DESIGN DECISIONS

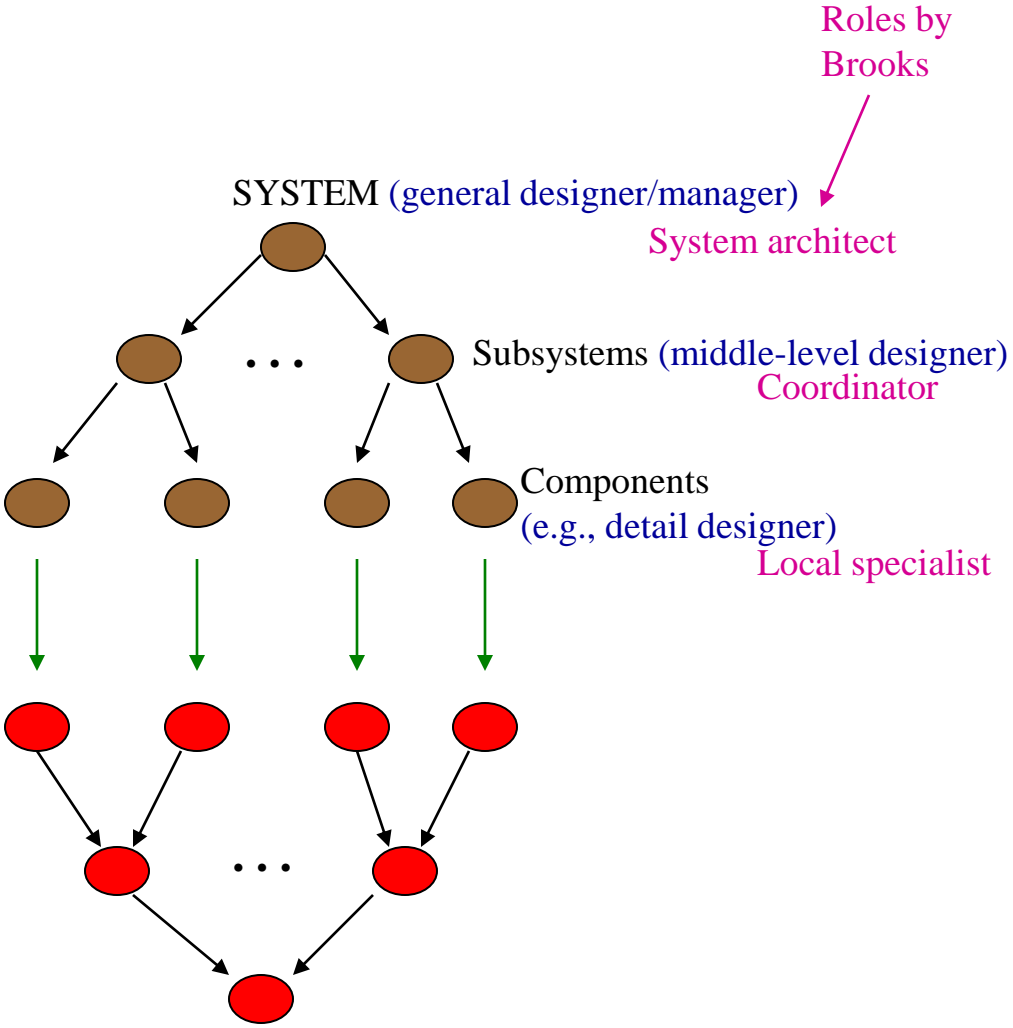


# Cascade-like design flow



PROCESS  
(Top-Down):  
partitioning of:  
\*system  
\*requirements

PROCESS  
(Bottom-Up  
design):  
\*generation  
\*evaluation  
\*selection  
\*synthesis



## Levels of creativity (by G. Altshuller)

LEVEL 1. Usage of a well-known object (product, technology, decision, etc.)

LEVEL 2. Searching for & selection of the best object

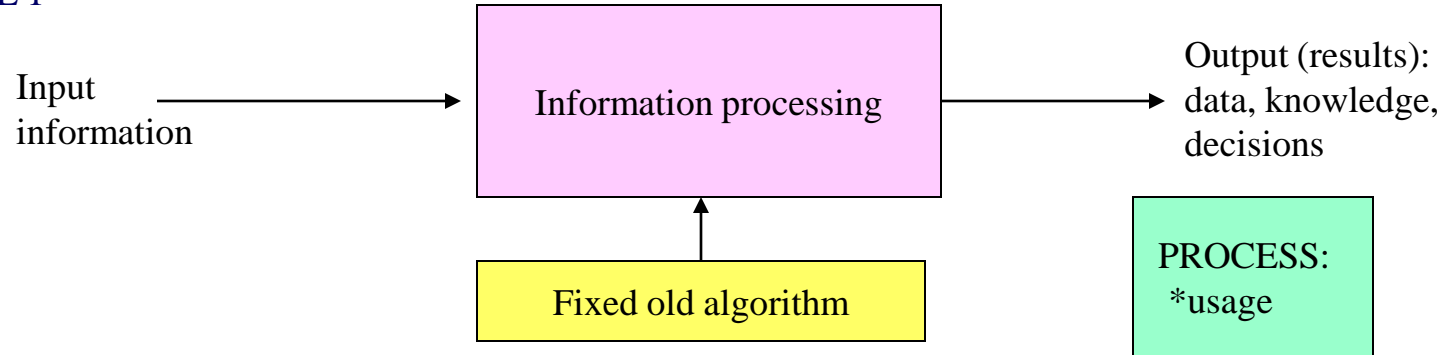
LEVEL 3. Improvement (modification) of an object

LEVEL 4. Design of a new object

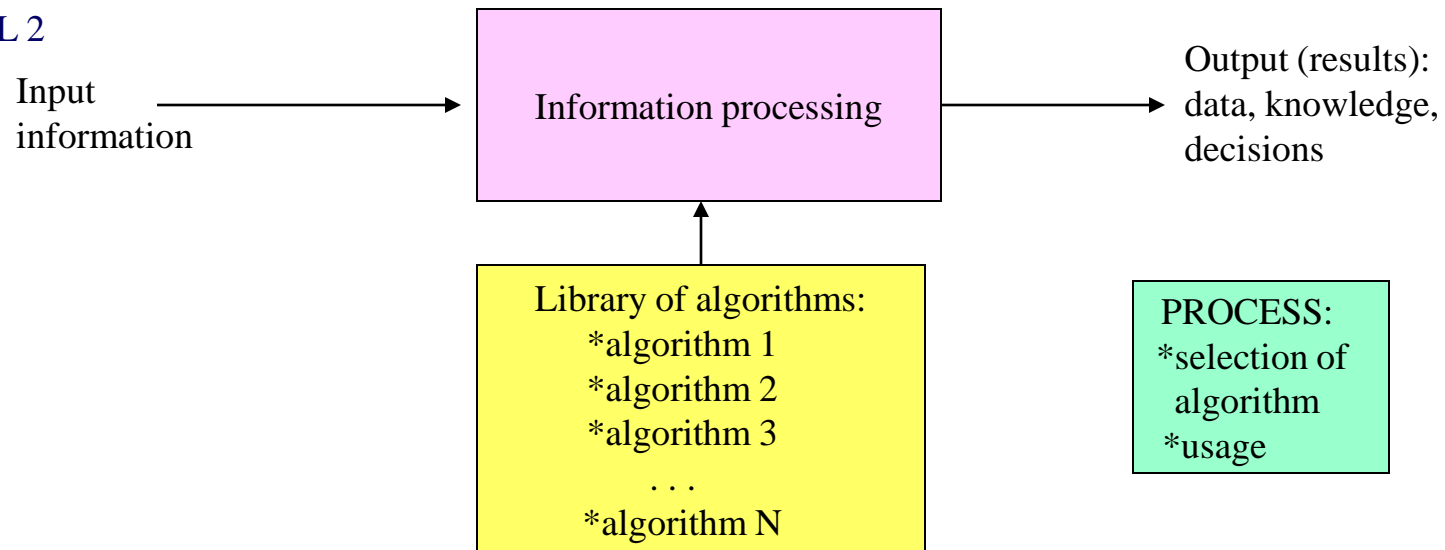
LEVEL 5. Design of a system of objects

## Illustration of creativity levels for processing

### LEVEL 1



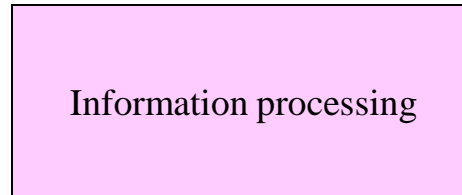
### LEVEL 2



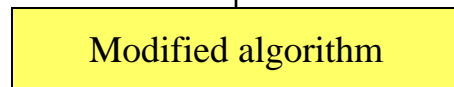
## Illustration of creativity levels for processing

### LEVEL 3

Input  
information



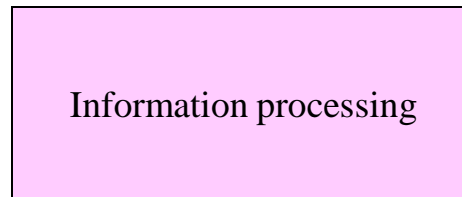
Output (results):  
data, knowledge,  
decisions



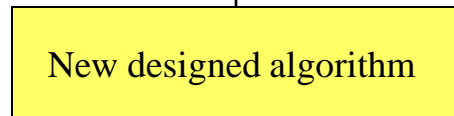
PROCESS:  
\*selection  
\*modification  
\*usage

### LEVEL 4

Input  
information



Output (results):  
data, knowledge,  
decisions



PROCESS:  
\*selection  
\*design of new  
algorithm  
\*usage

## Illustration of creativity levels for processing

### LEVEL 5

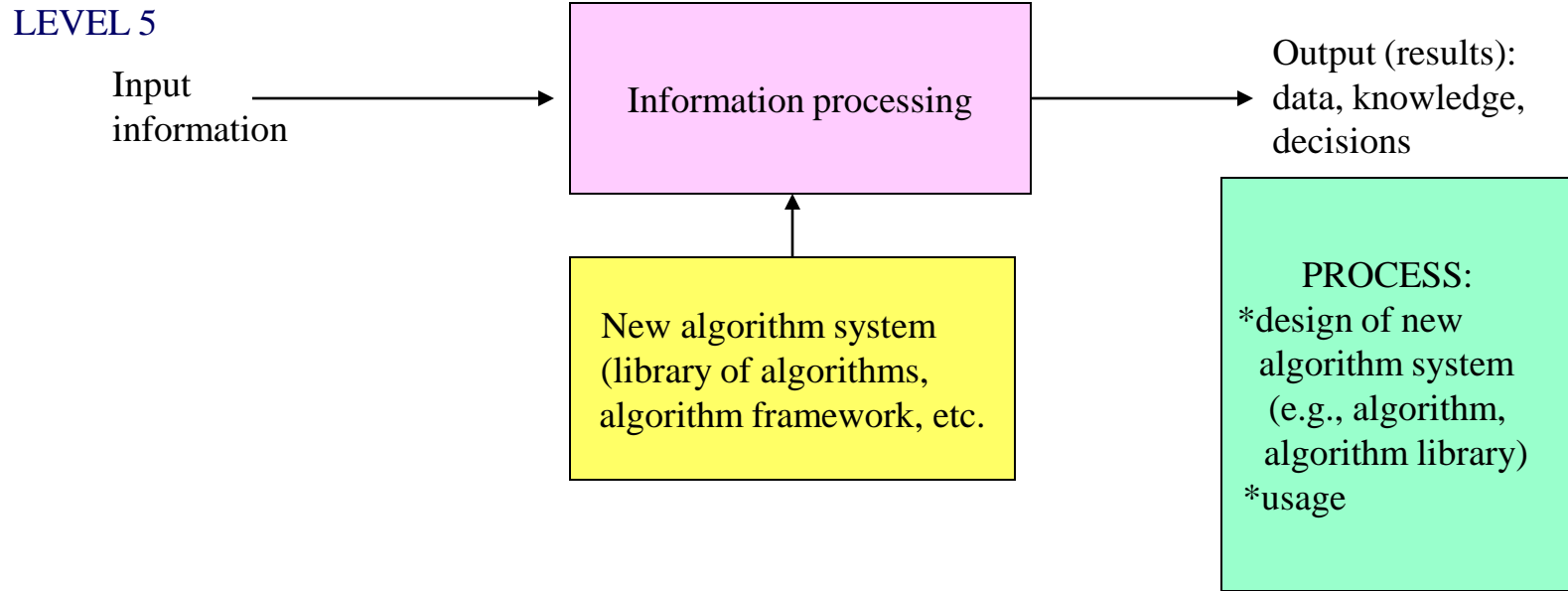
Input  
information

Information processing

Output (results):  
data, knowledge,  
decisions

New algorithm system  
(library of algorithms,  
algorithm framework, etc.)

**PROCESS:**  
\*design of new  
algorithm system  
(e.g., algorithm,  
algorithm library)  
\*usage



## Design problems (technological problems)

- 1.Design
- 2.Redesign (improvement, upgrade process)
- 3.Multistage design
- 4.Evaluation
- 5.Revelation of bottlenecks
- 6.Modeling of system evolution /development (& forecasting)

**LECTURE 7. Course: “Design of Systems: Structural Approach”**

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**L.7. Concurrent engineering and life cycle. Traditional hierarchical system design.**

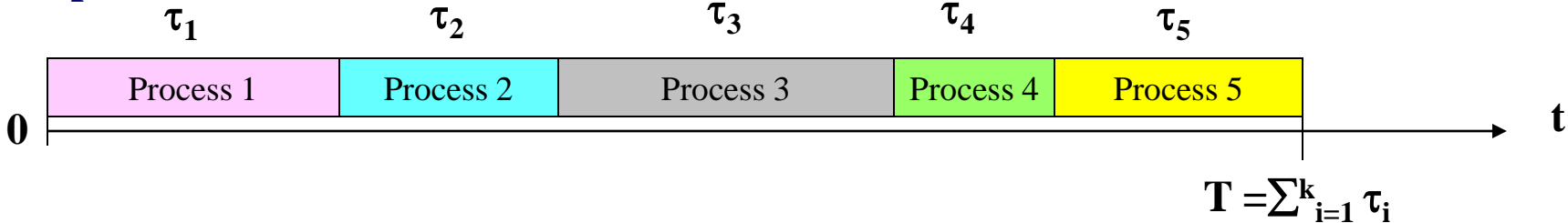
***PLAN:***

1. Concurrent engineering (concurrent processes, concurrency in life cycle)
2. Hierarchical system design
3. Some examples of applied systems (e.g., manufacturing systems)
4. Some problems in mobile communication systems

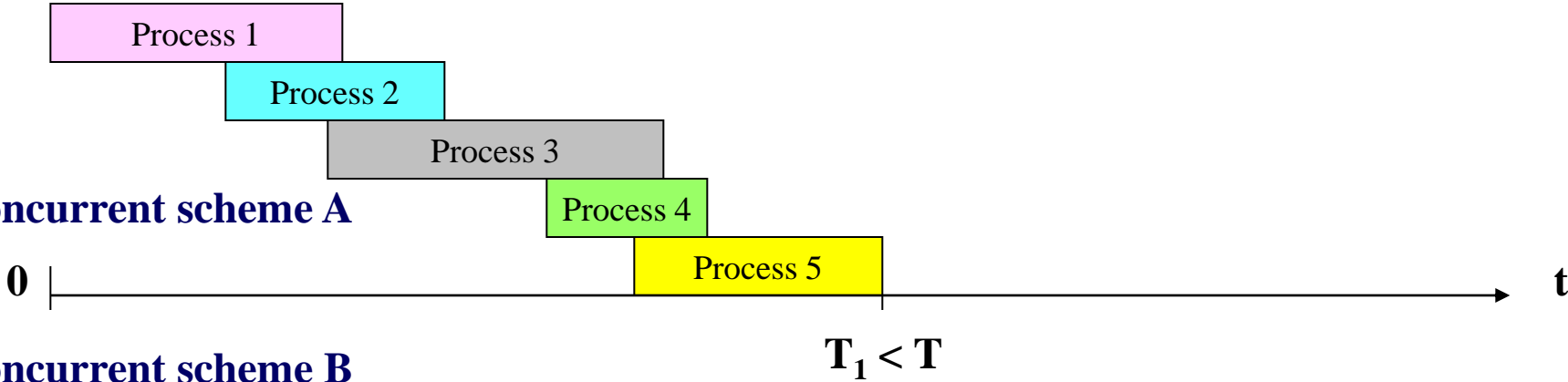
Sept. 17, 2004

Illustration for concurrent engineering

**Series process**



**Concurrent scheme A**



**Concurrent scheme B**





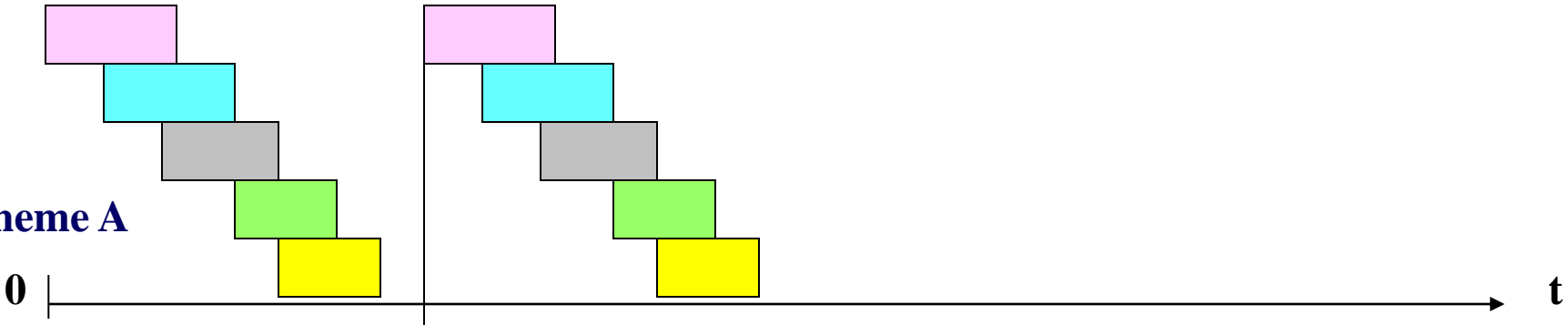
Concurrency in life cycle

Cycle 1

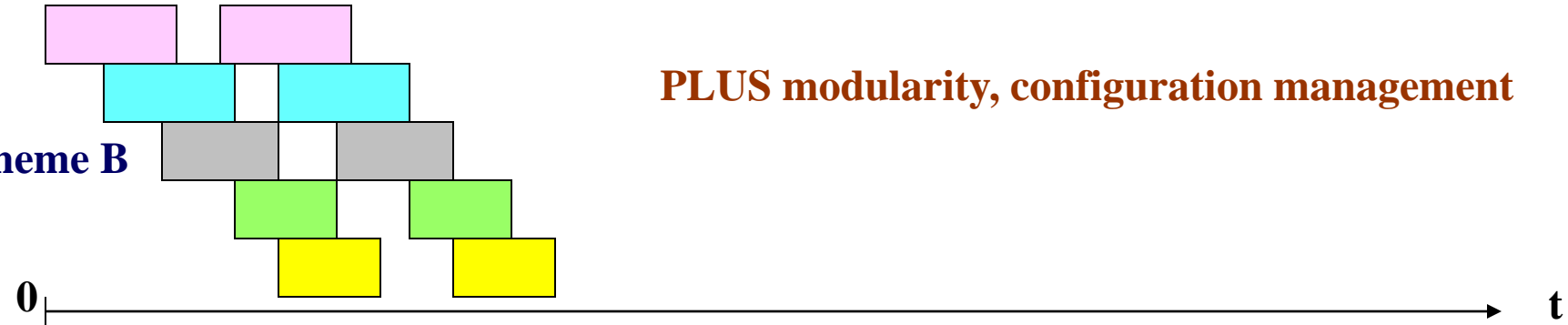
Cycle 2



Scheme A



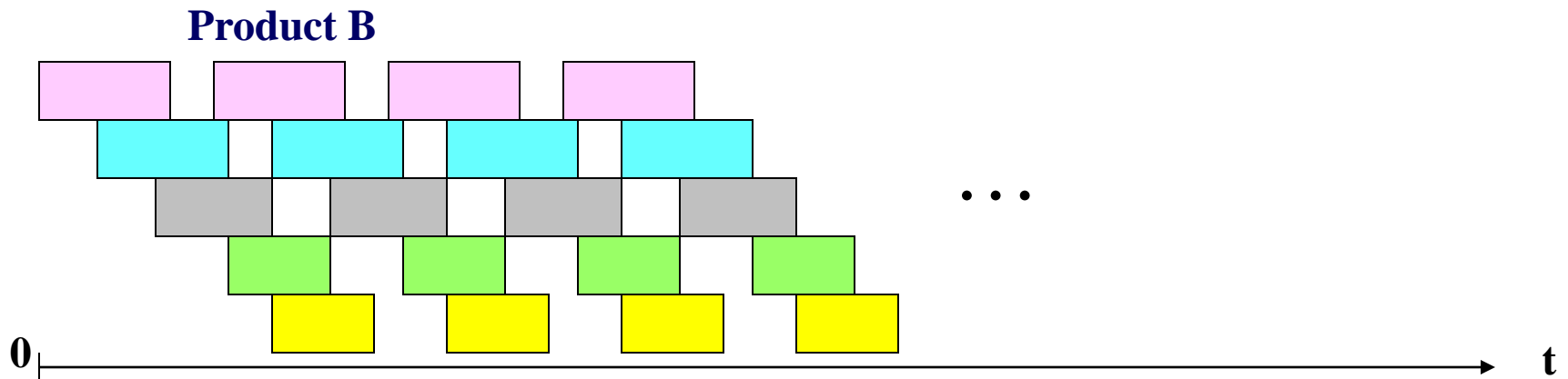
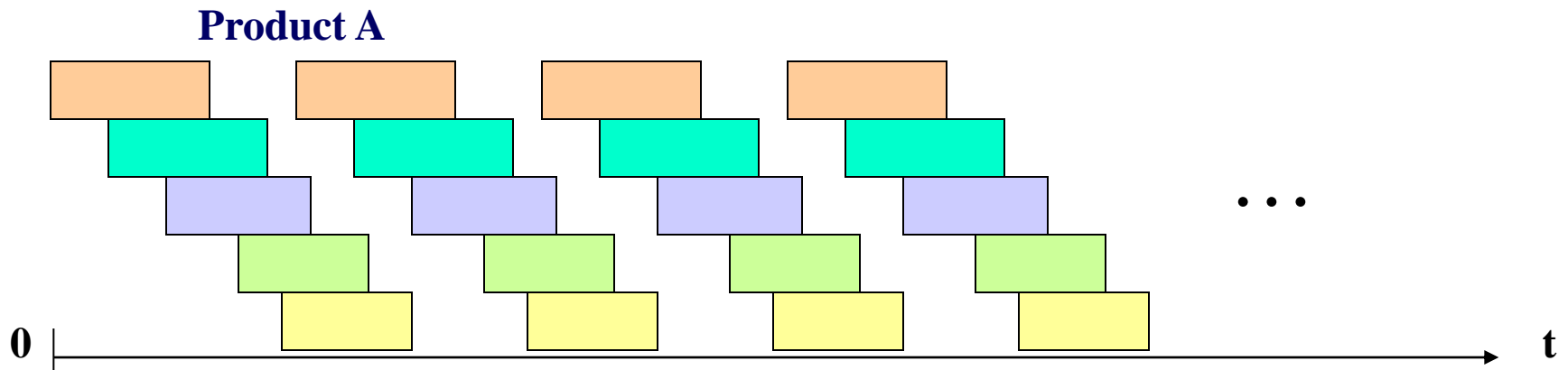
Scheme B



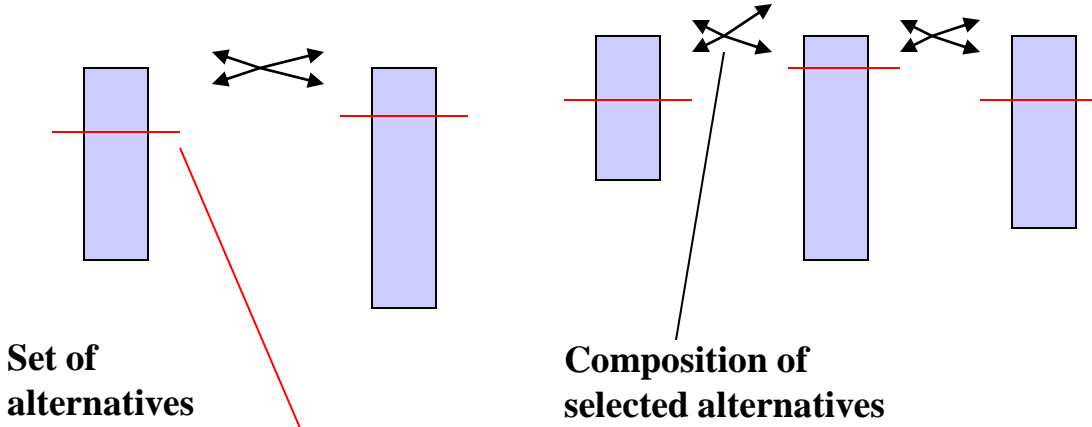
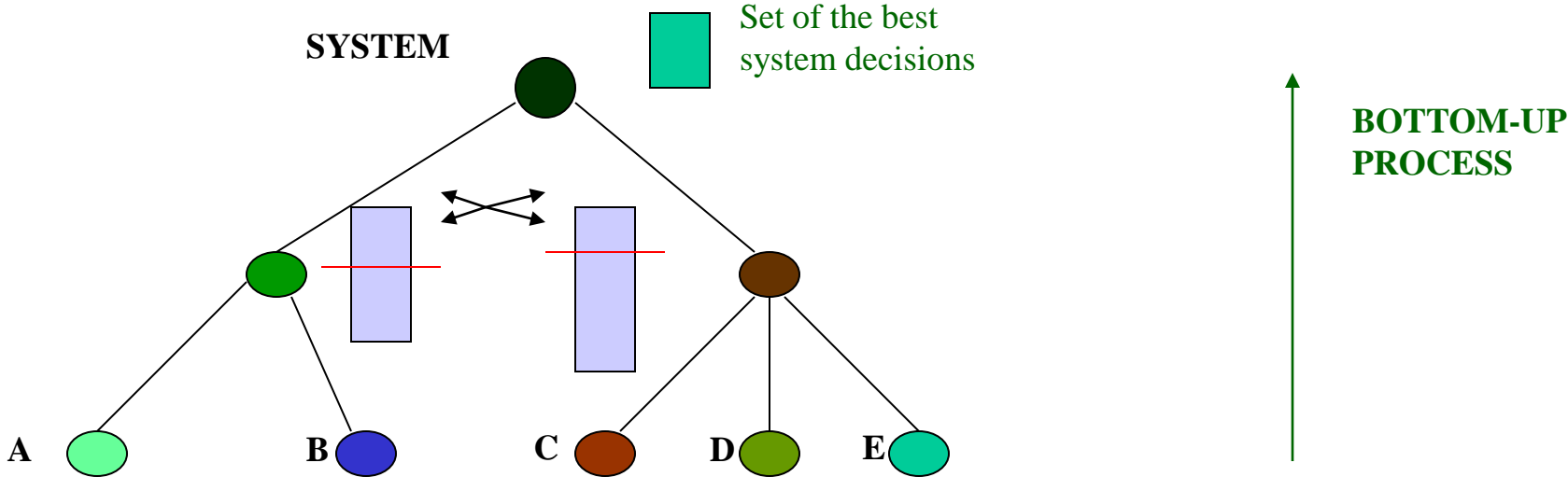
PLUS modularity, configuration management

## Concurrency in life cycle for several products

**ORGANIZATIONAL DIRECTION:**  
concurrency, modularity, configuration management, and coordination  
for products A, B, etc.



# Hierarchical system design



- Selection process:**
- 1. Constraints
  - 2. Multicriteria selection (ranking)
  - 3. Optimization approach
  - 4. Expert judgment

## Main approaches to system design

1. Multidisciplinary optimization (e.g., aerospace engineering, structural engineering)
2. Mixed integer non-linear programming (e.g., chemical engineering, process, systems engineering)
3. Non-linear multicriteria (multiobjective) optimization including evolutionary multiobjective optimization
4. Formal methods in design (e.g., mechanical engineering)
5. Global optimization methods
6. Grammatical design (i.e., grammar description of decomposable systems)
7. Methods of artificial intelligence (knowledge-based systems, neural networks, genetic algorithms, etc.) (e.g., computer engineering, VLSI design)
8. Parameter Space Investigation PSI (e.g., mechanical engineering, nuclear engineering)
9. Hierarchical system design (traditional engineering organizational methods, modular design methods, combinatorial synthesis)
10. Hybrid methods

**LECTURE 8-9. Course: “Design of Systems: Structural Approach”**

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**L.8. Principles of system analysis. Paradigm of decision making. Basic DM problems.**

**L.9. Kinds of scales, Pareto-efficient decisions, system evaluation, hierarchy of requirements.**

***PLAN:***

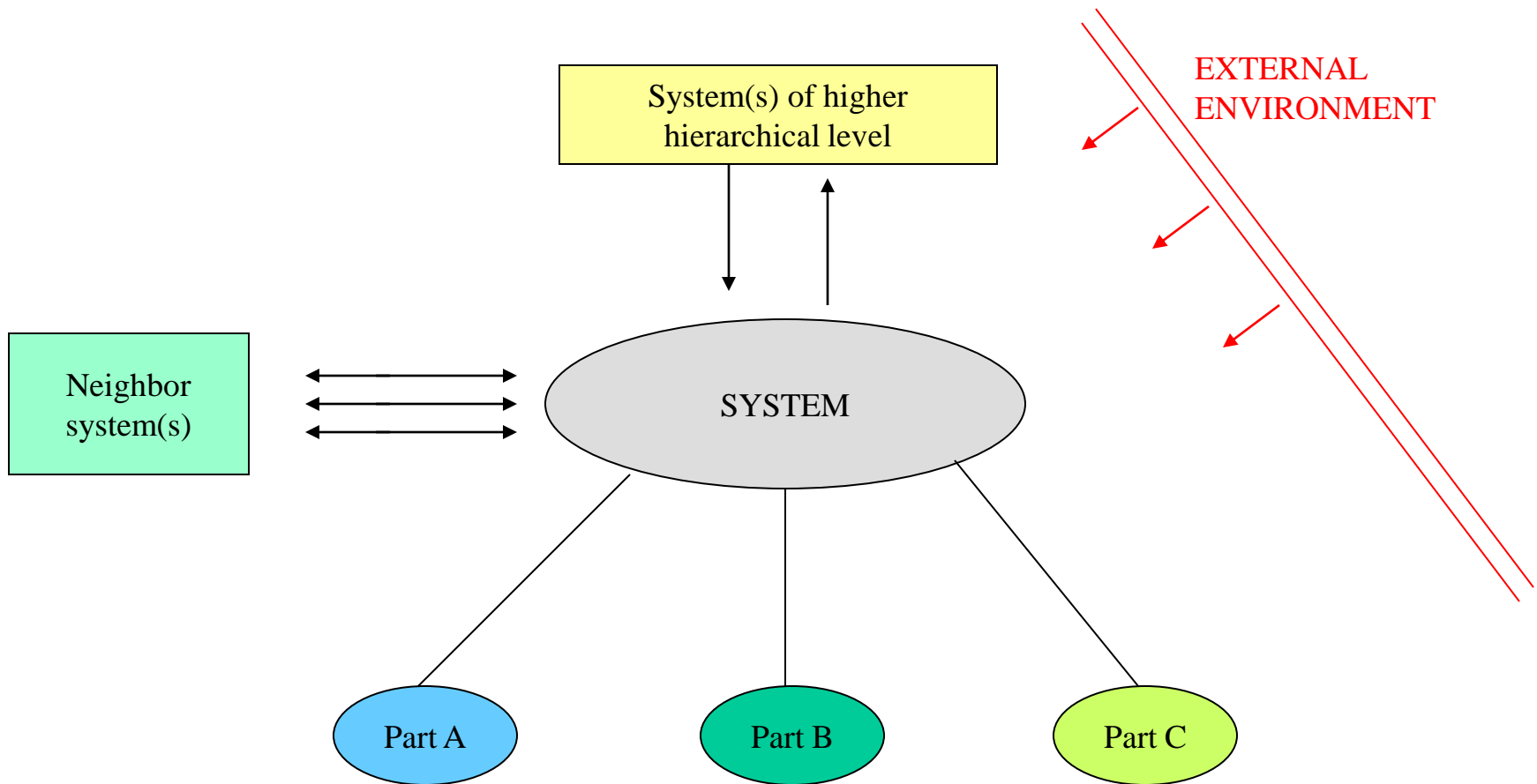
1.Principles of system analysis. 2.Paradigm of decision making. 3.Basic decision making problems.

4.Kinds of scales. 5.Pareto-effective decisions 6.Evaluation of systems

7.Hierarchy of requirements / criteria 8.Roles in decision making process. Example

Sept. 18, 2004

# A scheme for a system



## Principles of system analysis

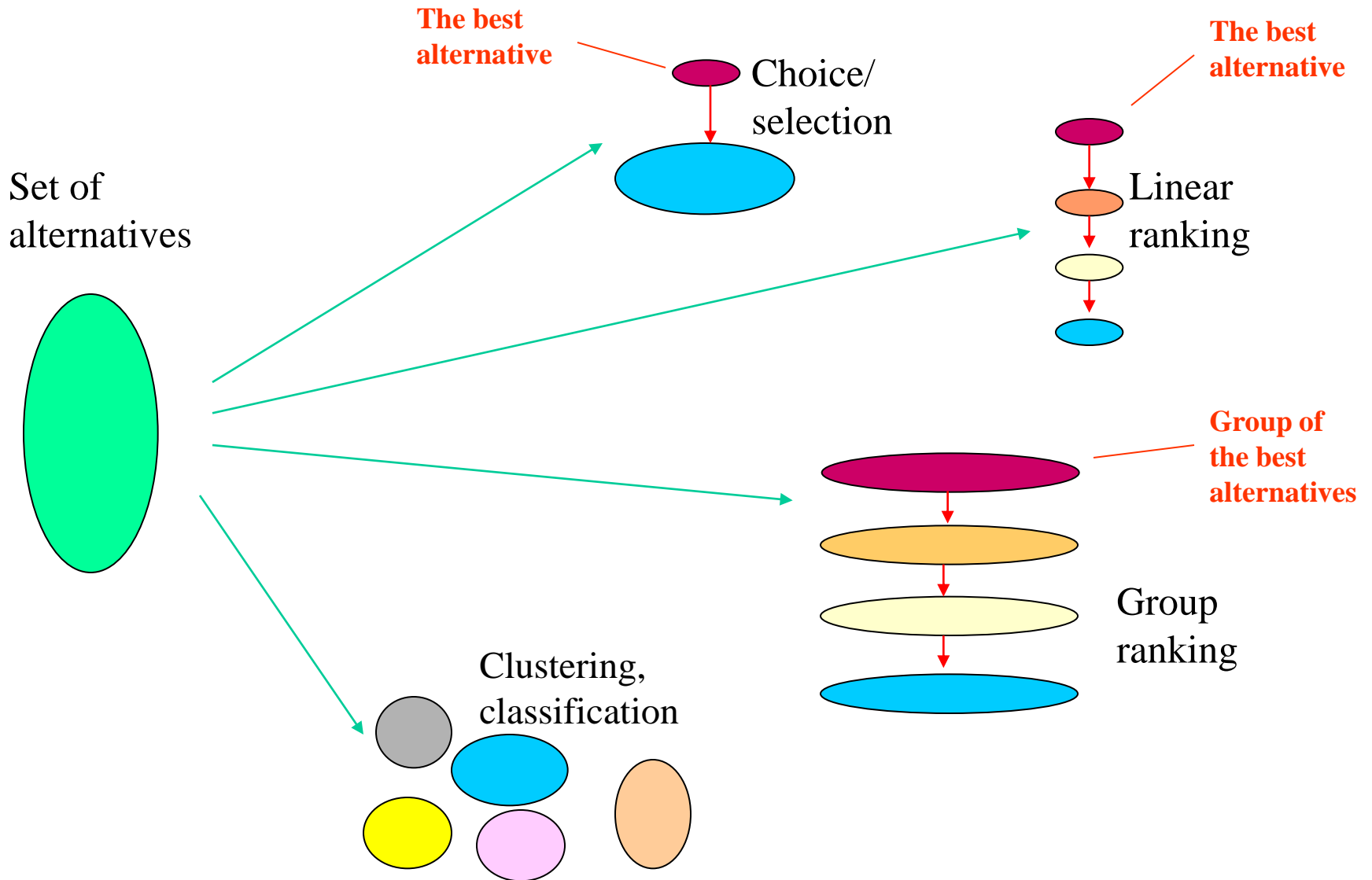
- 1.Examination of life cycle (e.g., R & D, manufacturing, testing, marketing, utilization & maintenance, recycling)
- 2.Examination of system evolution / development (i.e., dynamical aspects)
- 3.Examination of interconnection with environment (nature, community, other systems)
- 4.Examination of interconnection among system parts / components  
(physical parts, functions, information, energy, etc.)
- 5.Analysis of system changes (close to principle 2)
- 6.Revelation and study of main system parameters
- 7.Integration of various methods (decomposition, hierarchy, composition, etc.)
- 8.Investigation of main system contradictions (engineering, economics, ecology, politics, etc.)
- 9.Integration of various models and algorithms (e.g., physical experiments, mathematical modeling, heuristics, expert judgment)
- 10.Interaction among specialists from different professional domains and hierarchical levels (engineering, computer science, mathematics, management, social science, etc.)

## Decision Making Paradigm (stages) by Herbert A. Simon

- 1. Analysis of an applied problem (to understand the problem: main contradictions, etc.)**
- 2. Structuring the problem:**
  - 2.1. Generation of alternatives**
  - 2.2. Design of criteria**
  - 2.3. Design of scales for assessment of alternatives upon criteria**
- 3. Evaluation of alternatives upon criteria**
- 4. Selection of the best alternative (s)**
- 5. Analysis of results**



# Four Basic Decision Making Problems



# Kinds of Problems by Herbert A. Simon

## I. STANDARD PROBLEMS

## II. FORMULIZED PROBLEMS (models in mathematics as equations, optimization, etc.)

## III. ILL-STRUCTURED PROBLEMS

- \*human factors, information from expert(s) & decision maker(s)
- \*uncertainty

## IV. FORECASTING (decisions for the future)

Decision  
Making  
Problems



# Applied Decision Making Problems

## **1.LEVEL OF GOVERNMENT:**

- \*selection of research projects**
- \*investment into infrastructure (e.g., transport, communication, education)**
- \*selection of political decisions**

## **2.LEVEL OF COMPANY:**

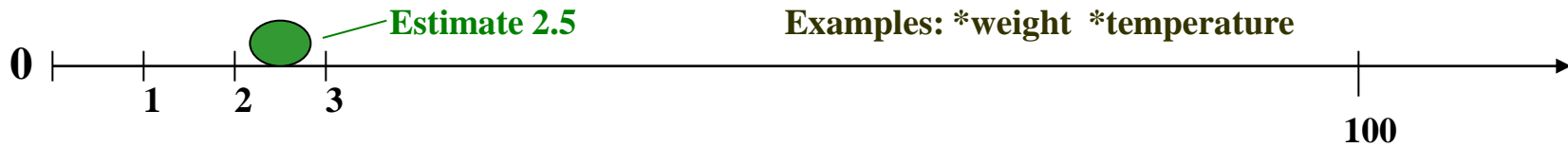
- \*selection of product**
- \*selection of market**
- \*selection of personnel**
- \*selection of partners**
- \*selection of place for new plants, etc.**

## **3.LEVEL OF PRIVATE LIFE:**

- \*selection of apartment**
- \*selection of university / college**
- \*selection of car**
- \*selection of bank program**
- \*selection of place for vacation, etc.**

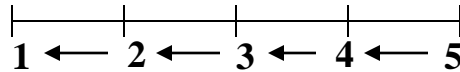
# Kinds of Scales

## 1. Quantity: quantitative scale

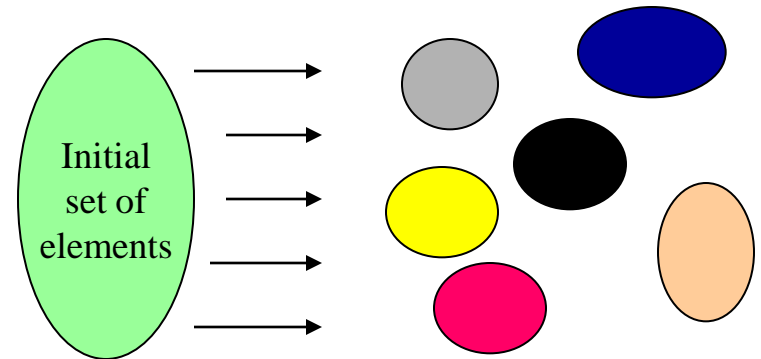


## 2. Quality: qualitative scales (levels, ordering, class)

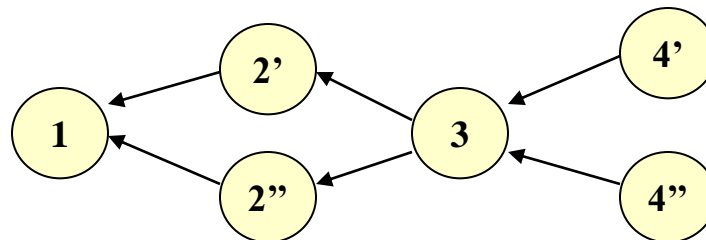
### 2a. Ordinal scale



### 2b. Nominal scale (for classes, clusters)



### 2c. Scale as partial order (generalization)



## Description of decision making problem

Alternatives  $A=(A_1, \dots, A_i, \dots, A_n)$  and criteria  $C=(C_1, \dots, C_j, \dots, C_k)$ ,  
 $\forall A_i$  a vector of estimates  $z_i = (z_{i1}, \dots, z_{ij}, \dots, z_{ik})$

Matrix of estimates is:

$$Z = \begin{array}{|c|} \hline z_{11}, \dots, z_{1j}, \dots, z_{1k} \\ \hline \dots \\ \hline z_{i1}, \dots, z_{ij}, \dots, z_{ik} \\ \hline \dots \\ \hline z_{n1}, \dots, z_{nj}, \dots, z_{nk} \\ \hline \end{array} \begin{array}{l} \longrightarrow P(A_1) \\ \longrightarrow P(A_i) \\ \longrightarrow P(A_n) \end{array}$$

Our goal is to get a “priority” for each alternatives:  $P(A_i)$

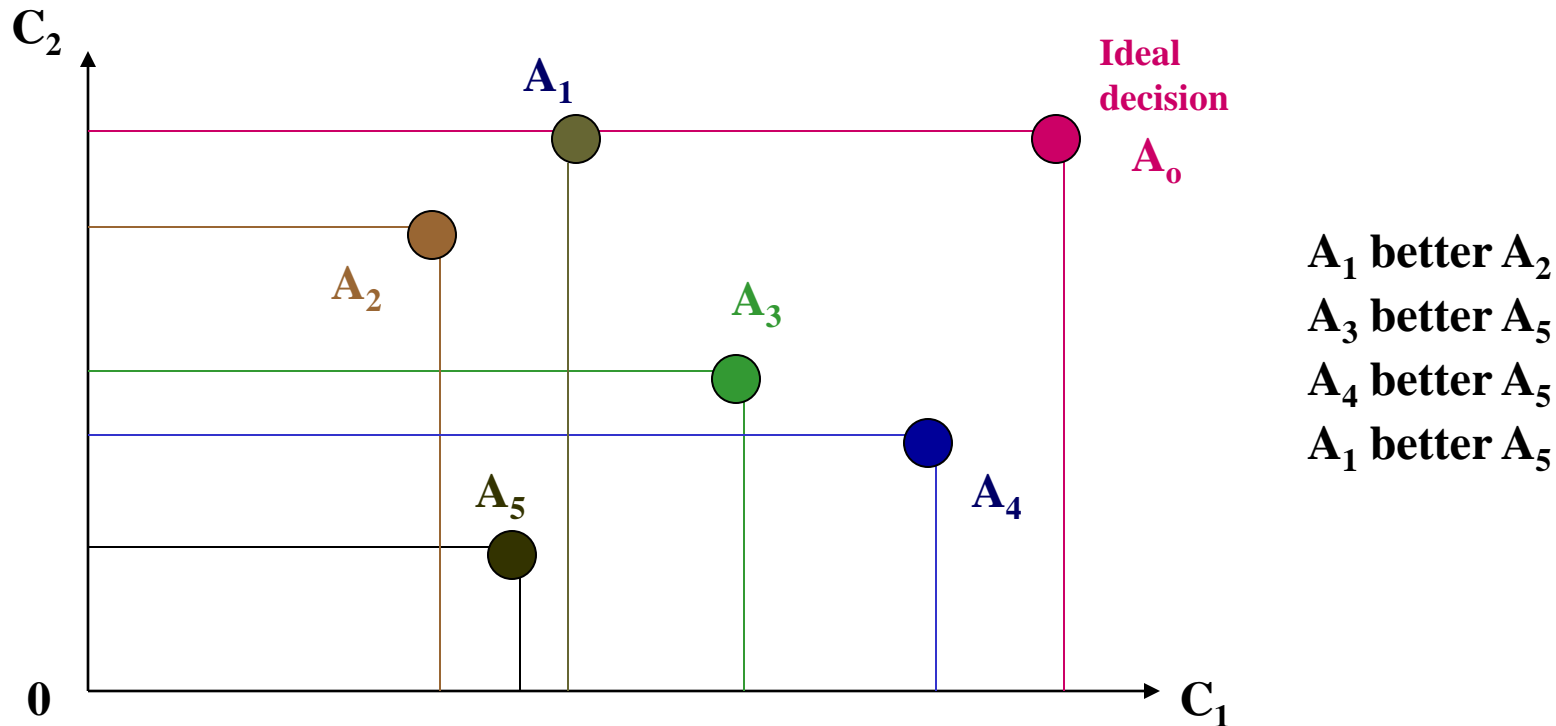
Evaluation of  $P(A_i)$  can be based on the following:

1. Quantitative scale
2. Ordinal scale
3. Scale as partial order

## Pareto-effective (Pareto-optimal) decisions

### PARETO RULE:

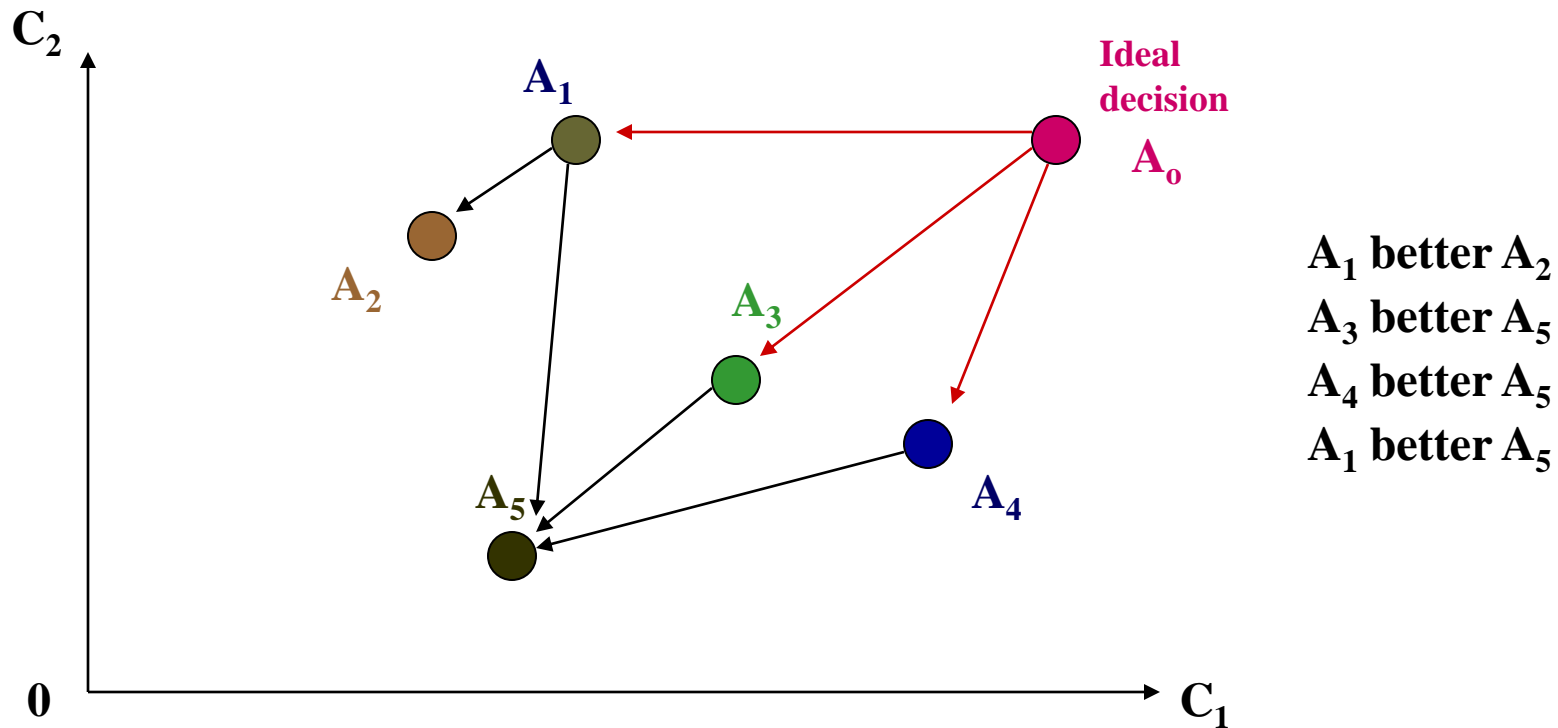
Alternative  $X=(x_1, \dots, x_j, \dots, x_k)$  and alternative  $Y=(y_1, \dots, y_j, \dots, y_k)$ ,  
 $X$  is better than  $Y$  if  $\forall j \ x_j \geq y_j$  and  $\exists i \ (1 \leq i \leq k)$  such that  $x_i > y_i$



$A_1, A_3, A_4$  are incomparable and have no dominating elements (only  $A_0$ )

$A_1, A_3, A_4$  are Pareto-effective decisions for set  $\{A_1, A_2, A_3, A_4, A_5\}$

## Partial order on alternatives



$A_1, A_3, A_4$  are incomparable and have no dominating elements (only  $A_0$ )

$A_1, A_3, A_4$  are Pareto-effective decisions for set  $\{A_1, A_2, A_3, A_4, A_5\}$

**Evaluation of a complex system can be based on the following:**

**1. Quantitative scale**

**2. Ordinal scale**

**3. Scale as partial order (including special discrete spaces)**



## Hierarchy of requirements / criteria

**1. Ecology, politics**

**2. Economics, marketing**

**3. Technology (e.g., manufacturing issues, maintenance issues)**

**4. Engineering**

## Main roles in decision making process

### **1.DECISION MAKER (DM)**

**(to make the resultant decision, to evaluate alternatives, etc.)**

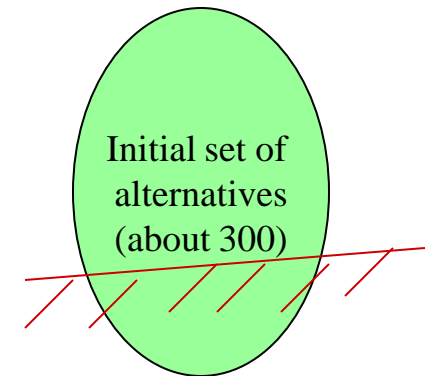
### **2.SUPPORT SPECIALIST**

**(to organize the decision making procedure including support of all stages)**

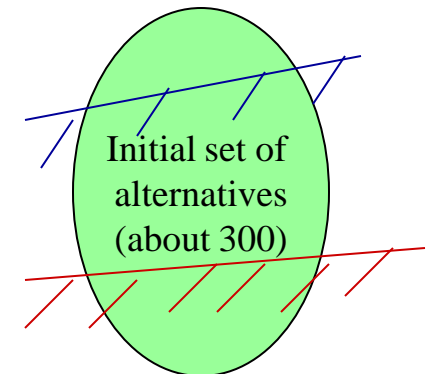
### **3.EXPERT(s) (to evaluate alternatives)**

Phases of decision making process: Example for selection for the best company (P. Humphreys)

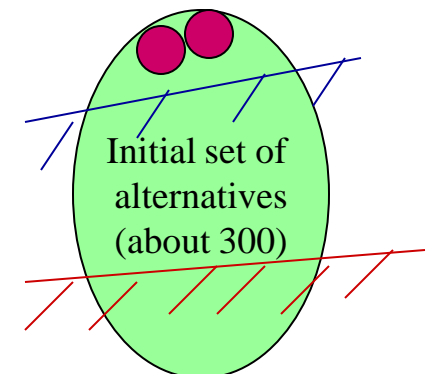
**Phase 1.** Analysis of initial requests (i.e., alternatives)  
& deletion of the worst material (about 1/3 of the requests)



**Phase 2.** Design of a special method for multicriteria selection, evaluation of alternatives upon criteria, selection of a group (a part) of the best alternatives (about 20...30) (group of experts)



**Phase 3.** Choice of the best alternative(s) (about 1...3):  
special procedure of expert judgment (group of Decision Makers)



## **LECTURE 10. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

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### **L.10. Multicriteria decision making.**

#### ***PLAN:***

1. Multicriteria decision making: \*utility function, \* method of pair comparison, \*method of incomparability (“equivalence”) levels, \*outranking technique (ELECTRE), \*AHP, etc.
2. Integration of results obtained on the basis of several methods (or subsystems of criteria)

Sept. 24, 2004

## Examples of utility functions

**Alternatives**  $A=(A_1, \dots, A_i, \dots, A_n)$  and **criteria**  $C=(C_1, \dots, C_j, \dots, C_k)$ ,  
 $\forall A_i$  a vector of estimates  $z_i = (z_{i1}, \dots, z_{ij}, \dots, z_{ik})$ ,  $\mu_j$  is a weight of criterion  $j$

**Arithmetic**  $F_a = \sum_{j=1}^k \mu_j z_j / z_{jb}$

**Geometrical**  $F_g = \prod_{j=1}^k (z_j / z_{jb})^{\mu_j}$

**Quadratic**  $F_q = \sum_{j=1}^k \mu_j (z_j / z_{jb})^2$

**Harmonic**  $F_h = 1 / ( \sum_{j=1}^k \mu_j (z_j / z_{jb}) )$

**Power**  $F_p = \sum_{j=1}^k \mu_j (z_j / z_{jb})^k$

**General**  $F_o = \sum_{j=1}^k \mu_j \varphi (z_j / z_{jb})$

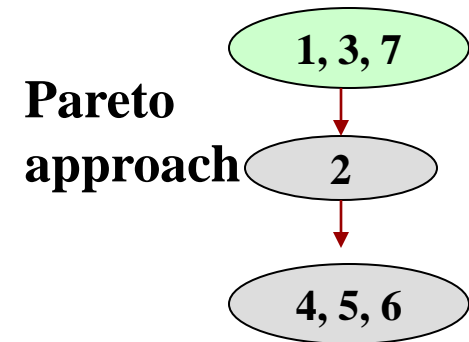
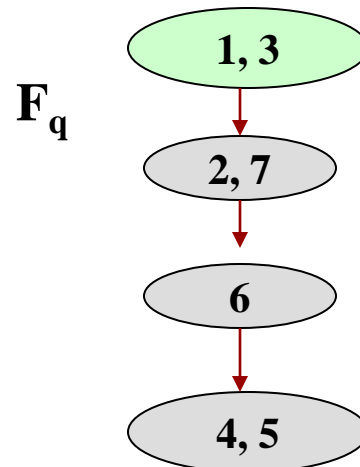
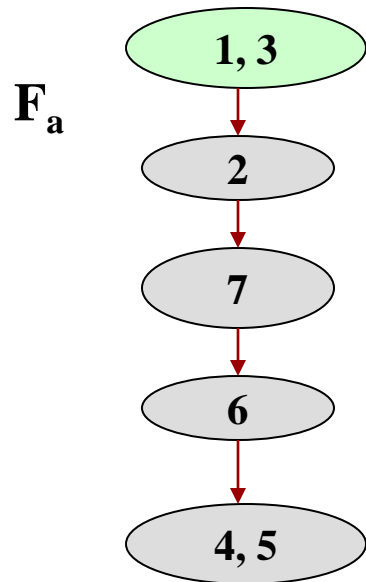
where  $\varphi$  is a differential function,

$z_{jb}$  is an estimate-base

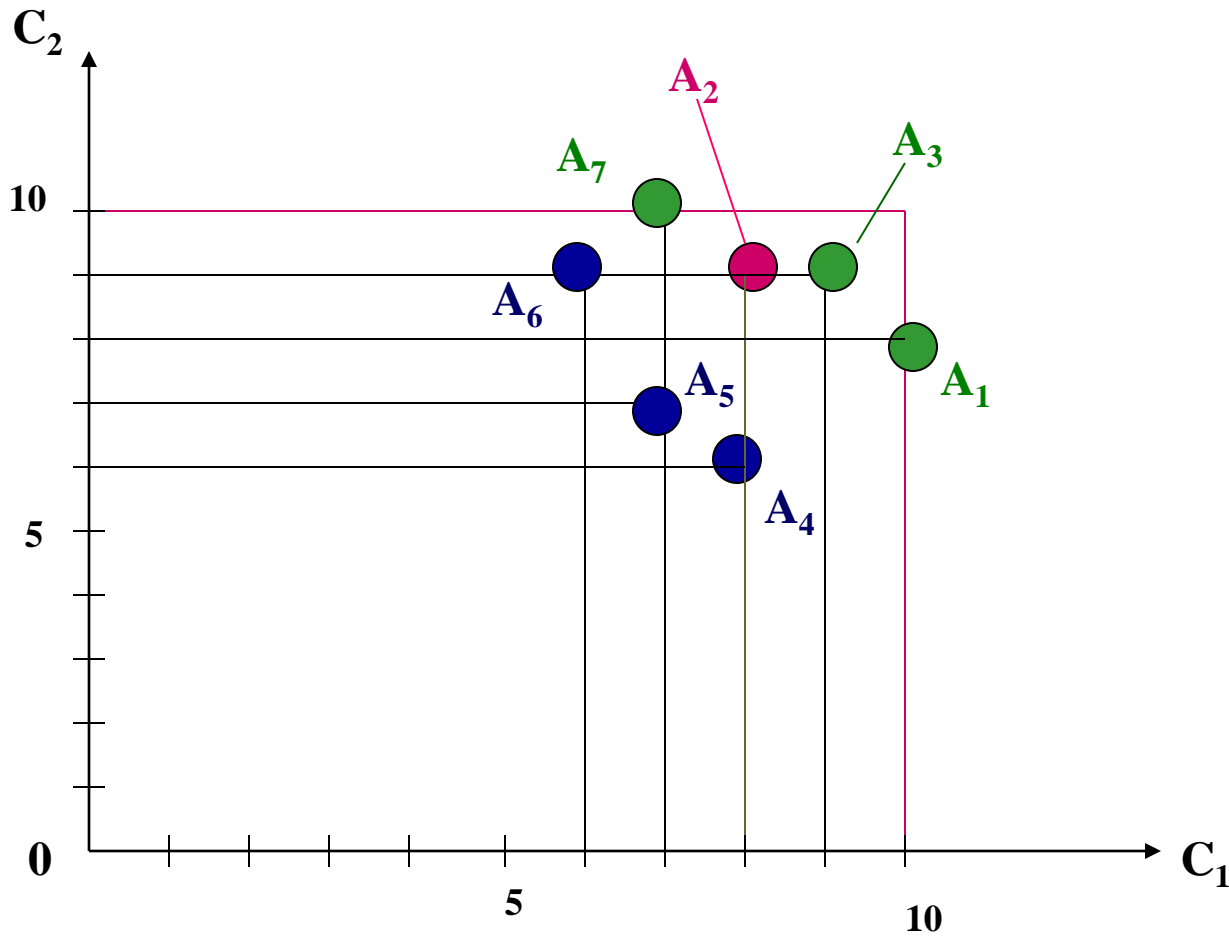
## Illustrative numerical example for multicriteria ranking

	Mathematics $C_1$	Sport $C_2$	$F_a$	$F_q$	Pareto approach
--	----------------------	----------------	-------	-------	--------------------

$A_1$	10	8	18 / 1	164 / 1	1
$A_2$	8	9	17 / 2	145 / 2	2
$A_3$	9	9	18 / 1	162 / 1	1
$A_4$	6	8	14 / 5	100 / 4	3
$A_5$	7	7	14 / 5	98 / 4	3
$A_6$	9	6	15 / 4	117 / 3	3
$A_7$	10	7	16 / 3	149 / 2	1

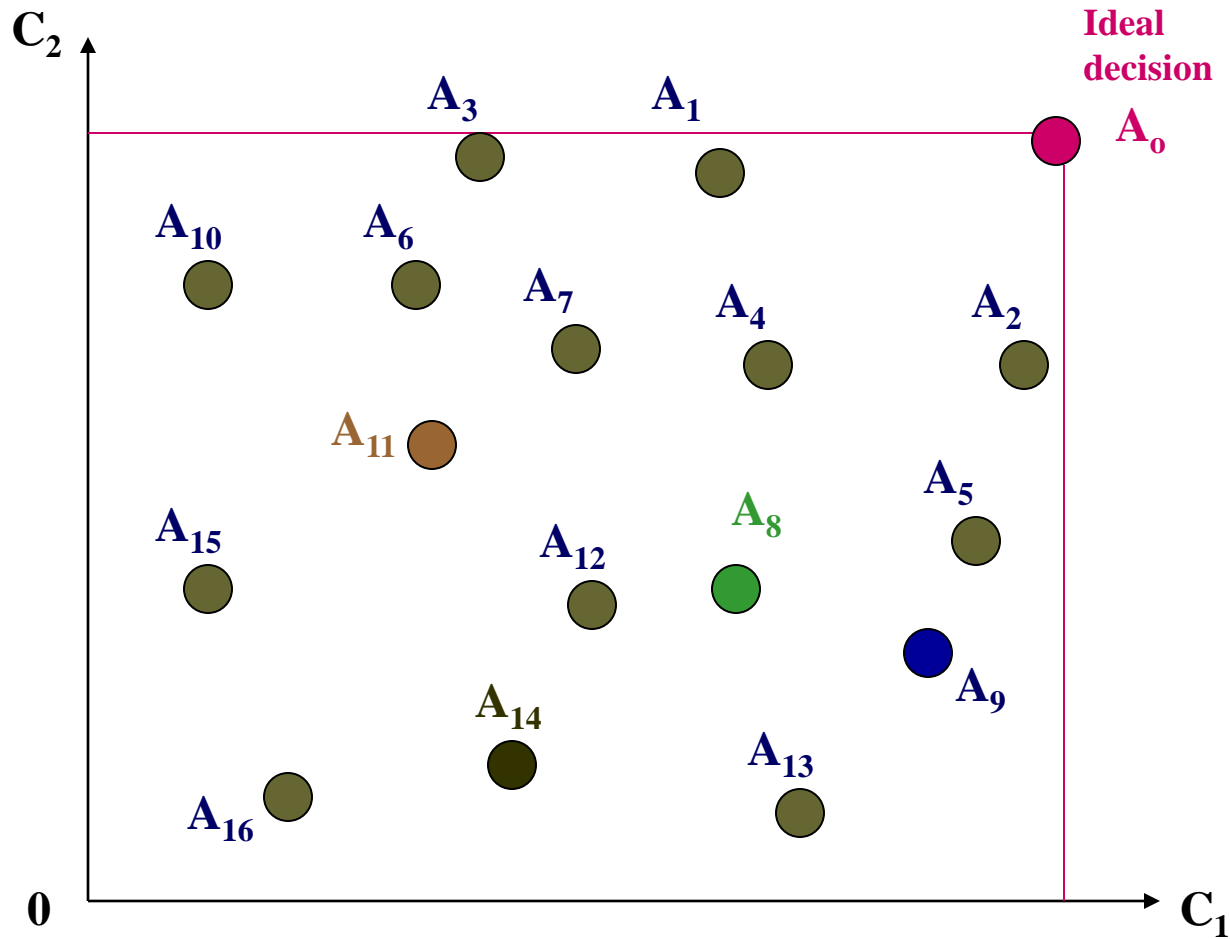


Pareto-approach for example above



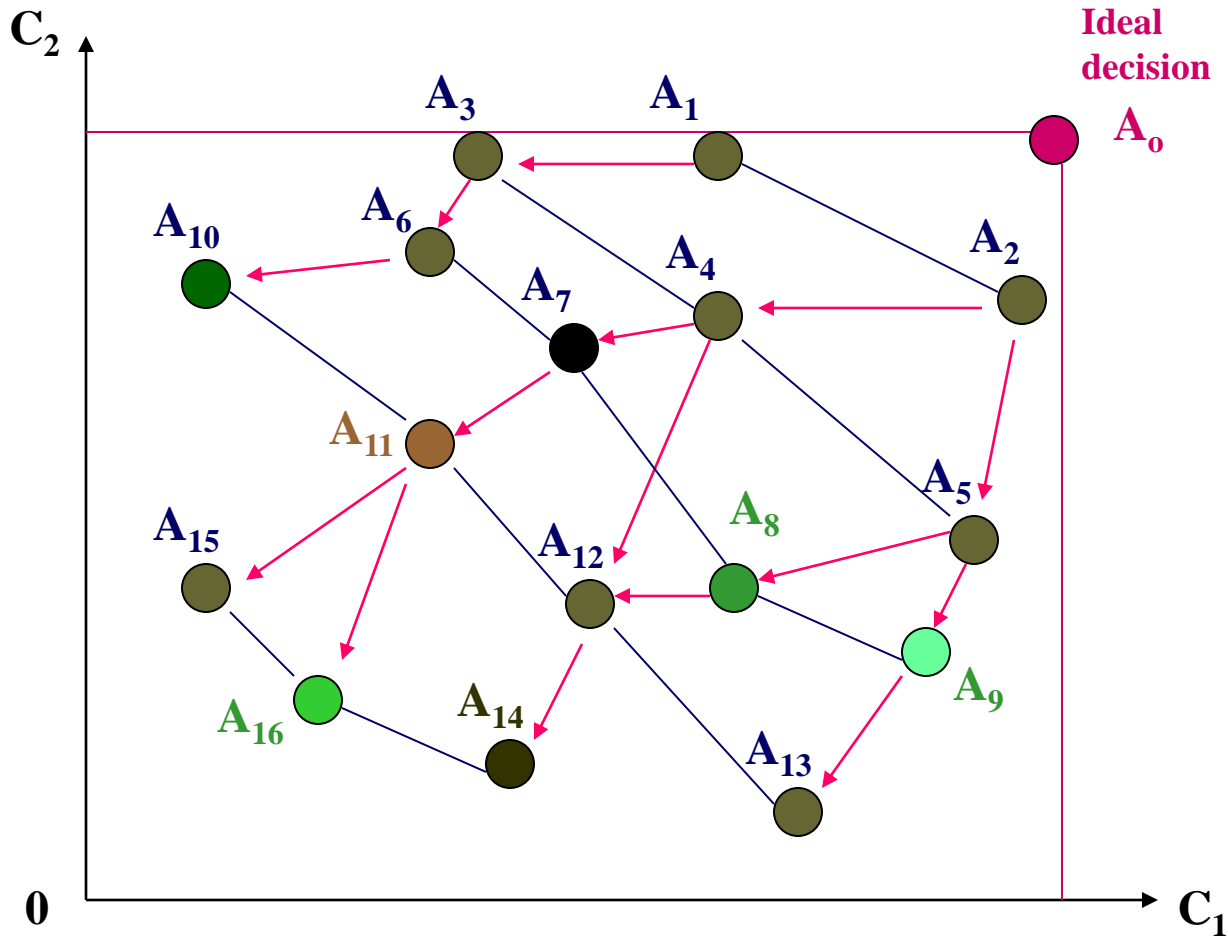
- $A_3$  better  $A_2$
- $A_3$  better  $A_5$
- $A_1$  better  $A_4$
- $A_7$  better  $A_5$
- $A_2$  better  $A_4$
- $A_2$  better  $A_6$
- $A_3$  better  $A_6$
- $A_3$  better  $A_4$
- $A_1$  better  $A_5$
- $A_7$  better  $A_6$
- $A_2$  better  $A_5$

Method of “equivalence” (incomparability) levels: Initial Alternatives



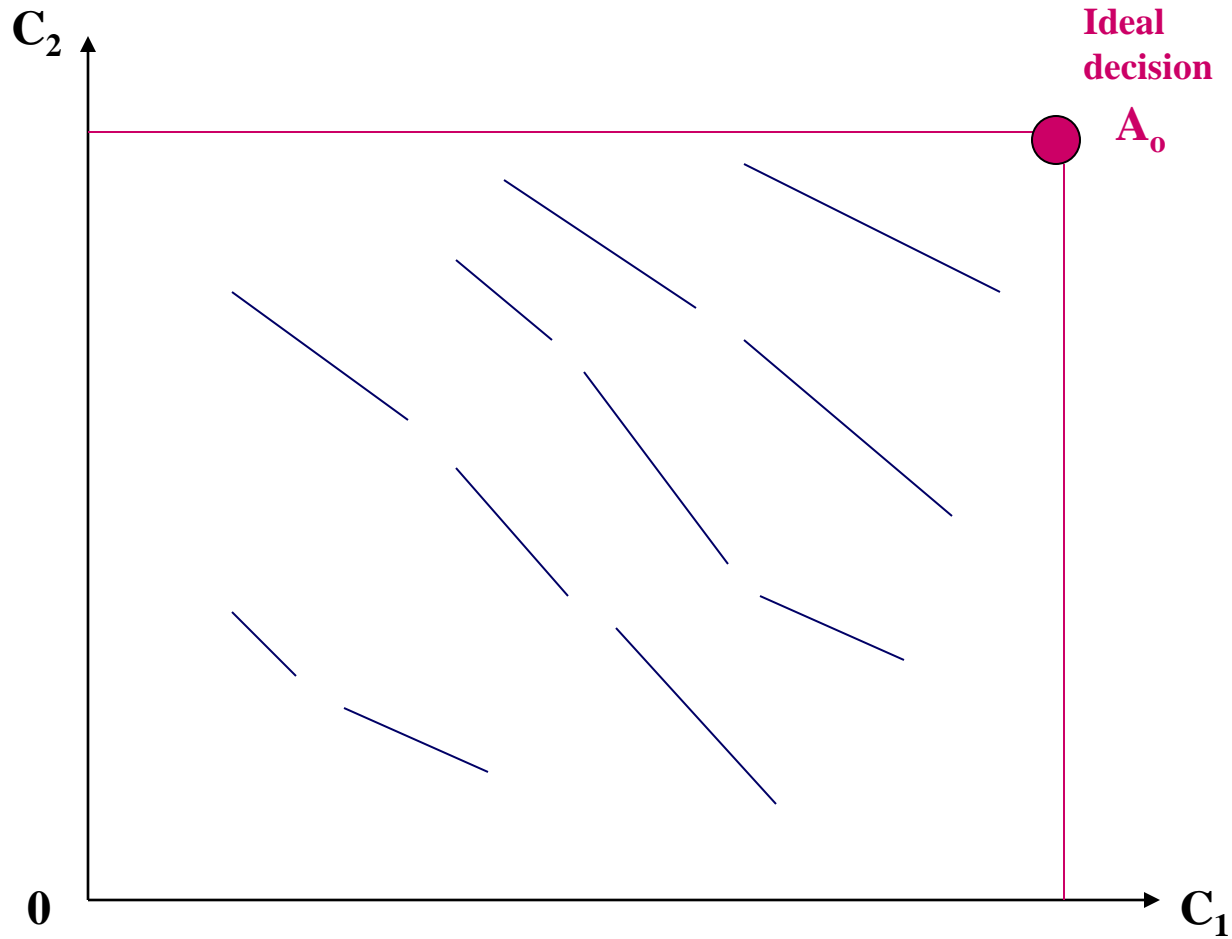


Method of “equivalence” (incomparability) levels: Pairwise Comparison

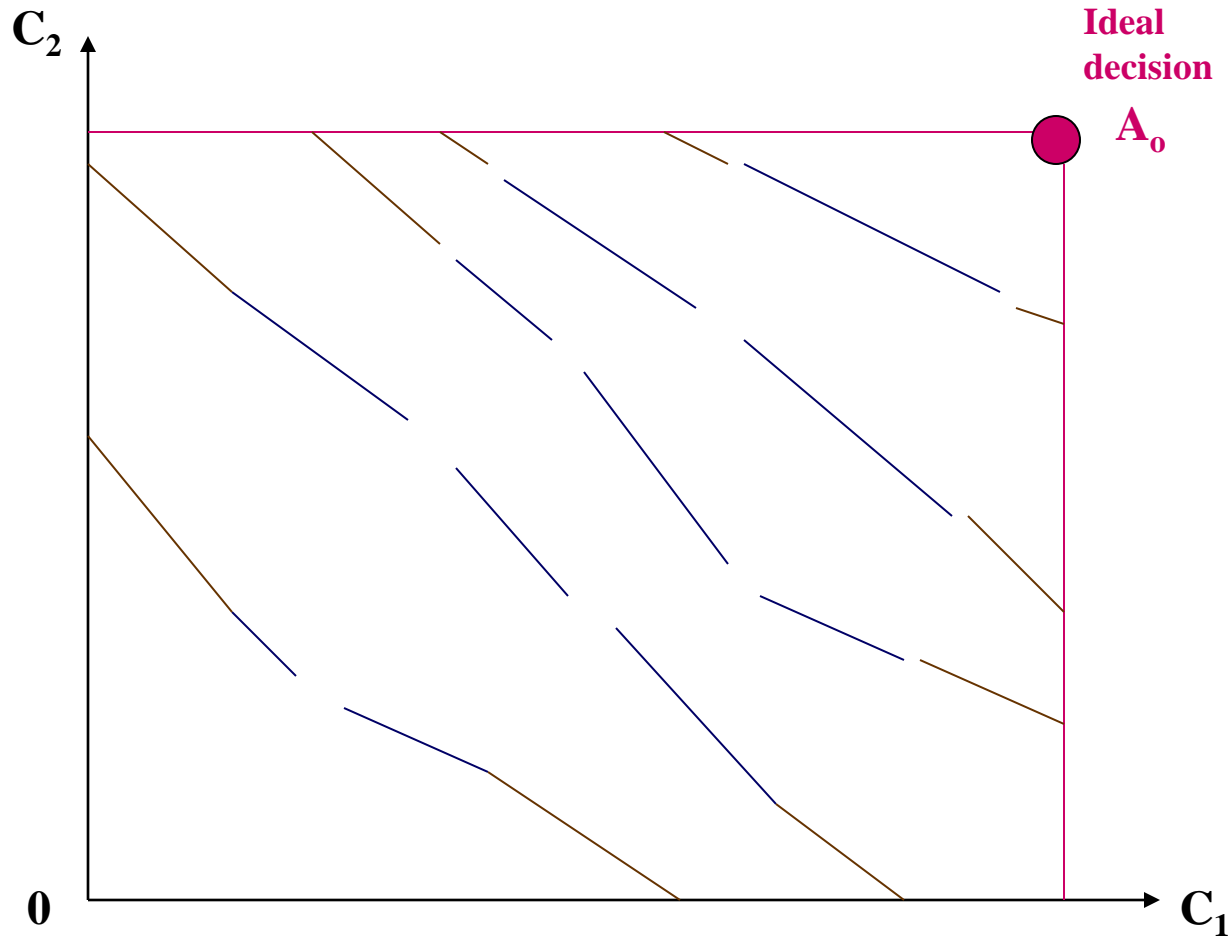


**Pairwise Comparison:**  
**1. Dominance**  
**2. Incomparability**

Method of “equivalence” (incomparability) levels: basis or layers of incomparability



Method of “equivalence” (incomparability) levels: extended layers of incomparability



Method of “equivalence” (incomparability) levels: evaluation of new alternatives

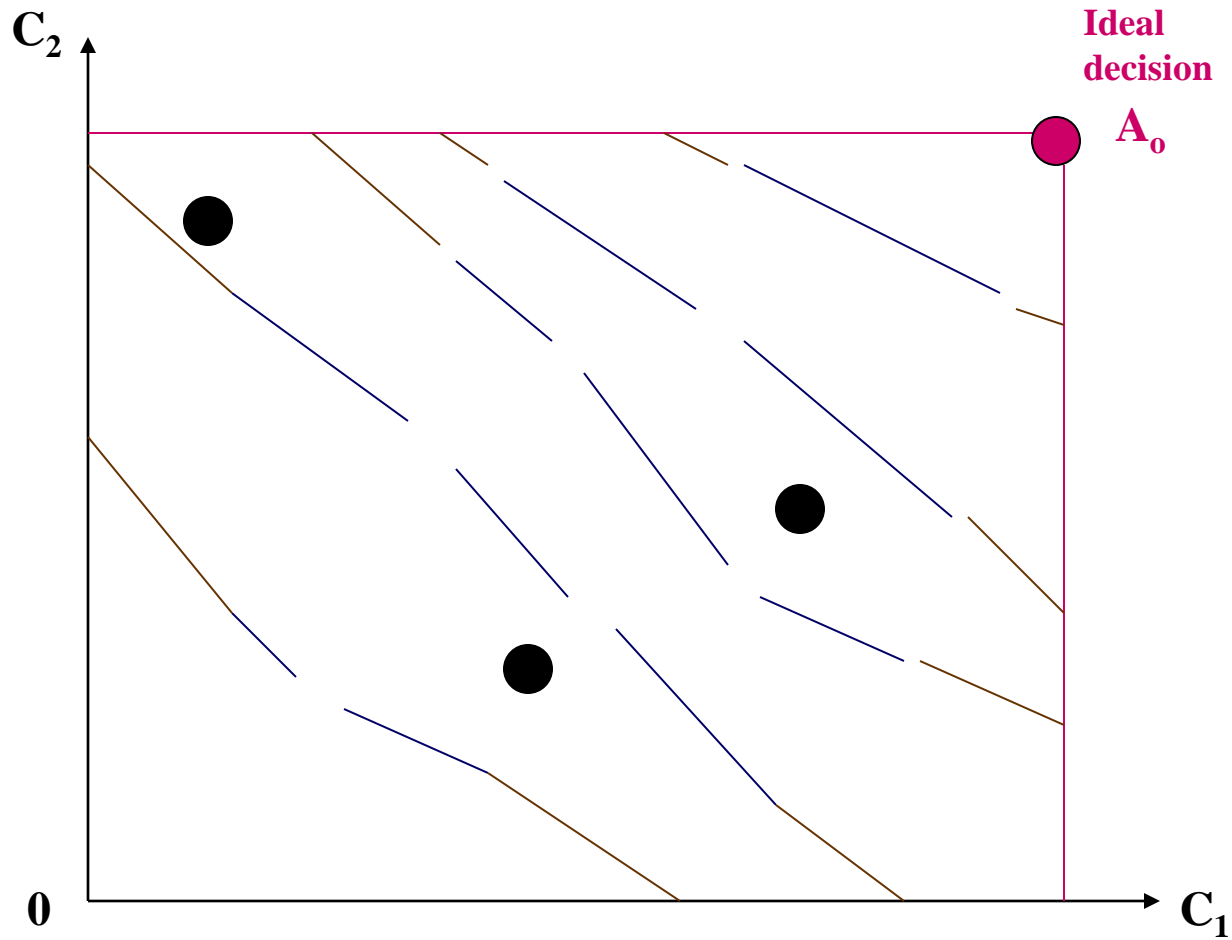


Illustration for arithmetic utility function: layers of incomparability

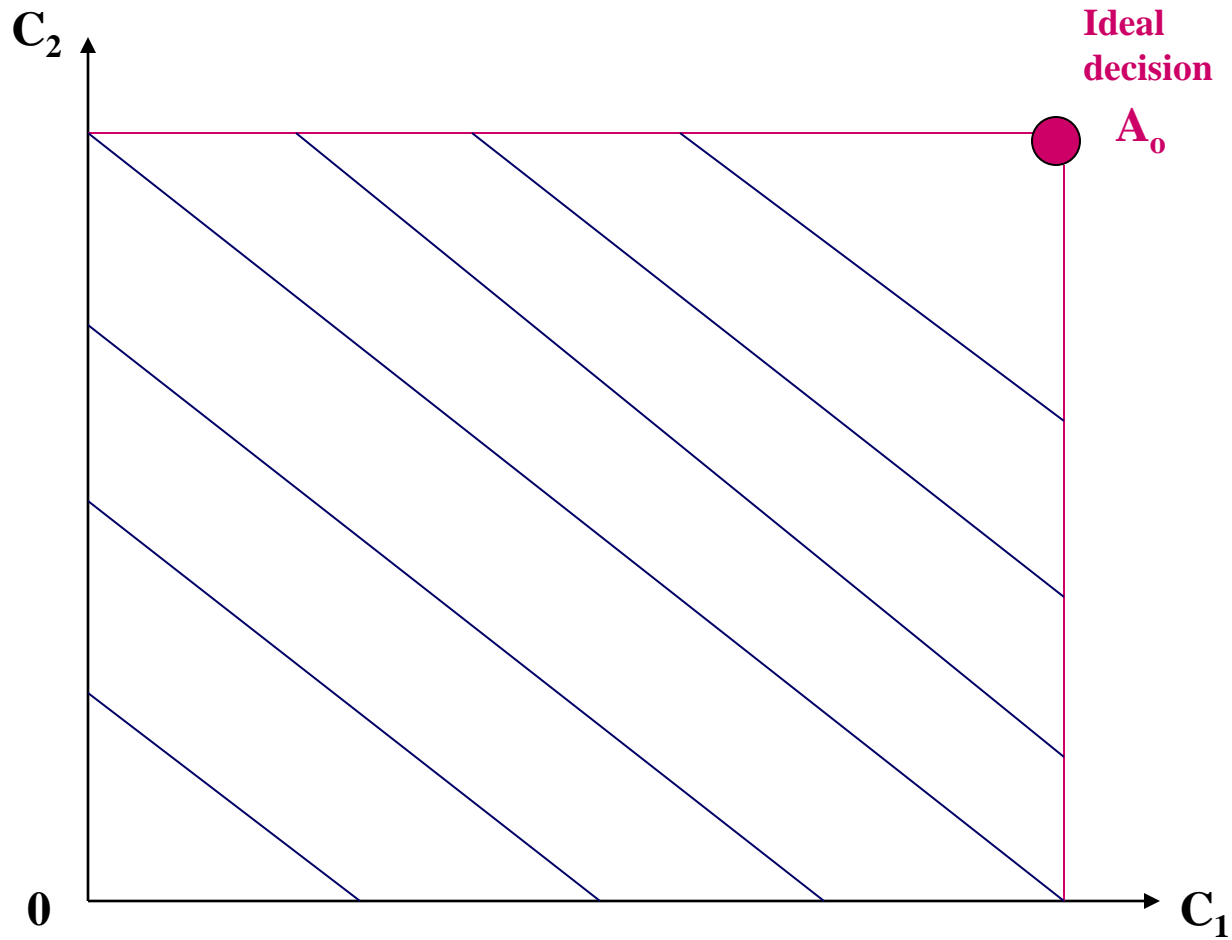


Illustration for quasi-quadratic utility function: layers of incomparability

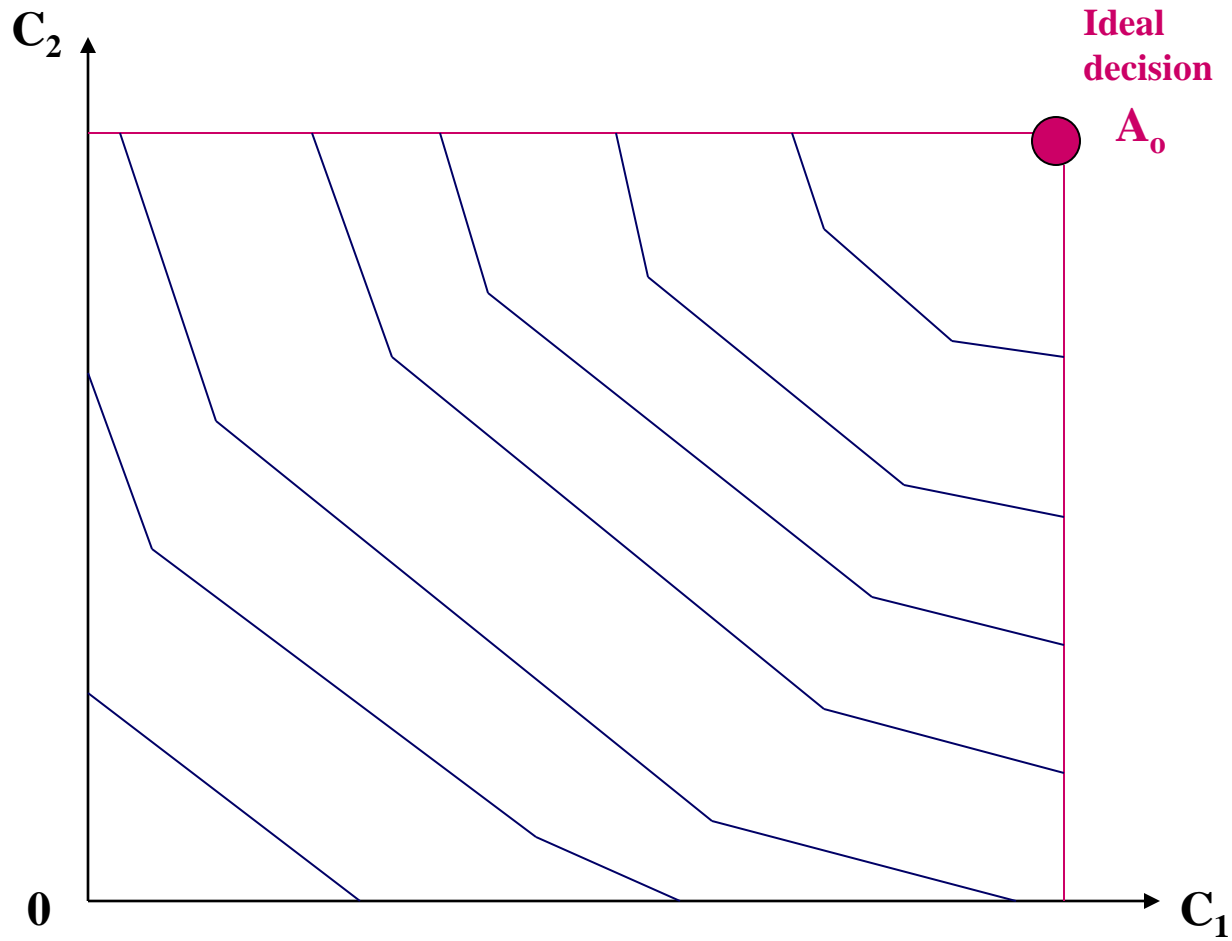
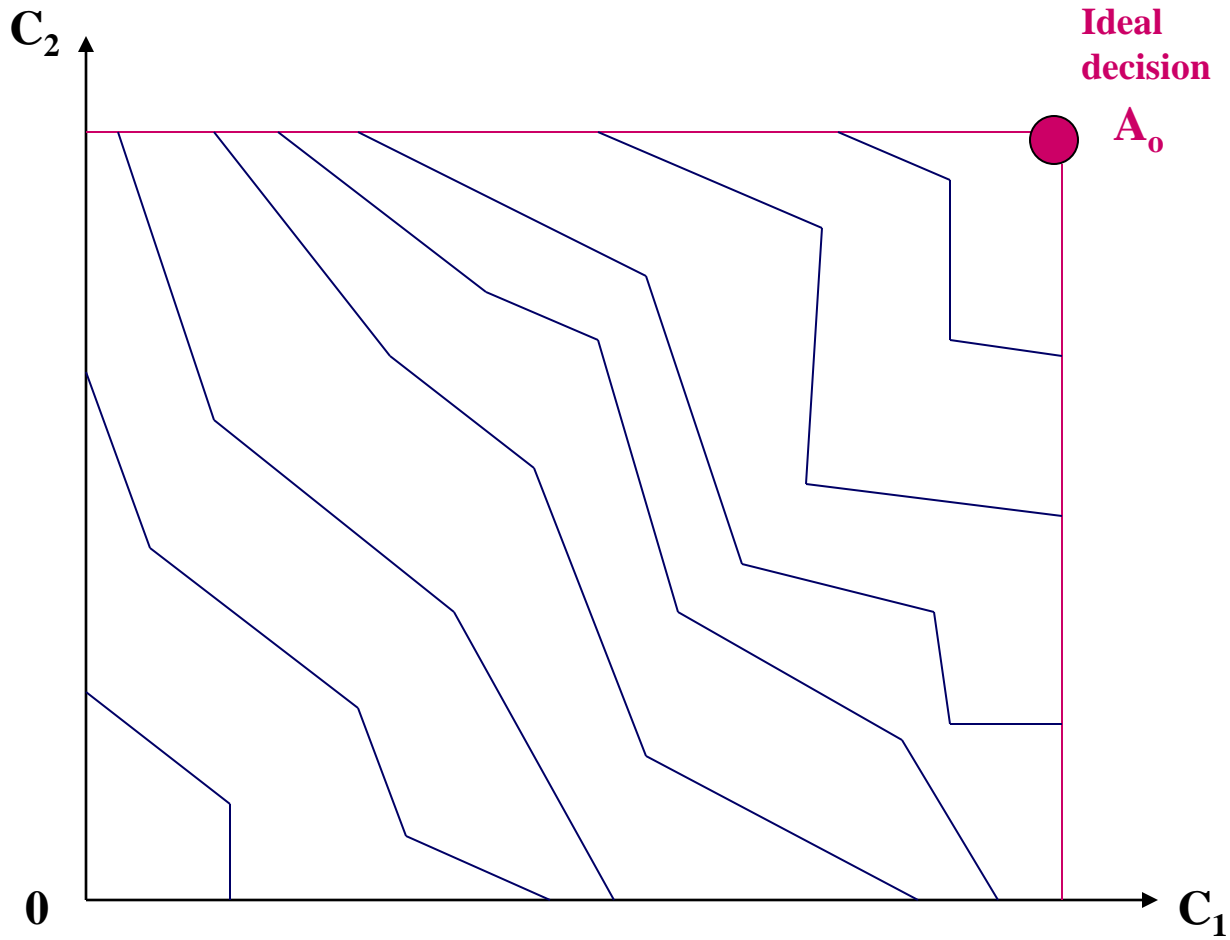


Illustration example for a “complex” situation of incomparability layers



Outranking technique (method ELECTRE) by B. Roy

Alternatives  $A=(A_1, \dots, A_i, \dots, A_n)$  and criteria  $C=(C_1, \dots, C_j, \dots, C_k)$ ,  
 $\forall A_i$  a vector of estimates  $z_i = (z_{i1}, \dots, z_{ij}, \dots, z_{ik})$ ,  $\mu_j$  is a weight of criterion  $j$

$\forall$  pair  $A_u, A_v \in A$  to compute:

Coefficient of “concordance”

$$\alpha_{uv} = ( 1 / \sum_{j=1}^k \mu_j ) \sum_{(j \in X(u, v))} \mu_j$$

Coefficient of “discordance”

$$\beta_{uv} = 0 \quad \text{if } |Y(u, v)| = 0 \quad \text{else} \\ \max_j (( \mu_j |z_{uj} - z_{vj}| ) / ( d_j \sum_{j=1}^k \mu_j ))$$

$X(uv) = \{ j \mid z_{uj} \geq z_{vj} \}$ ,  $Y(uv) = \{ j \mid z_{uj} < z_{vj} \}$ ,  $d_j$  is scale size

**RULE:**  $A_u$  better  $A_v$  if  $( \alpha_{uv} \geq p ) \& ( \beta_{uv} \leq q )$   
where  $p, q$  are thresholds (e.g.,  $p = 0.9$  and  $q = 0.2$  )



Illustrative numerical example for multicriteria ranking

	$C_1$ 0.1	$C_2$ 0.1	$C_3$ 0.15	$C_4$ 0.4	$C_5$ 0.25	Criteria {j} weight $\mu_j$
$A_1$	10	8	8	10	4	
$A_2$	1	9	7	5	3	
$A_3$	0	9	10	6	1	
$A_4$	10	2	14	3	2	
$A_5$	7	7	5	8	3	
$d_j$	11	8	10	8	4	

$$u = 1, v = 3$$



$$X(1,3) = \{ 1,4,5 \}$$

$$Y(1,3) = \{ 2,3 \}$$

$$\alpha_{13} = (1 / 1) (0.1 + 0.4 + 0.25) = 0.75$$

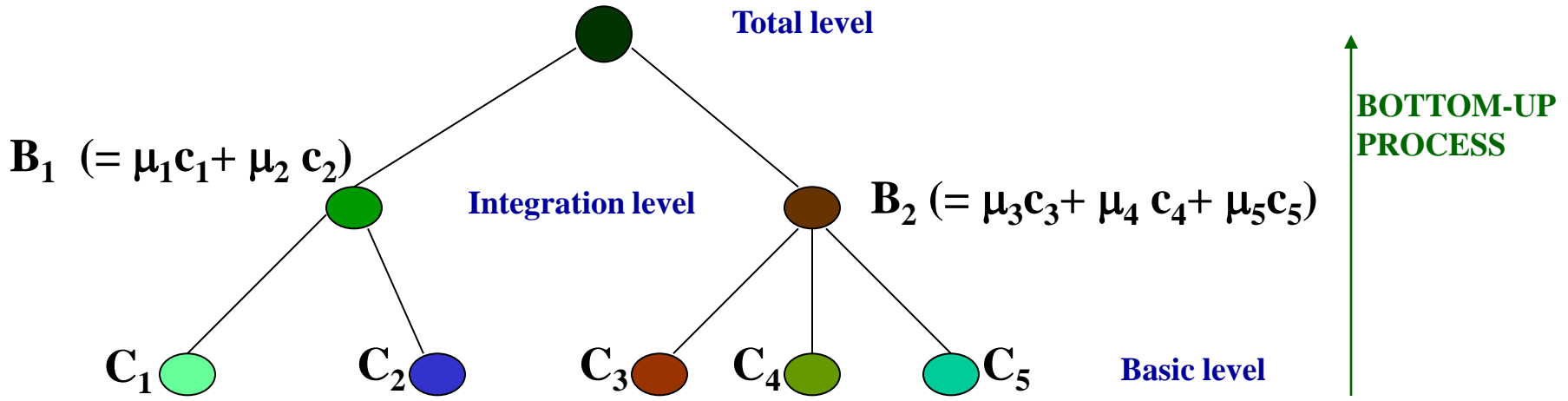
$$\beta_{13} = \max \{ (0.1 (9-8) / 8), (0.15 (10 - 8) / 10) \} = \max \{ 0.125, 0.03 \} = 0.125$$

Version of Result 1:  $p = 0.7$   $q = 0.3 \Rightarrow A_1$  better  $A_3$

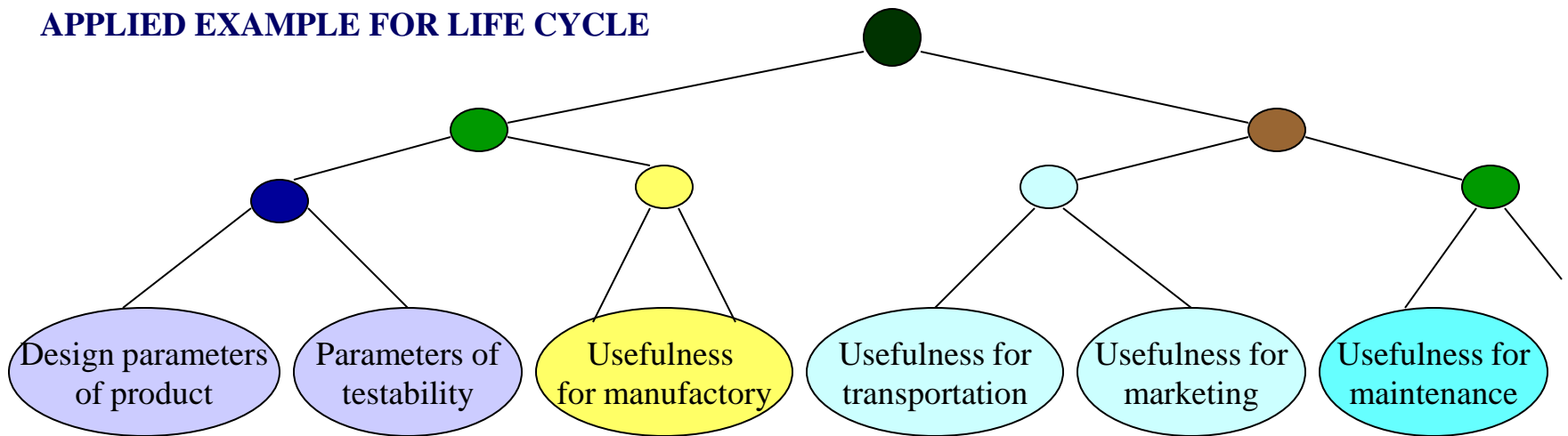
Version of Result 2:  $p = 0.8$   $q = 0.2 \Rightarrow$  incomparable ones

Analytic Hierarchy Process (T.L. Saaty)

$$J^* (= \lambda_1 b_1 + \lambda_2 b_2)$$

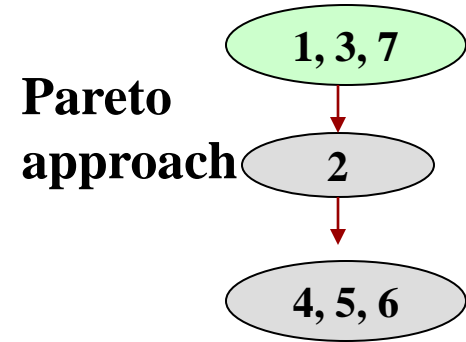
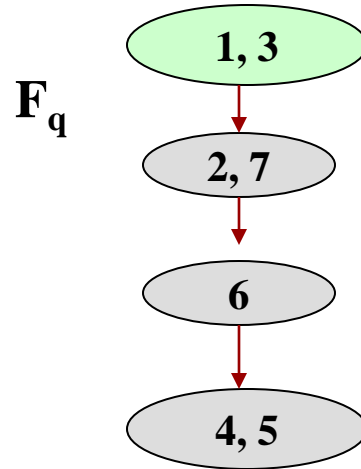
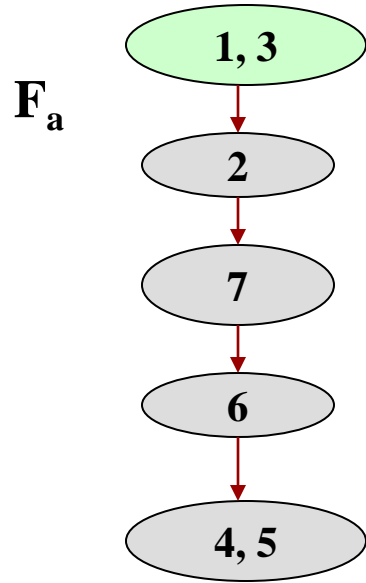


APPLIED EXAMPLE FOR LIFE CYCLE

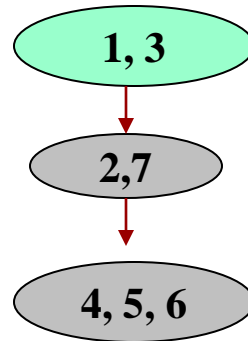


# Integration (aggregation) of results

Previous example:



Intuitive integration:



## Integration (aggregation) approaches

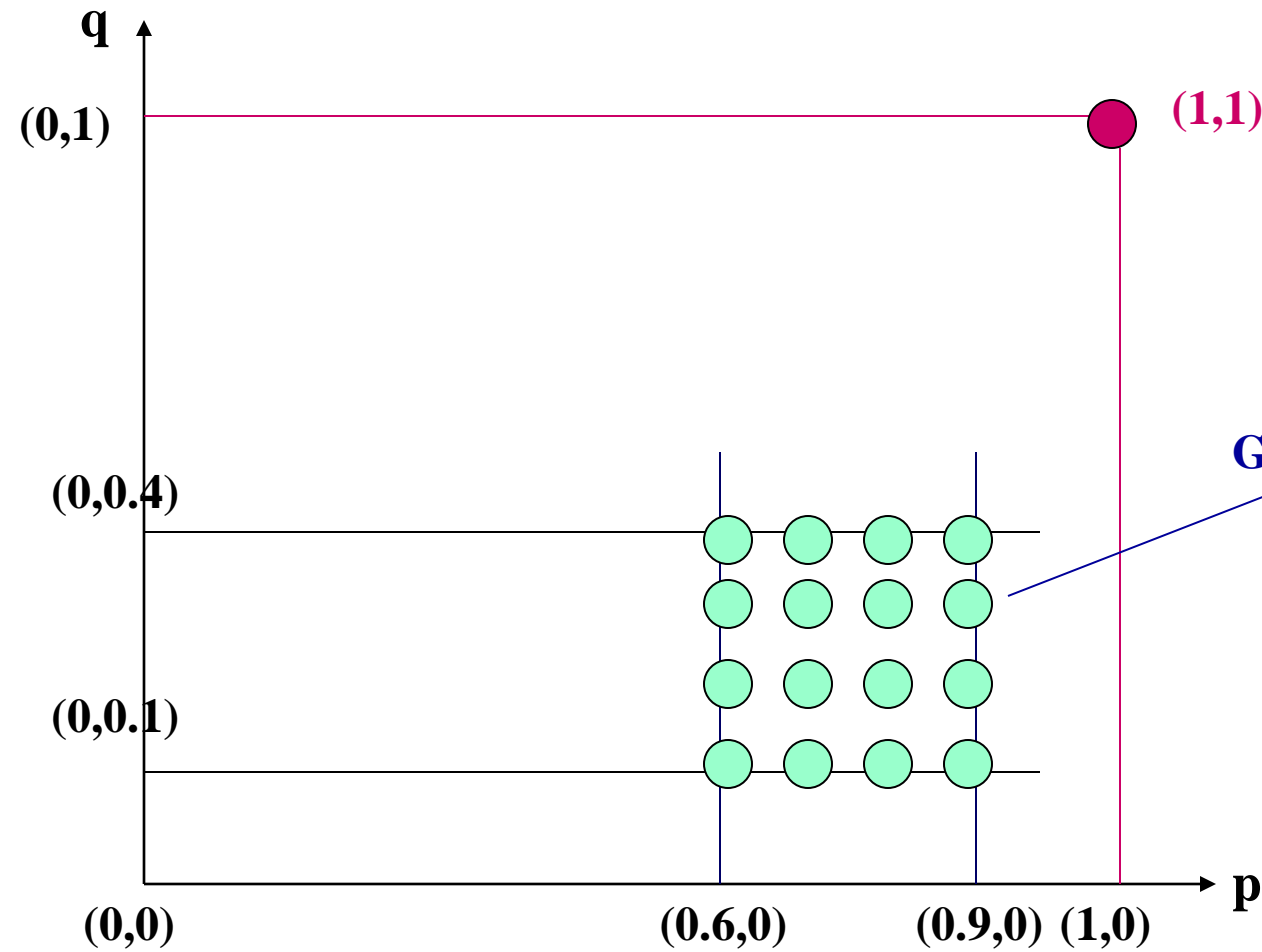
**1.Election rules**

**2.Election rules & deletion of “margin results”**

**3.Multicriteria approaches above**

**4.Membership function (including fuzzy results)**

Integration (aggregation) approach: Example for usage of ELECTRE (M.Sh. Levin, DSS COMBI)



Grid of thresholds

- SOLVING SCHEME:**
1. Method ELECTRE (for each threshold pair)
  2. Ranking (obtaining layers)
  3. Aggregation of results

**LECTURE 11-12. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

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**L.11. Framework of decision making, examples.**

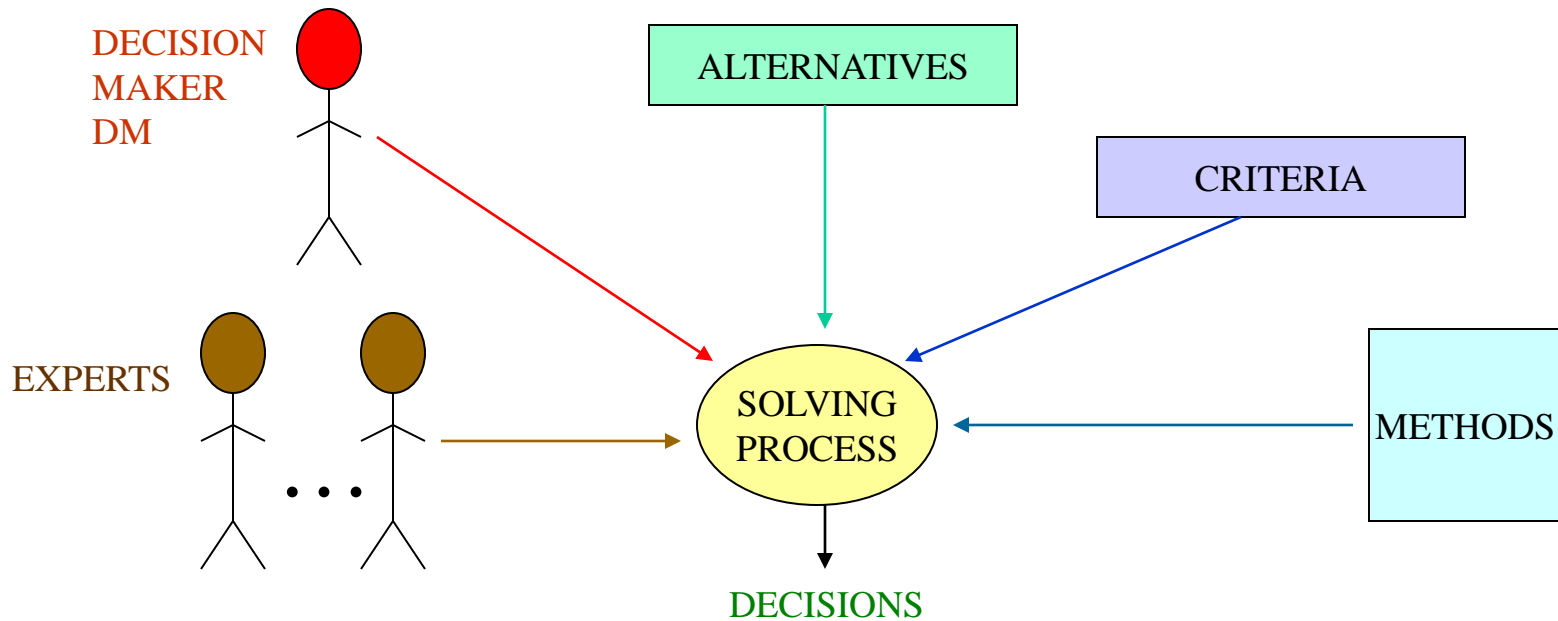
**L.12. Function, mapping. Optimization models.**

***PLAN:***

1. Framework of multicriteria decision making
2. Partitioning the procedure of multicriteria decision making
3. Numerical examples: \*partitioning of initial problem \*aggregation of results
4. Mapping

Sept. 25, 2004

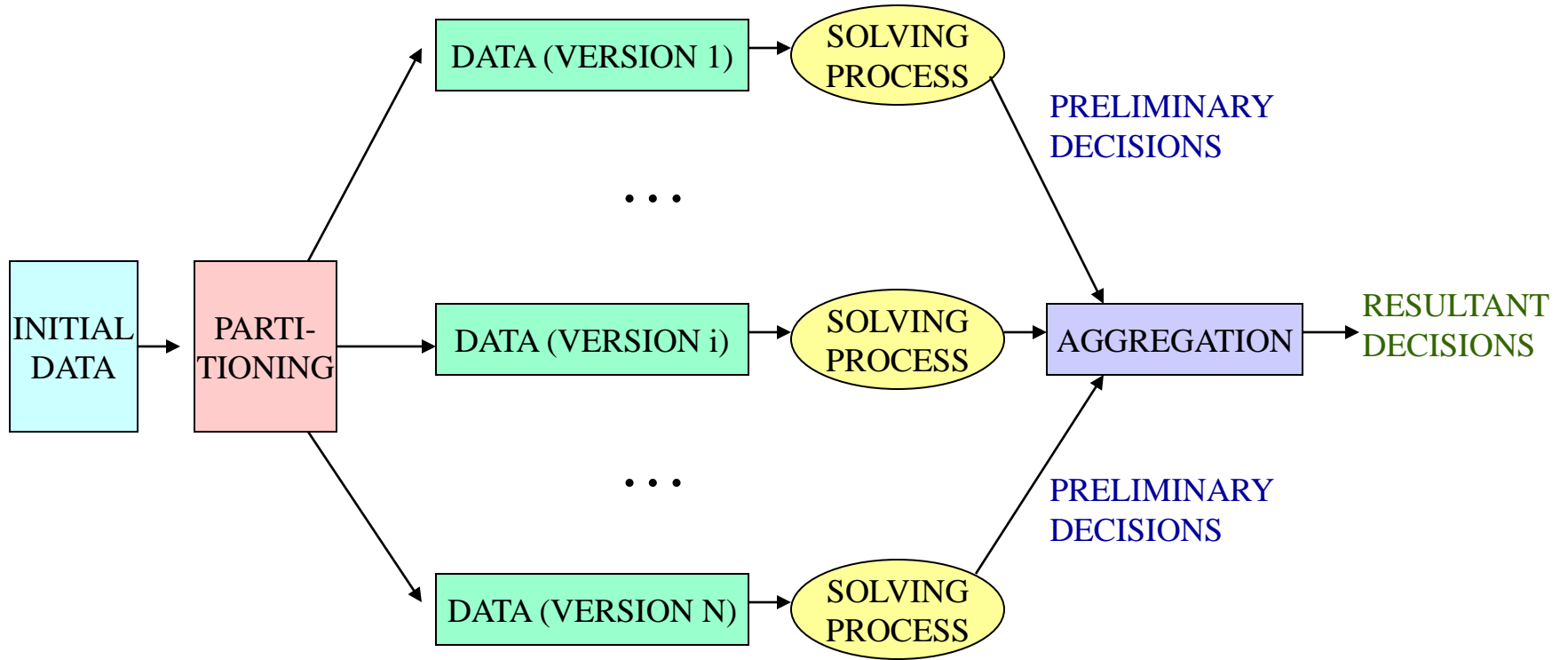
## Framework of multicriteria decision making & its partitioning (parallelization)



### APPROACHES FOR PARTITIONING OF DECISION MAKING FRAMEWORK:

1. By criteria
2. By alternatives
3. By experts
4. By methods
5. Hybrid approaches

# Parallel Processing Scheme





## Example: Partitioning by criteria groups

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	TOTAL
A <sub>1</sub>	10	8	9	7	6	1	3	3
A <sub>2</sub>	8	7	7	9	10	3	1	3
A <sub>3</sub>	10	7	6	7	9	3	2	2
A <sub>4</sub>	9	9	9	8	9	1	1	1
A <sub>5</sub>	7	10	10	8	8	1	2	2

FINAL AGGREGATION

Example: Partitioning by alternative groups

PRELIMINARY STAGE

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	1 <sup>st</sup> step	
A <sub>1</sub>	10	8	9	1	<div style="border-left: 1px solid blue; border-right: 1px solid blue; height: 100px; position: relative;"> <div style="position: absolute; top: 50%; left: 50%; transform: translate(-50%, -50%);"> <span style="color: blue;">↙</span> </div> <div style="position: absolute; bottom: 50%; left: 50%; transform: translate(-50%, -50%);"> <span style="color: green;">↘</span> </div> </div>
A <sub>2</sub>	8	7	7	3	
A <sub>3</sub>	10	7	6	3	
A <sub>4</sub>	9	9	9	1	
A <sub>5</sub>	7	10	10	1	
A <sub>6</sub>	10	10	6	1	<div style="border-left: 1px solid green; border-right: 1px solid green; height: 100px; position: relative;"> <div style="position: absolute; top: 50%; left: 50%; transform: translate(-50%, -50%);"> <span style="color: green;">↙</span> </div> </div>
A <sub>7</sub>	6	8	9	2	
A <sub>8</sub>	7	7	9	3	
A <sub>9</sub>	9	8	9	1	

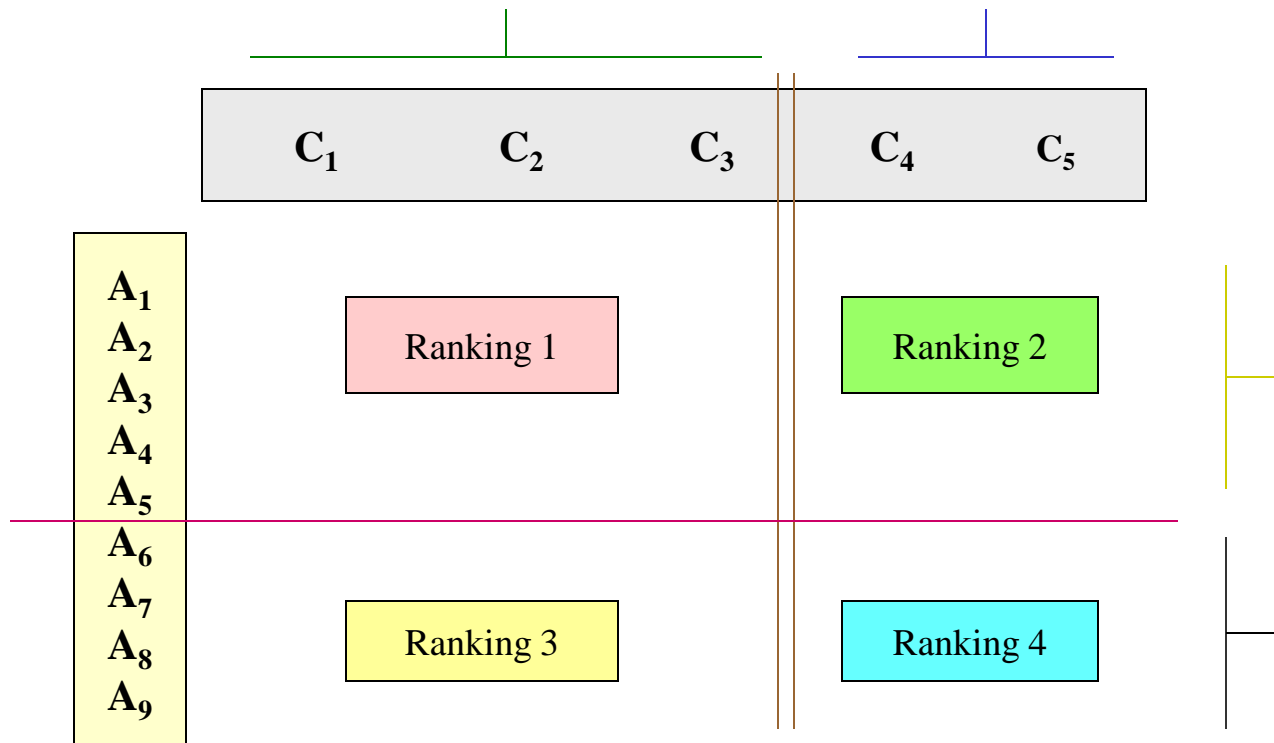
FINAL STAGE

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	
A <sub>1</sub>	10	8	9	2
A <sub>4</sub>	9	9	9	3
A <sub>5</sub>	7	10	10	1
A <sub>6</sub>	10	10	6	1
A <sub>9</sub>	9	8	9	4

Example: Joint partitioning by criteria groups & alternative groups

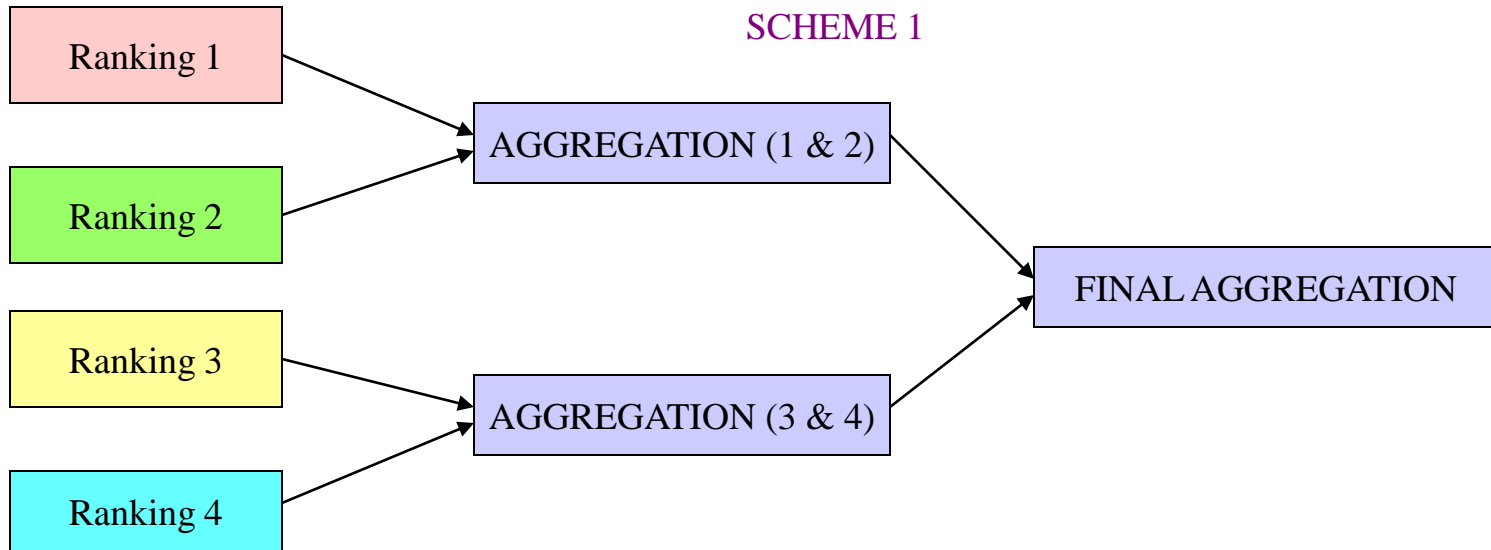
	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>
A <sub>1</sub>	10	8	9	7	6
A <sub>2</sub>	8	7	7	9	10
A <sub>3</sub>	10	7	6	7	9
A <sub>4</sub>	9	9	9	8	9
A <sub>5</sub>	7	10	10	8	8
A <sub>6</sub>	10	10	6	7	9
A <sub>7</sub>	6	8	9	10	8
A <sub>8</sub>	7	7	9	10	10
A <sub>9</sub>	9	8	9	9	9

# Possible schemes for joint partitioning by criteria groups & alternative groups

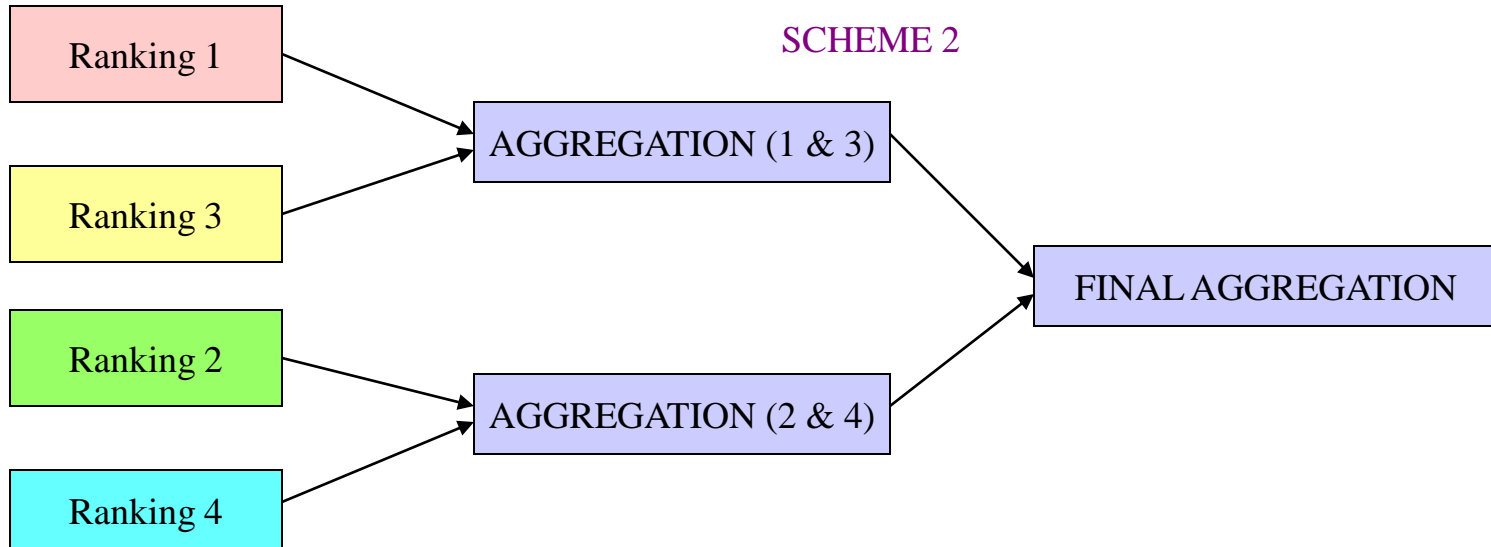


Possible schemes for joint partitioning by criteria groups & alternative groups

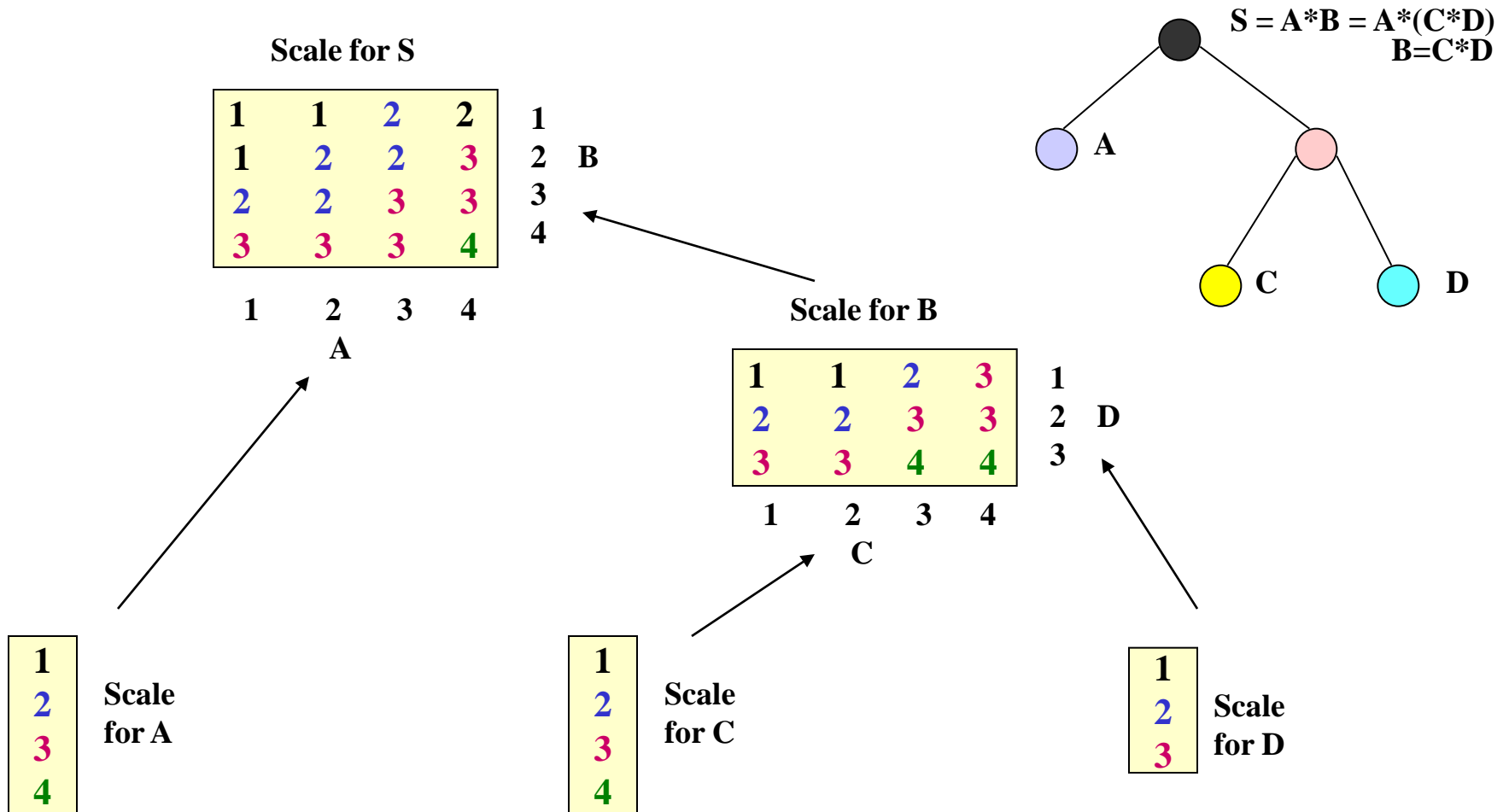
SCHEME 1



SCHEME 2



Example: Integration tables (Glotov & Paveljev)

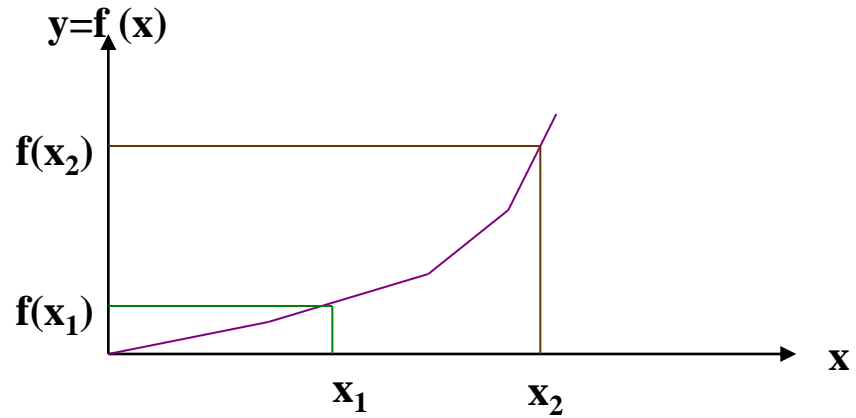


**EXAMPLE:** Basic estimates are the following: 4 for A, 3 for C, 1 for D; intermediate estimate for B is 2; resultant estimate for S is 3.

**NOTE:** multidimensional integration tables are possible (and useful) too.

# Mapping & Optimization Models

$x \in \mathbf{R}$



$x \in X \subseteq \mathbf{R}$



$X$

MAPPING  $y=f(x)$



$Y$

$y \in Y \subseteq \mathbf{R}$

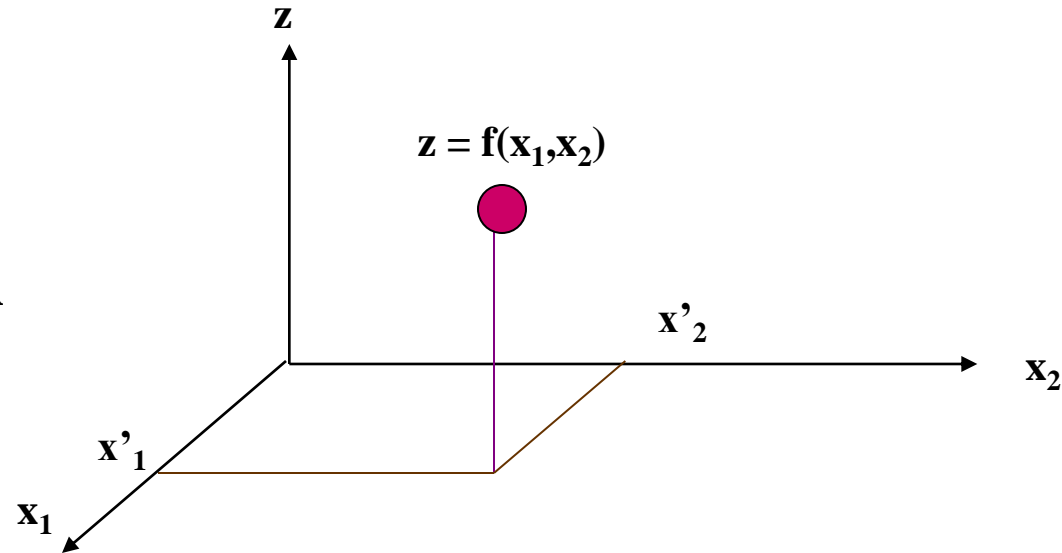


## Mapping & Optimization Models

$$z = f(x_1, x_2) \in \mathbb{R}^2$$

$$(x_1, x_2) \in X \subseteq \mathbb{R}^2$$

$$f(x_1, x_2) \Rightarrow Z \subseteq \mathbb{R}$$



**GENERALLY:**

$$(x_1, \dots, x_m) \in X \subseteq \mathbb{R}^m$$

$$(y_1, \dots, y_n) \in Y \subseteq \mathbb{R}^n$$





## Mapping & Optimization Models

$$\mathbf{x} \in \mathbf{X} \subseteq \mathbf{R}$$

$\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2]$  (admissible domain)

$f(\mathbf{x})$  is objective function

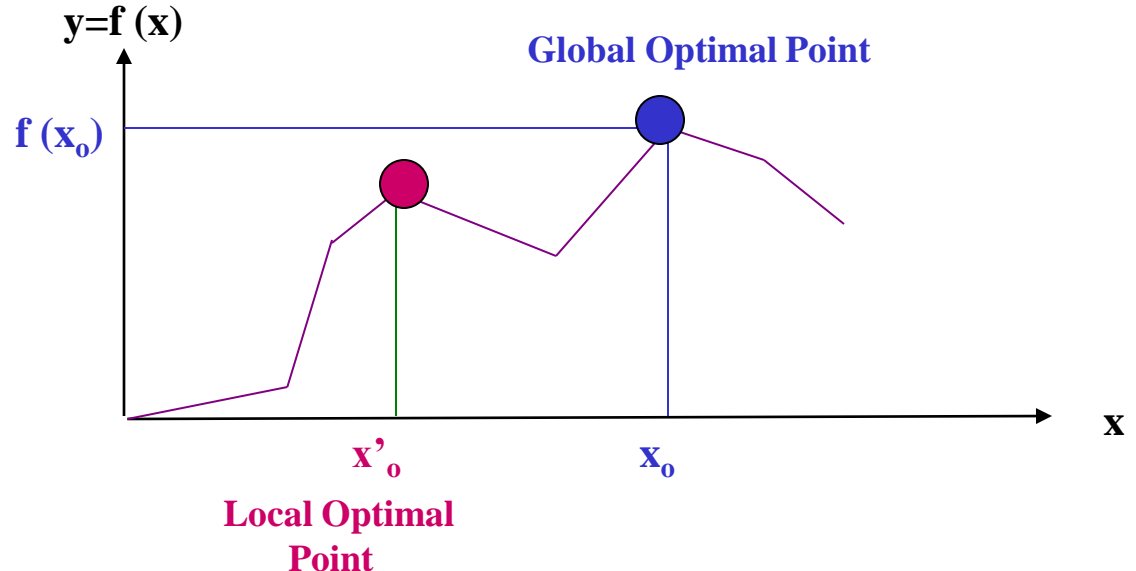
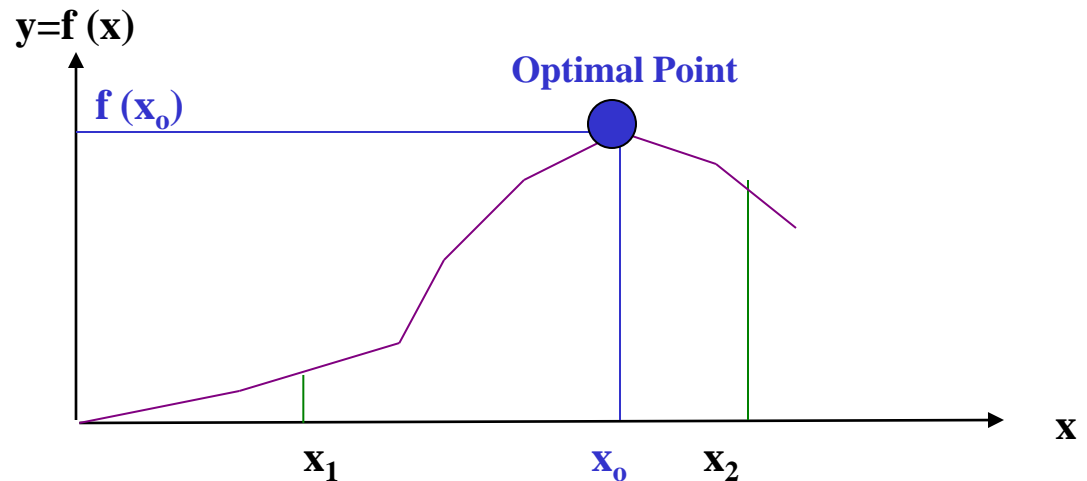
$$\begin{array}{l} \max f(\mathbf{x}) \\ \text{subject to} \\ \mathbf{x} \in \mathbf{X} \end{array}$$

$$\begin{array}{l} \max f(\mathbf{x}) \\ \text{subject to} \\ \mathbf{x} \geq \mathbf{x}_1 \\ \mathbf{x} \leq \mathbf{x}_2 \end{array}$$

**GENERALLY:**

$$\begin{array}{l} \max f(\mathbf{x}) \\ \text{subject to} \\ \varphi_1(\mathbf{x}) \leq 0 \\ \dots \\ \varphi_k(\mathbf{x}) \leq 0 \end{array}$$

$\varphi_j(\mathbf{x})$  is constraint function ( $1 \leq j \leq k$ )



## Illustration table for optimization models

Objective function $f(\mathbf{x})$	Constraint $\varphi_j(\mathbf{x})$	Type of model	Method
Linear	Linear	Linear	*simplex *ellipsoid method *method of Karmarkar
Quadratic	Linear	Quadratic	*simplex *ellipsoid
Convex	Linear	Convex	*gradient method *ellipsoid method
...	...	...	...

### **NOTES:**

- 1. Objective function can be examined as vector-like one too (multi-objective optimization)**
- 2. Constraints can be examined as binary relations too**
- 3. In discrete optimization discrete spaces are examined**
- 4. In stochastic optimization all parameters / functions can be stochastic ones**
- 5. It is possible to take into account uncertainty by two ways:**
  - \*stochastic parameters / functions**
  - \*parameters / functions on the basis of fuzzy sets**

## LECTURE 13. Course: “Design of Systems: Structural Approach”

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### L.13. Basic models of combinatorial optimization I.

#### *PLAN:*

1. Basic combinatorial optimization problems:

\*knapsack problem, \*solving schemes for multicriteria knapsack problem, \*multiple choice problem.

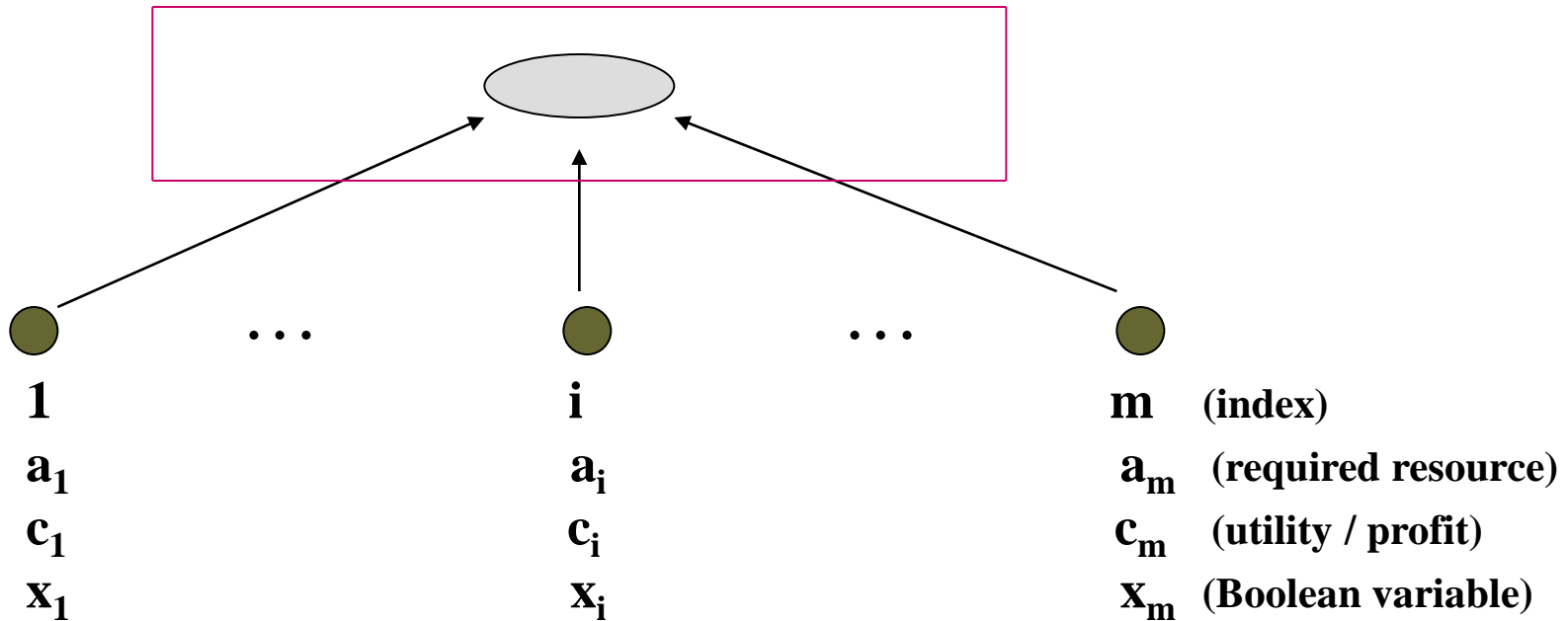
2. Algorithms: \*types of solutions (exact, approximate), \*types of algorithms (polynomial and enumerative algorithms),

3. Complexity of problems.

4. Global approaches and local techniques

Oct. 1, 2004

## Knapsack problem



$$\begin{aligned} \max \quad & \sum_{i=1}^m c_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^m a_i x_i \leq b \\ & x_i \in \{0, 1\}, \quad i = 1, \dots, m \\ & \text{possible additional constraints} \\ & \sum_{i=1}^m a_{ik} x_i \leq b_k, \quad k = 1, \dots, l \end{aligned}$$

- 1.Ordering by decreasing of  $c_i / a_i$  (algorithm by Danzig, heuristic)***
- 2.Branch-And-Bound method***
- 3.Dynamic programming (exact solution)***
- 4.Dynamic programming (approximate solving scheme)***
- 5.Probabilistic methods***
- 6.Hybrid schemes***

## Simple versions of knapsack problem

1.  $c_i = c_0$  (*equal utilities*)
2.  $a_i = a_0$  (*equal required resources*)

### **Polynomial algorithm:**

1. *ordering by non-decreasing of  $a_i$*
2. *ordering by non-increasing of  $c_i$*

## Extended versions of knapsack problem

1. Knapsack problem with objective function as *min*
2. Knapsack problem with several “knapsacks”
3. Knapsack problem with additional structural (logical) constraints over elements (e.g., some kinds of trees)
4. Multi-objective knapsack problem
5. Knapsack problem with fuzzy parameters



Heuristic solving scheme for multicriteria (multiple objective) versions of knapsack problem

**ALGORITHM SCHEME** (case of *linear ranking*):

**STEP 1.** Multicriteria ranking of elements (to obtain *linear ranking*)

**STEP 2.** Series selection of elements

(the best element, the next element, etc.)

After each selection: testing the resource constraint ( $\leq b$ ).

*If the constraint is not right it is necessary to delete the last selected element and to STOP.*

*Else: to STEP 2.*

**STOP.**



## Heuristic solving scheme for multicriteria (multiple objective) versions of knapsack problem

**ALGORITHM SCHEME** (case of *group ranking*):

**STEP 1.** Multicriteria ranking of elements (to obtain *group ranking*)

**STEP 2.** Series selection of elements

(elements of the best group, elements of the next group, etc.)

After each selection: testing the resource constraint ( $\leq b$ ).

*If the constraint is not right it is necessary to go to STEP 3.*

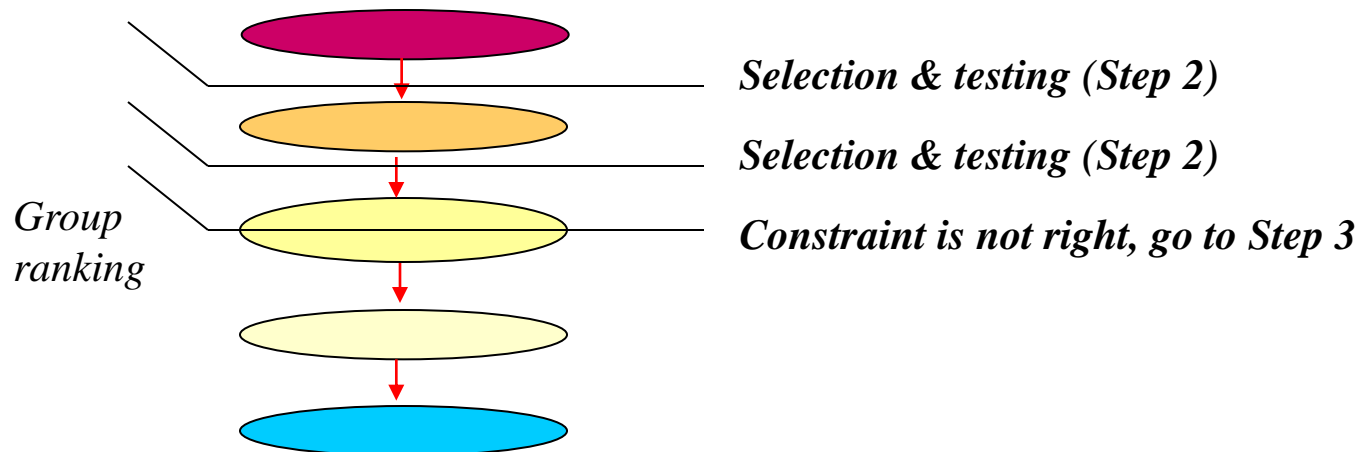
*Else: to STEP 2.*

**STEP 3.** Solving for the last analyzed element group the special case of knapsack problem (with equal utilities) as series selection of elements from the list (non-increasing by  $a_i$ ).

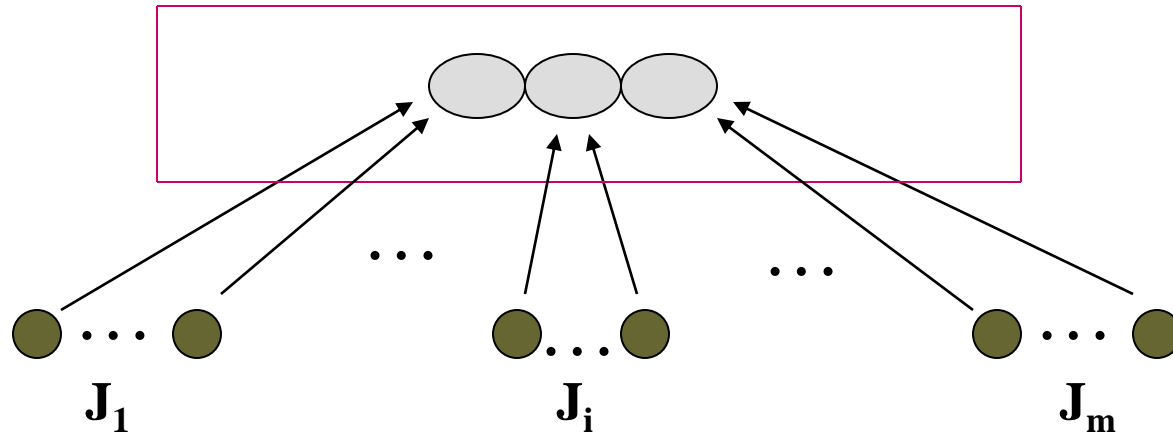
Here constraint is the following:  $\leq b - \sum_{(i \in Q)} a_i$

(where Q is a set of selected elements from the previous groups)

**STOP.**



## Multiple choice problem



$$\forall i \quad |J_i| = q_i, \quad j = 1, \dots, q_i$$

$$\begin{aligned} \max \quad & \sum_{i=1}^m \sum_{j=1}^{q_i} c_{ij} x_{ij} \\ \text{s.t.} \quad & \sum_{i=1}^m \sum_{j=1}^{q_i} a_{ij} x_{ij} \leq b \\ & \sum_{j=1}^{q_i} x_{ij} \leq 1, \quad i = 1, \dots, m \\ & x_{ij} \in \{0, 1\}, \quad i = 1, \dots, m, \quad j = 1, \dots, q_i \end{aligned}$$

- 1. *Ordering by decreasing of  $c_{ij} / a_{ij}$  (heuristic)***
- 2. *Branch-And-Bound method***
- 3. *Dynamic programming (exact solution)***
- 4. *Dynamic programming (approximate solving scheme)***
- 5. *Probabilistic methods***
- 6. *Hybrid schemes***

## Illustration for dynamic programming

Series Design of a Solution:

1. From START point to END point
2. From END point to START point

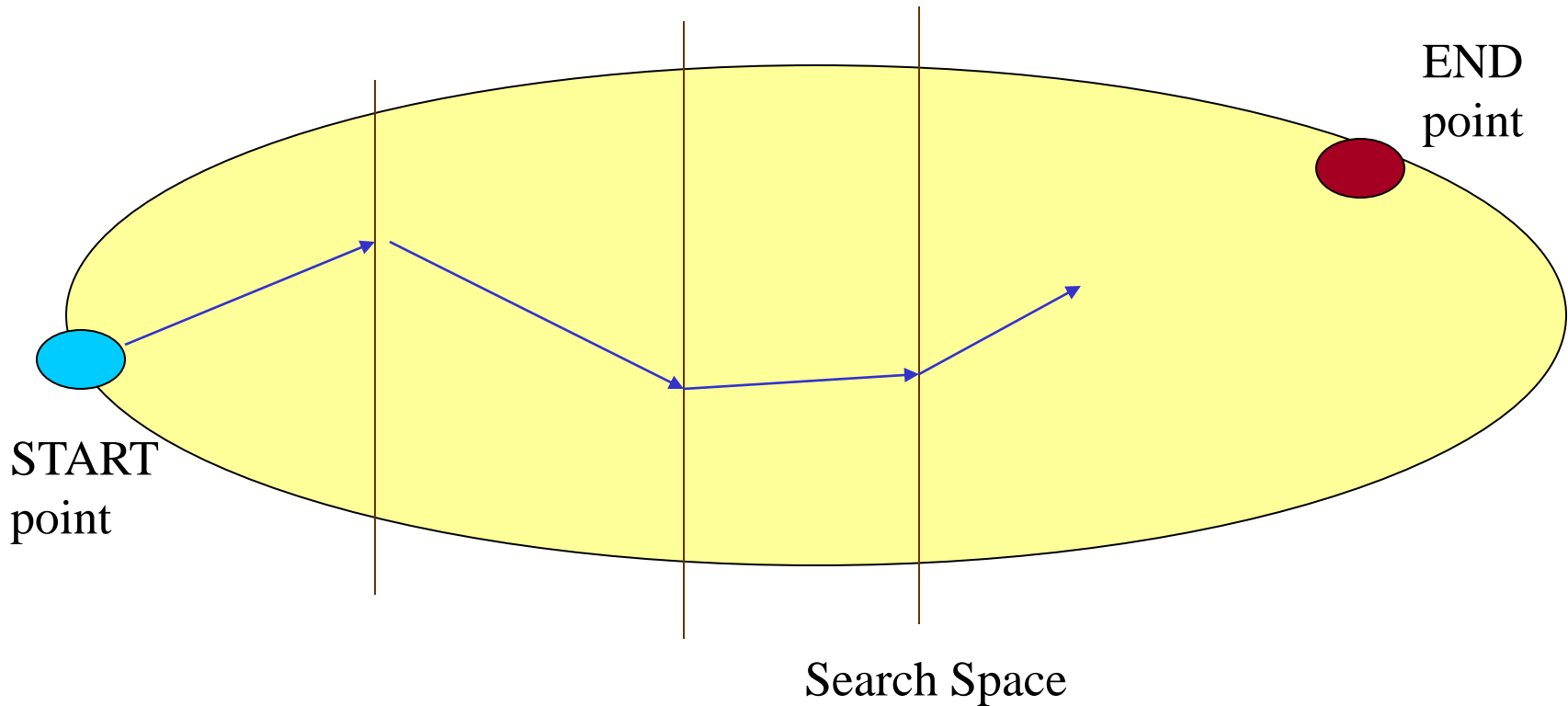
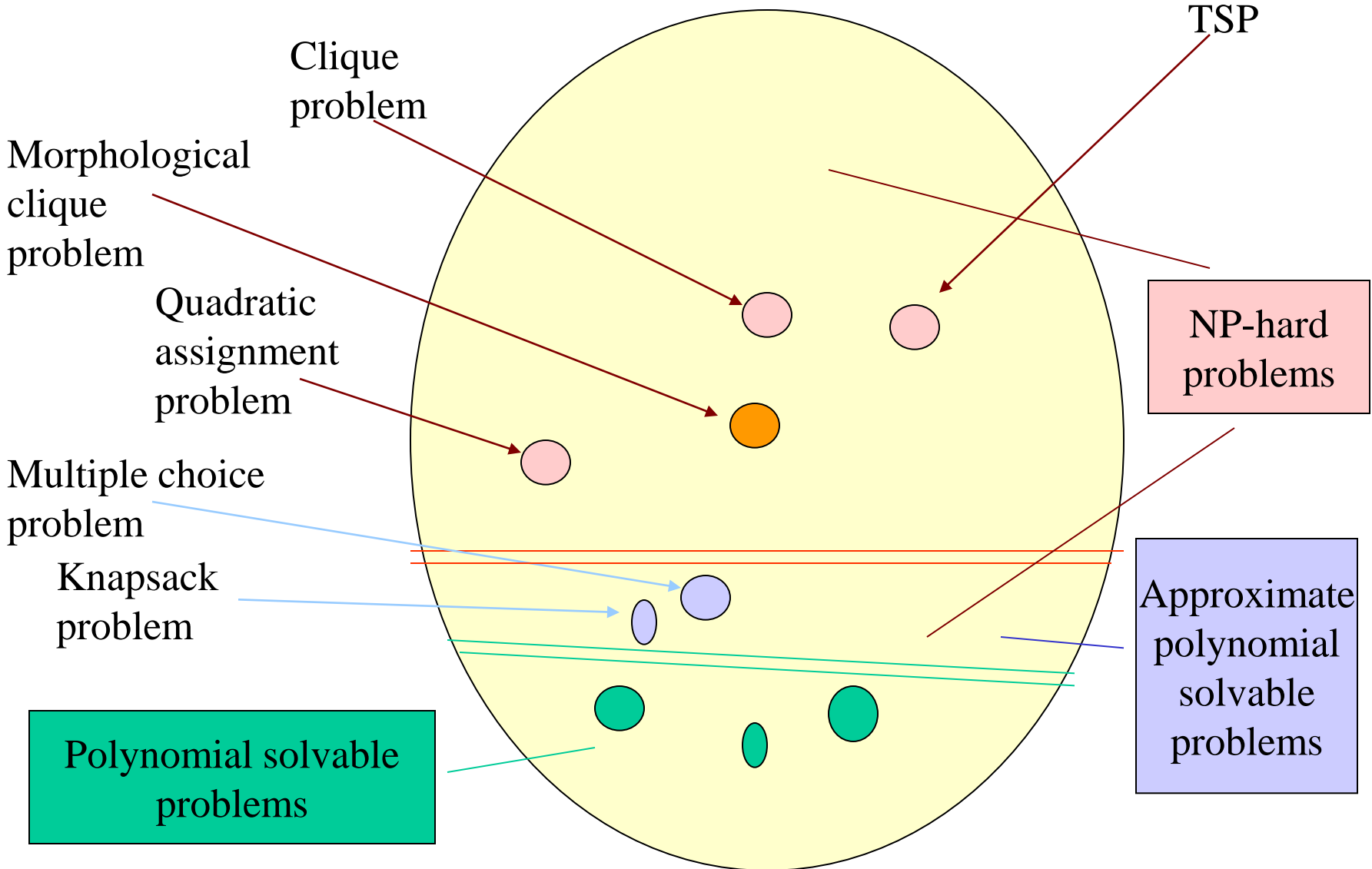


Illustration for complexity of combinatorial optimization problems



## Classification of algorithms

### **BY EXACTNES OF RESULT (solution):**

- 1.Exact solution**
- 2.Approximate solution (for worst case):**
  - \*limited error (absolute error) \*limited error (relative error) \*other situations**
- 3.Approximate solution (statistically)**
- 4.Heuristic (without an estimate of exactness)**

### **BY COMPLEXITY OF SOLVING PROCESS (e.g., number of steps):**

- 1.Polynomial algorithms (of length of input, for example:  
 $O(n \log n)$ ),  $O(n)$ ,  $O(1)$ ,  $O(n^2)$**
- 2.Polynomial approximate schemes (for a specified exactness / limited error, for example:  $O(n^2/\epsilon)$  where  $\epsilon \in [0,1]$  is a relative error for objective function)**
- 3.Statistically good algorithms (statistically polynomial ones)**
- 4.Enumerative algorithms**

...

### **BASIC ALGORITHM RESOURCES:**

- 1.Number of steps (computing operations)**
- 2.Required volume of memory**
- 3.Required number of interaction with specialists (oracle)  
(to get additional information)**
- 4.Required communication between processors (for multi-processor algorithms)**

## Global approaches and local techniques

### **GLOBAL APPROACHES:**

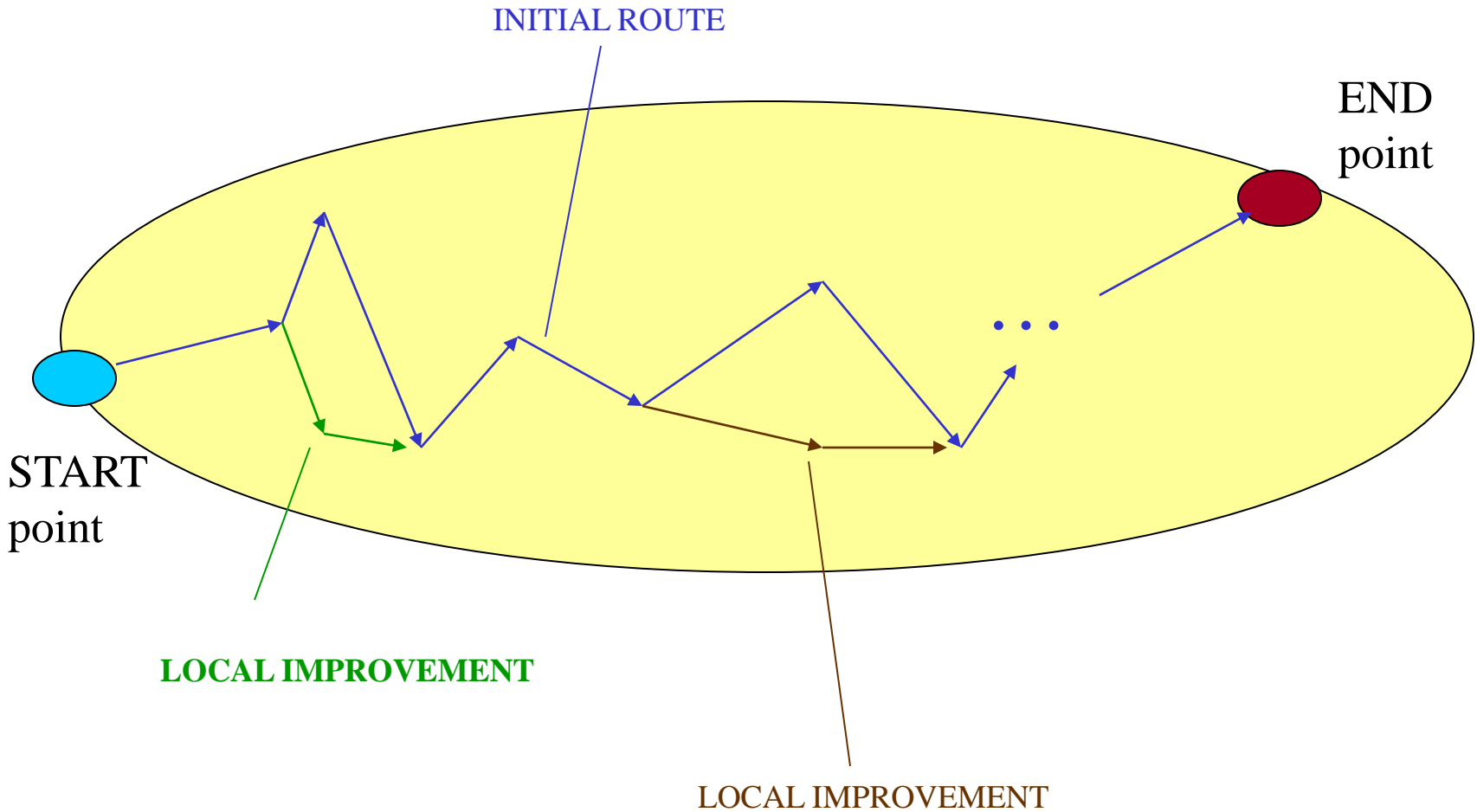
- 1.Partitioning into subproblems**
- 2.Decomposition (extension of an obtained “good” local solutions)  
(examples: dynamic programming, Branch-And-Bound)**
- 3.Grid method with deletion of “bad points”**
- 4.Approximation approach (i.e., approximation of initial problem or its part(s) by more simple construction(s))**

### **LOCAL TECHNIQUES:**

- 1.Local optimization as improvement of a solution or its part**
- 2.Probabilistic steps**
- 3.Greedy approach (selection of the “simple” / “close” / etc. step)**
- 4.Recursion**



Illustration for improvement of a solution (local optimization)



**LECTURE 14-15. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Institute of Physics and Technology (University)**

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**L.14. Basic models of combinatorial optimization II.**

**L.15. Scheme of multicriteria design PSI**

***PLAN:***

1. Basic combinatorial optimization problems:

\*integer nonlinear programming (special formulation & applied example),

\*packing problems & bin-packing problem (illustration),

\*scheduling problems (problem and algorithm for assembly process, 3 examples for one-machine scheduling,

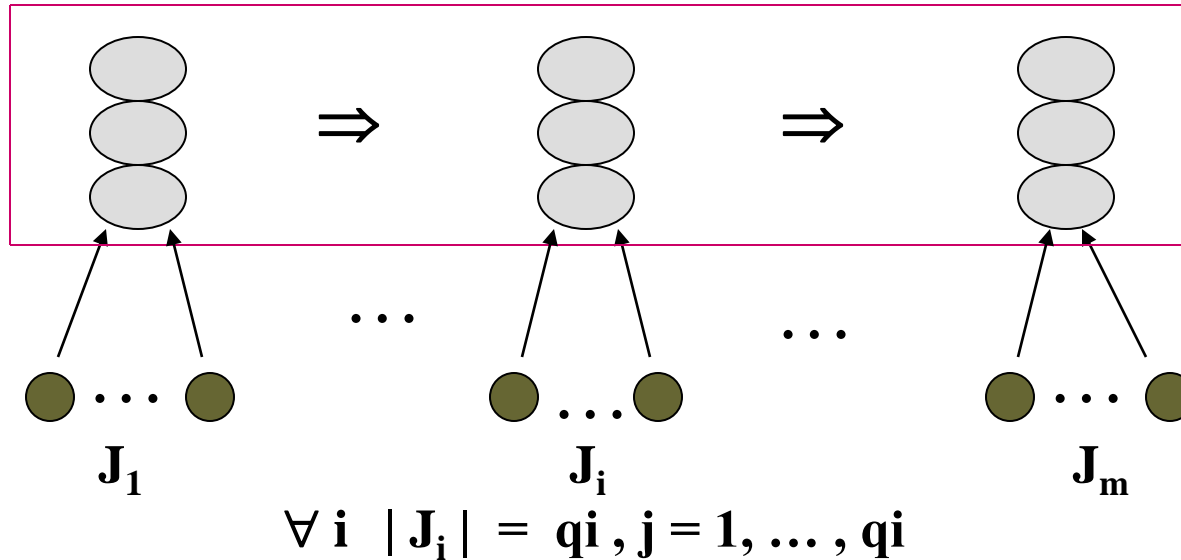
\*maximal clique problem (illustration)

2. Scheme of multicriteria design (PSI – parameter space investigation)

Oct. 2, 2004

# Integer Nonlinear programming (modular design of series system from the viewpoint of reliability)

(by Berman & Ashrafi)



$$\max \quad \prod_{i=1}^m (1 - \prod_{j=1}^{q_i} (1 - p_{ij} x_{ij}))$$

$$s.t. \quad \sum_{i=1}^m \sum_{j=1}^{q_i} d_{ij} x_{ij} \leq b$$

$$\sum_{j=1}^{q_i} x_{ij} \geq 1, \quad i = 1, \dots, m$$

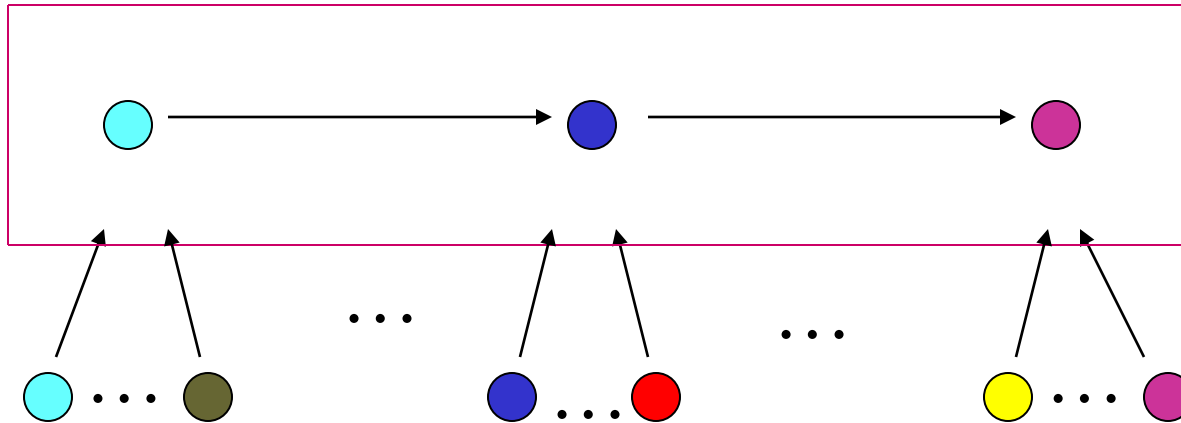
$$x_{ij} \in \{0, 1\}, \quad i = 1, \dots, m, \quad j = 1, \dots, q_i$$

$p_{ij}$  is reliability,  $d_{ij}$  is cost

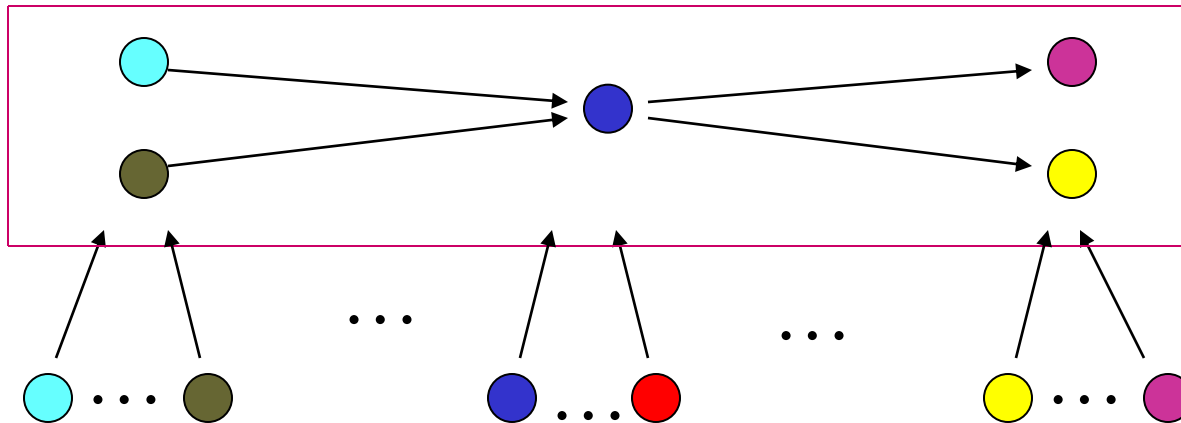
Integer Nonlinear programming (modular design of series system from the viewpoint of reliability)

(by Berman & Ashrafi)

**EXAMPLE 1**  
(series scheme)



**EXAMPLE 2**  
(parallel-series scheme)



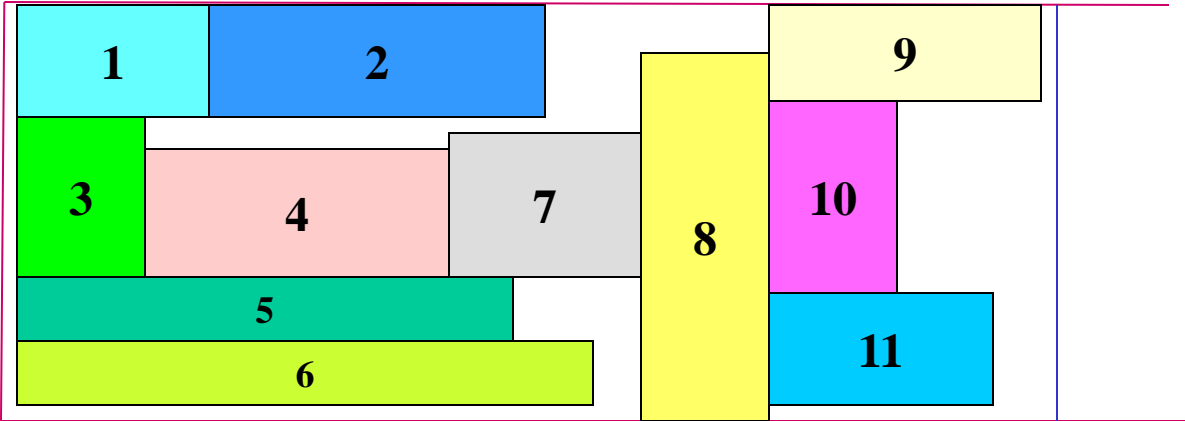
**1. *Branch-And-Bound* method**

**2. *Dynamic programming***

**4. *Heuristics (e.g., reducing the problem to a continuous one)***

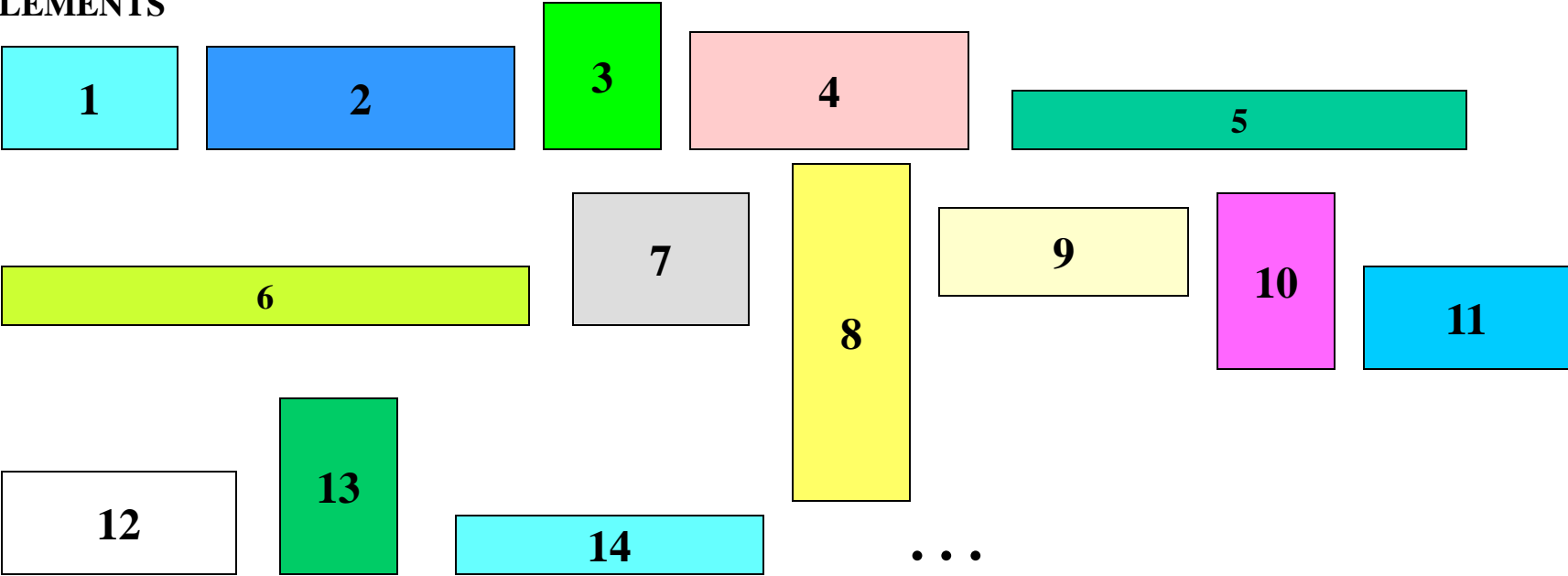
Packing problem (illustration)

REGION FOR  
PACKING



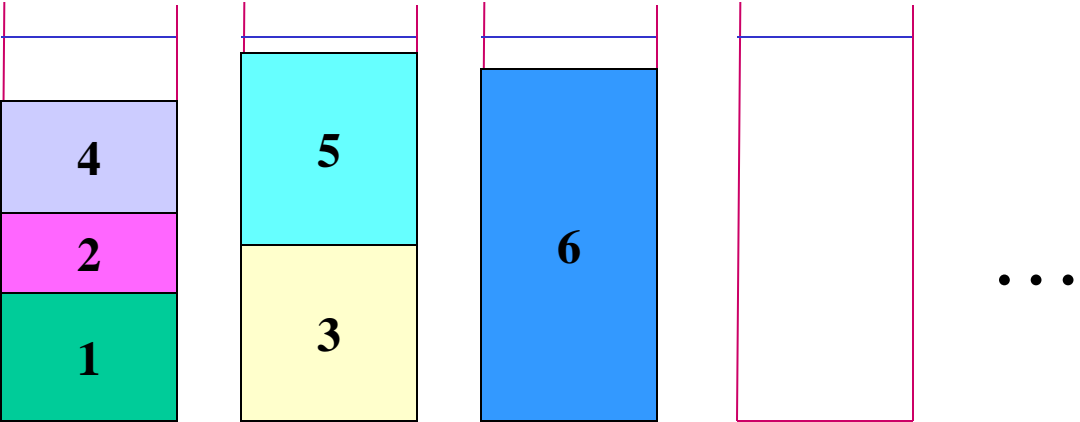
**GOALS:**  
\*Maximum of  
packed  
elements  
\*Minimum of  
free space

ELEMENTS



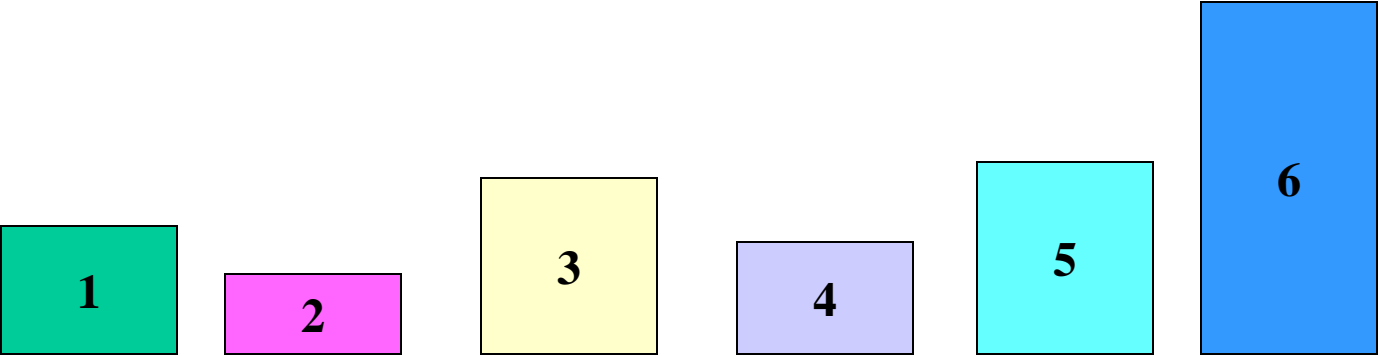
Bin-packing problem (illustration)

CONTAINERS FOR PACKING



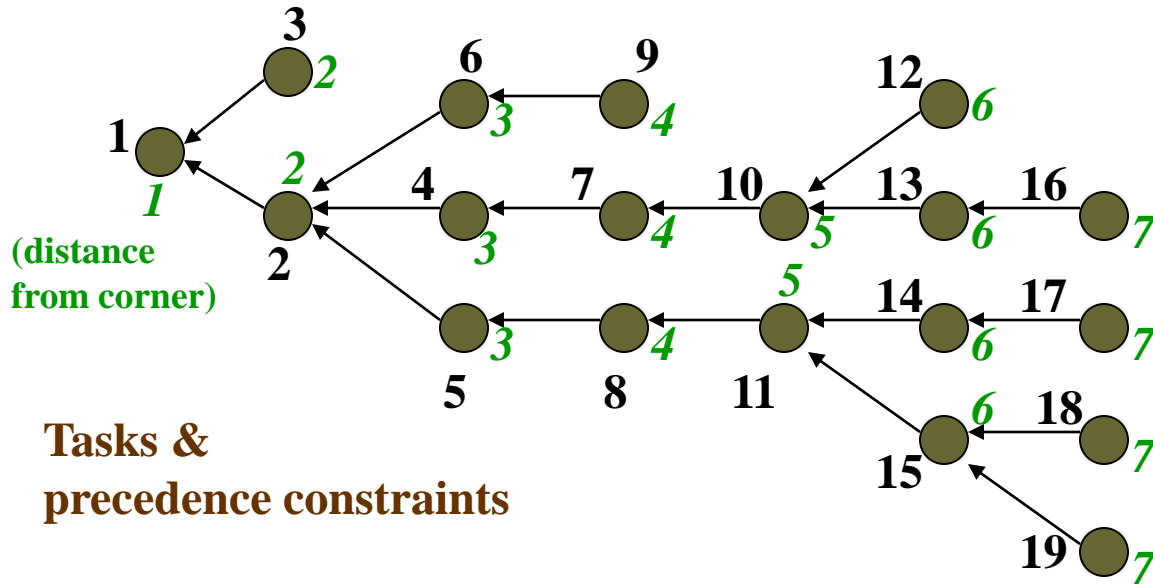
**GOAL:**  
Usage of minimal number of containers

ELEMENTS

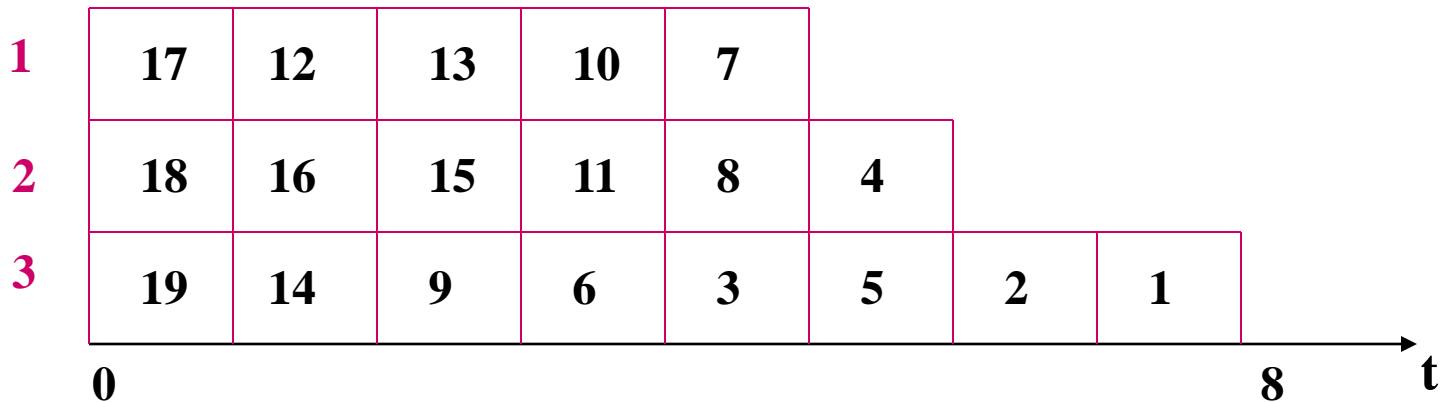


Scheduling problems: illustrative example for assembly process (algorithm of longest tails)

**GOAL:**  
Minimal total complete time



**3 processors:**





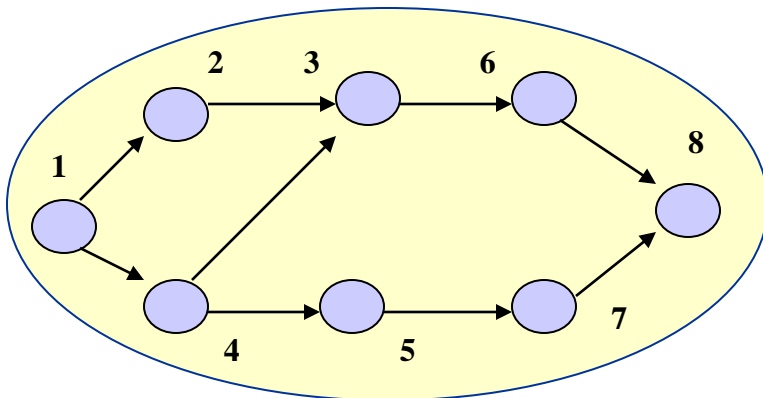
Simple scheduling problems for one machine (processor): Problem Formulation

Initial set of elements:  $R = \{ 1, \dots, i, \dots, n \}$

Schedule (linear ordering):  $S = \langle s[1], \dots, s[i], \dots, s[n] \rangle$   
 $s[i]$  is the element number on position  $i$  in schedule  $S$   
 $f(S)$  is a real-value positive objective function

**Problem is:**

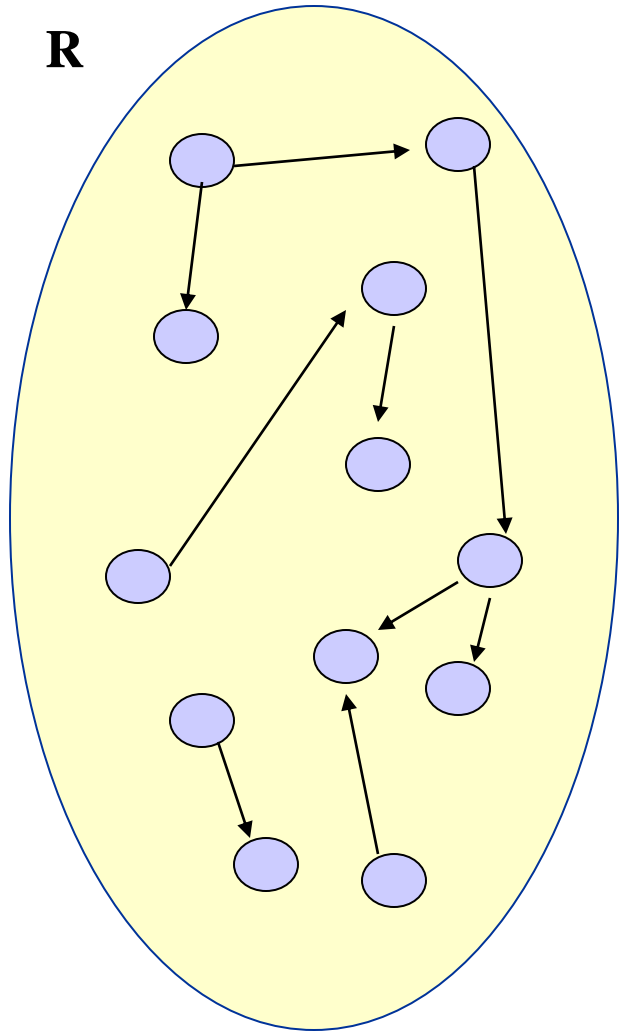
**Find optimal schedule  $S^*$ :  $f(S^*) = \min f(S) \quad \forall S$**



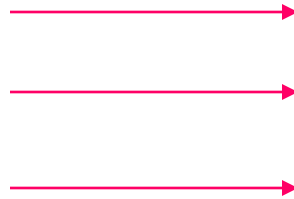
**Precedence constraints:  $G = (R, E)$   
(usually: free-cycle)**

**$E$  is mapping  $R$  into  $R$**

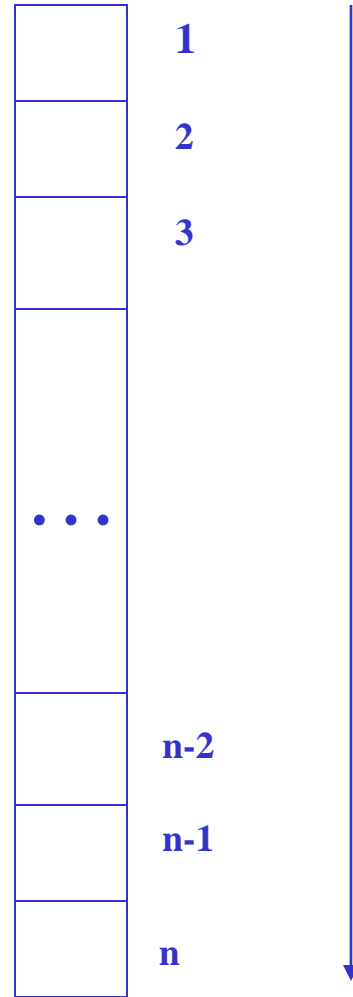
Basic Illustrative Figure



mapping



**R**  $\Rightarrow$  **S**



Basic Schedule Problem [Hardy, Littlewood, Polya, 1930]

Sequence (a) = {  $a_1, a_2, \dots, a_n$  }

Sequence (b) = {  $b_1, b_2, \dots, b_n$  }

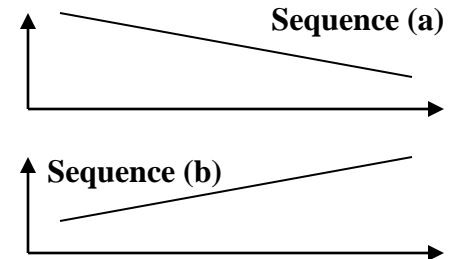
$\Rightarrow$

$$S = \langle b(s[1]), b(s[2]), \dots, b(s[n]) \rangle$$

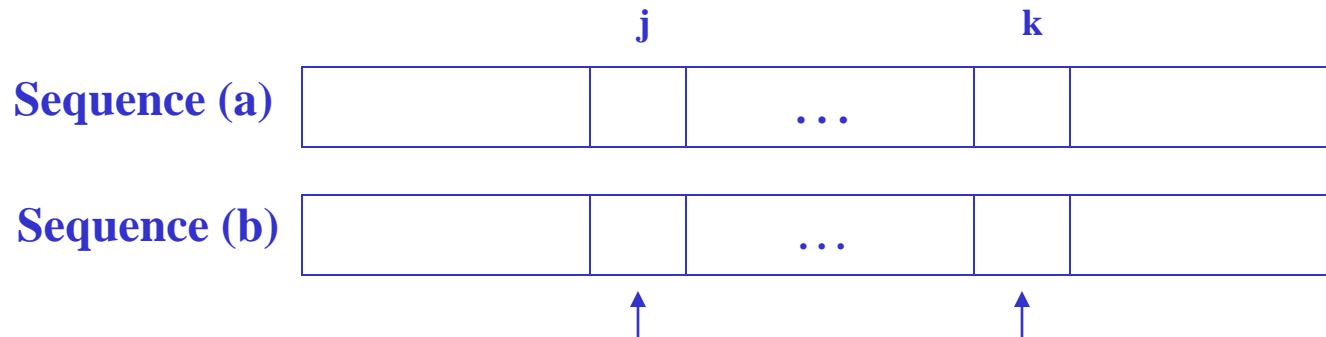
$$f(S) = \sum_{i=1}^n a_i b(s[i]) \Rightarrow \min$$

**LEMMA:**  $S^*$  is an optimal schedule if

- (1) sequence (a) is ordered under non-increasing
- (2) sequence (b) is ordered under non-decreasing



**PROOF:** Let  $a_1 > a_2 > \dots > a_n$  &  $b_1 < b_2 < \dots < b_n$   
 Let exist  $j$  and  $k$  that  $a_j > a_k$  &  $b_j > b_k$  ( $j < k$ )  
 Than we interchange  $b_j$  and  $b_k$  in sequence (b) and get:  
 $\Delta f = a_j b_k + a_k b_j - (a_j b_j + a_k b_k) = (a_j - a_k) (b_k - b_j) \leq 0$ .



## Basic Schedule Problem by Smith (1956)

$$R = \{ 1, \dots, i, \dots, n \}$$

$$S = \langle s[1], \dots, s[i], \dots, s[n] \rangle$$

$$f(S) = \sum_{i=1}^n f_i(C_i) \Rightarrow \min \quad (1)$$

$C_i$  is a completion time for job (task)  $i$   
 $f_i(C_i) = a_i C_i + b_i$  (penalty function,  $a_i > 0$ )

**THEOREM 1:**  $S^*$  is an optimal schedule ( $f(S^*) \leq f(S) \quad \forall S$ ) if

1. exists a real value function  $g(i,j)$  such that  $g(i,j) < g(j,i) \Rightarrow f(S) < f(S^*)$
2. in  $S^*$   $i < j$  if  $g(i,j) < g(j,i)$

**This is Problem 1 (P1)**

## Basic Schedule Problem by Tanaev (1966)

$$R = \{ 1, \dots, i, \dots, n \}$$

$$S = \langle s[1], \dots, s[i], \dots, s[n] \rangle$$

$$f(S) = \sum_{i=1}^n f_i \Rightarrow \min \quad (2)$$

$C_i$  is a completion time for job (task)  $i$

$$f_i(C_i) = a_i \exp(\lambda C_i) \quad (\lambda > 0)$$

**This is Problem 2 (P2)**

Algorithm for Problems 1 and 2: non-decreasing of the indicator

**Problem 1 P1:**  $\omega(i) = \tau_i / a_i$

**Problem 2 P2:**  $\omega(i) = a_i \exp(\lambda \tau_i) / (1 - \exp \lambda \tau_i)$

**NOTE:**

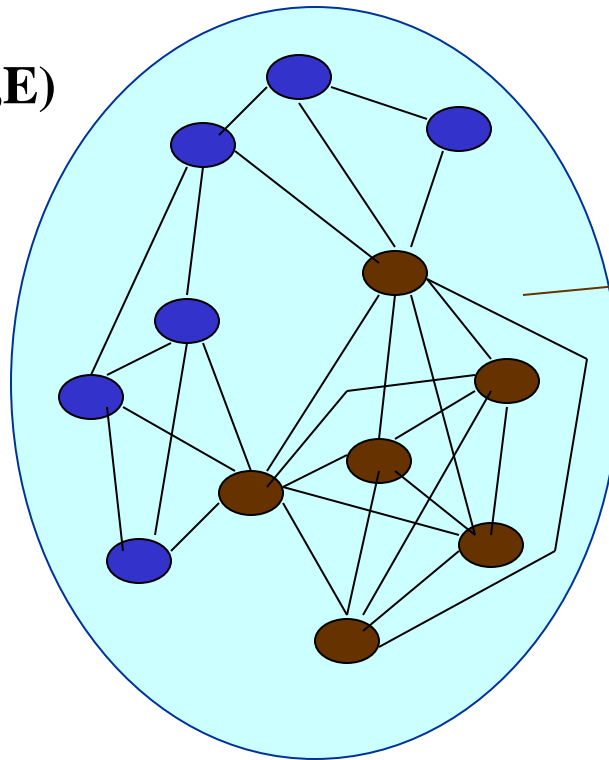
**Considered algorithms (on the basis of ordering) can be used (an extended version) in the case of precedence constraints as tree or parallel-series graph**

Maximal clique problem (illustration)

**Initial graph  $G = (R, E)$ ,  $R$  is set of vertices,  $E$  is set of edges**

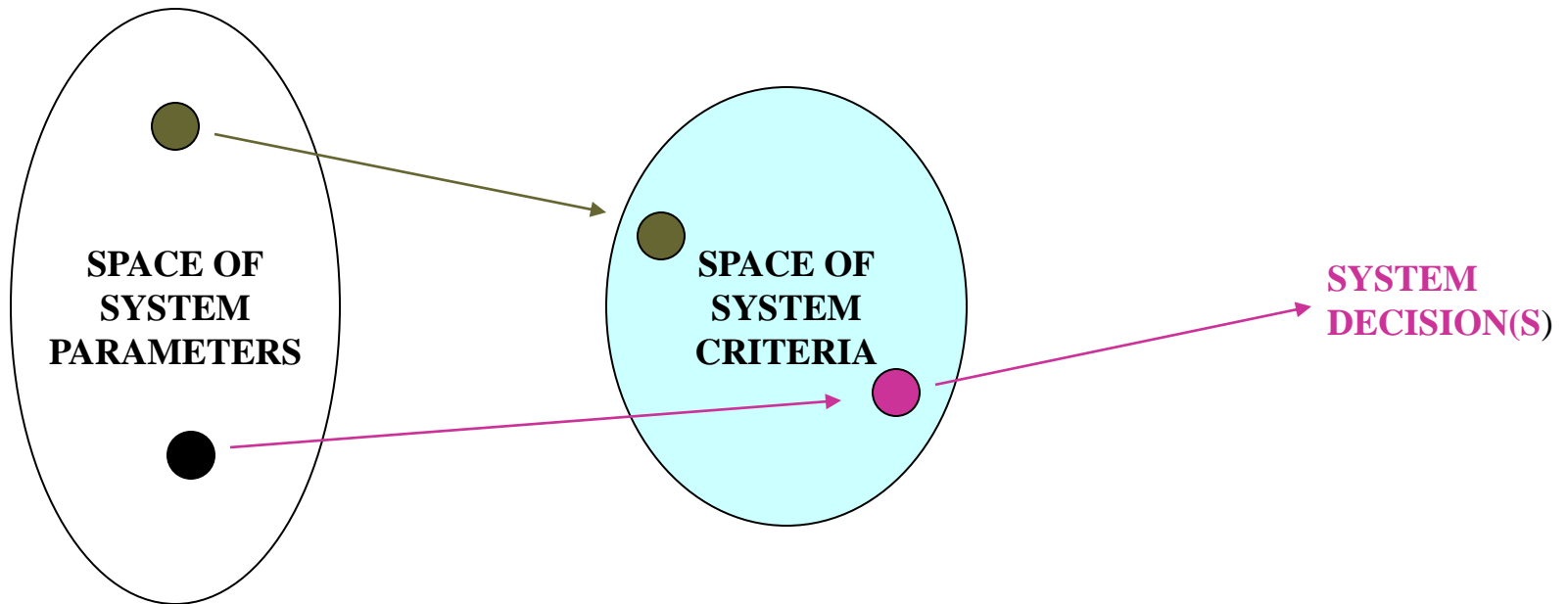
**Problem is: *Find the maximal (by number of vertices) clique (i.e., complete subgraph)***

**$G = (R, E)$**



**Clique  
consisting of  
6 vertices  
(maximal  
complete  
subgraph)**

## Scheme of multicriteria system design





## Hierarchy of requirements / criteria

**1. Ecology, politics**


**2. Economics, marketing**

**2. Technology (e.g., manufacturing issues, maintenance issues)**

**3. Engineering**

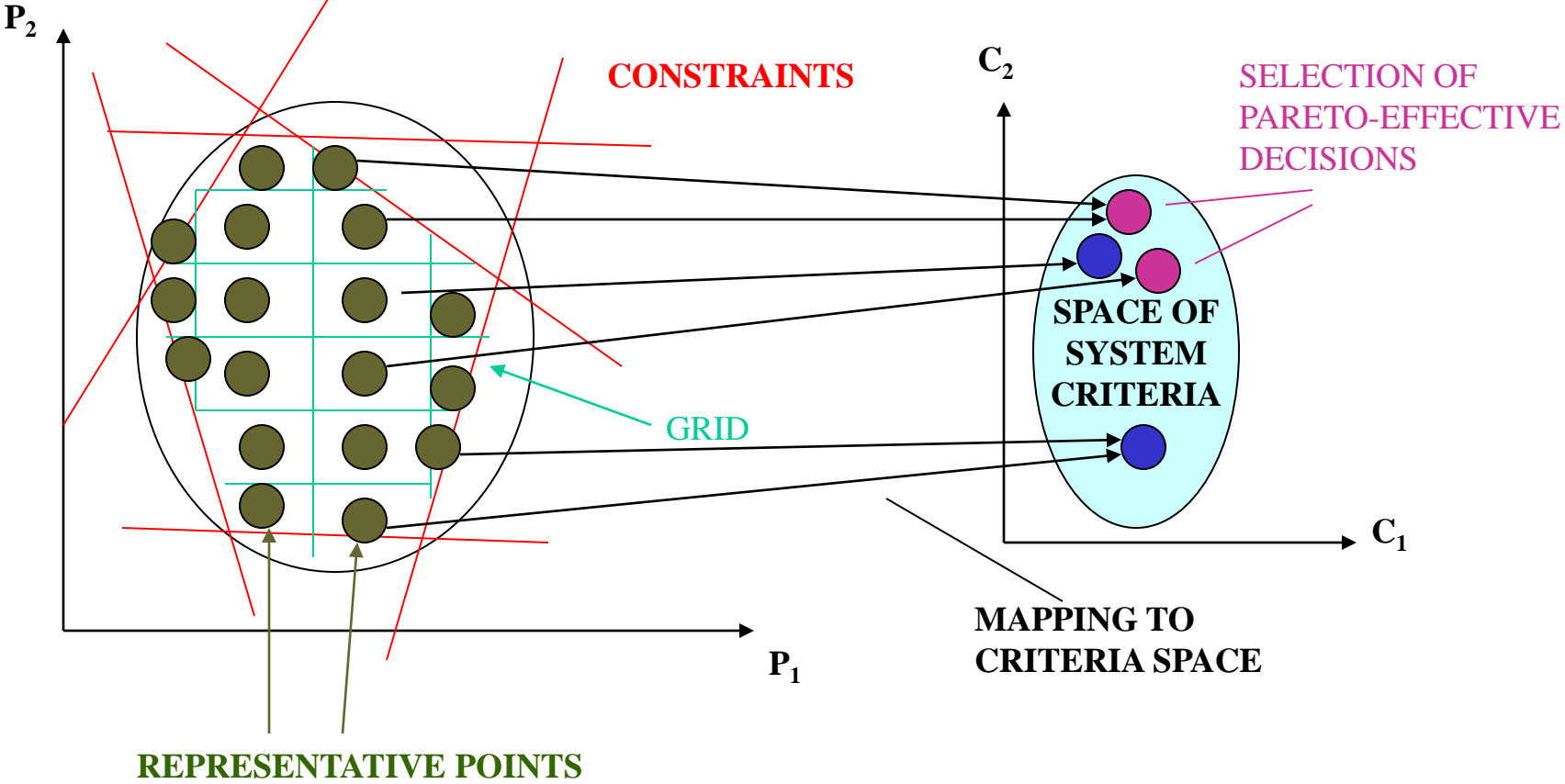


**SPACE OF  
SYTEM  
CRITERIA**



**SPACE OF  
SYTEM  
PARAMETERS**

Scheme of multicriteria design PSI (parameters space investigation) (Sobol & Statnikov)



**LECTURE 16. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Institute of Physics and Technology (University)**

**Mark Sh. Levin**

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**L.16. Interchange techniques, genetic algorithm, etc.**

***PLAN:***

1. Technical documentation (Russian experience as a set of basic documents)
2. Types of interchange techniques
3. Genetic algorithms
4. Multi-objective evolutionary optimization
5. Multidisciplinary optimization
6. Mixed Integer Nonlinear Programming

Oct. 8, 2004

## Technical documentation (Russian experience)

### **BASIC VERSION**

- 1. Preliminary “avan”-project**
- 2. ”Avan”-project**
- 3. Technical suggestion (proposal)**
- 4. Technical project**
- 5. Work-project**
- 6. Report on the 1<sup>st</sup>-stage of utilization  
(including suggestion on system improvement)**
- 7. Resultant report on utilization  
(including suggestion on system improvement)**

### **COMPRESSED VERSION**

- 1. Technical suggestion (proposal)**
- 2. Technical-work project**
- 3. Report on utilization**

## Types of 2-Exchange Technique (Illustration)

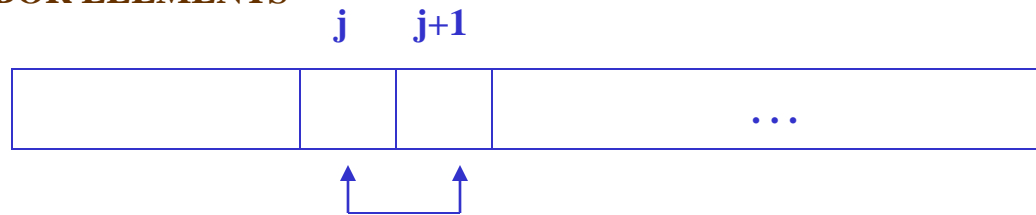
### BASIC VERSION

Sequence



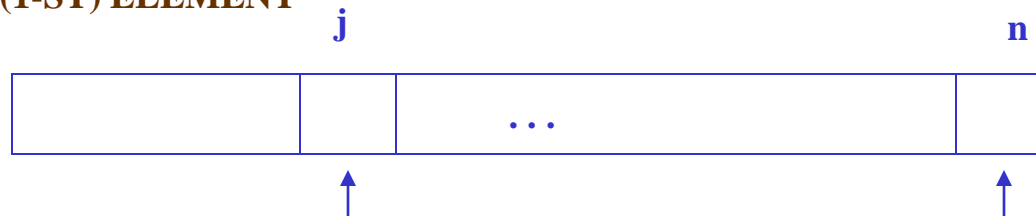
### VERSION FOR NEIGHBOR ELEMENTS

Sequence



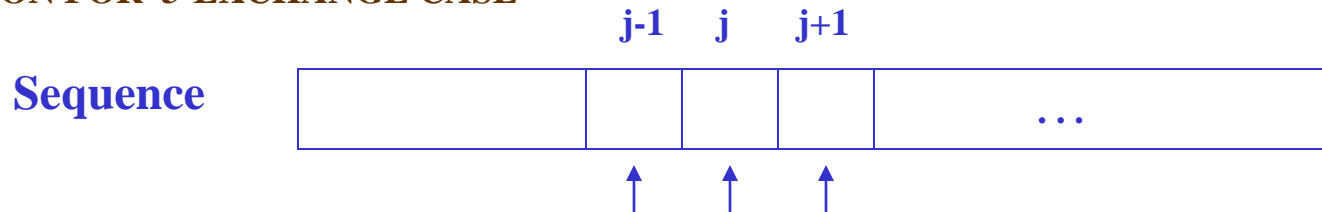
### VERSION WITH LAST (1-ST) ELEMENT

Sequence

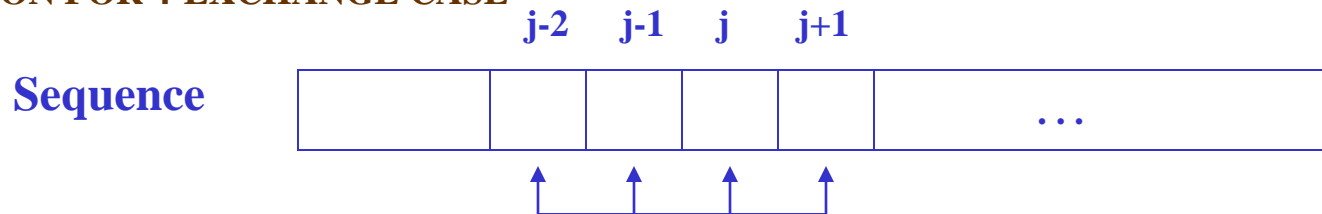


## 3-Exchange Technique & 4-Exchange Technique (Illustration)

### VERSION FOR 3-EXCHANGE CASE



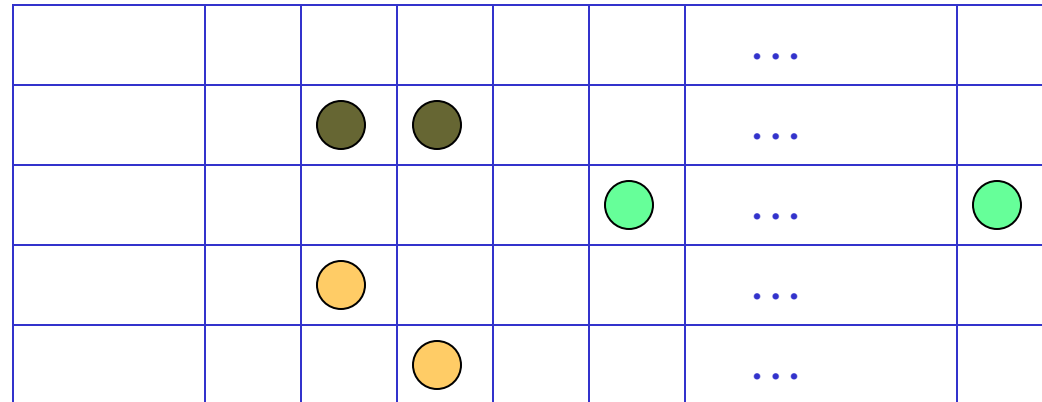
### VERSION FOR 4-EXCHANGE CASE



## Exchange Technique: Two-dimensional Case (Illustration)

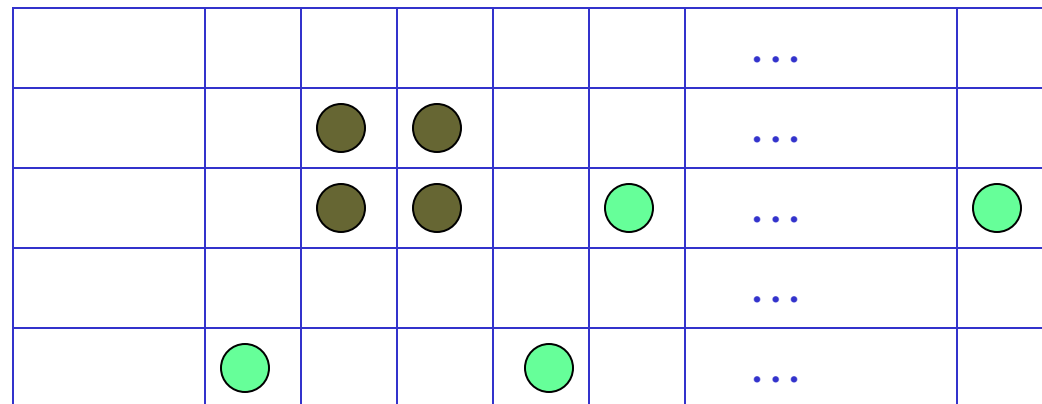
### 2-EXCHANGE CASE

Array



### 4-EXCHANGE CASE

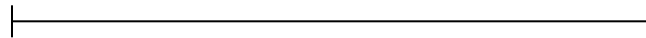
Array



**Basic knapsack problem is:**

$$\begin{aligned} \mathit{max} \quad & \sum_{i=1}^m c_i x_i \\ \mathit{s.t.} \quad & \sum_{i=1}^m a_i x_i \leq b \\ & x_i \in \{0, 1\}, \quad i = 1, \dots, m \end{aligned}$$

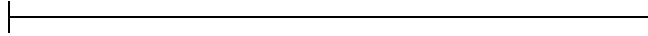
**Solution**  $\mathbf{x}_0 = (x_1, \dots, x_i, \dots, x_m)$





# Genetic algorithms (illustration for knapsack problem)

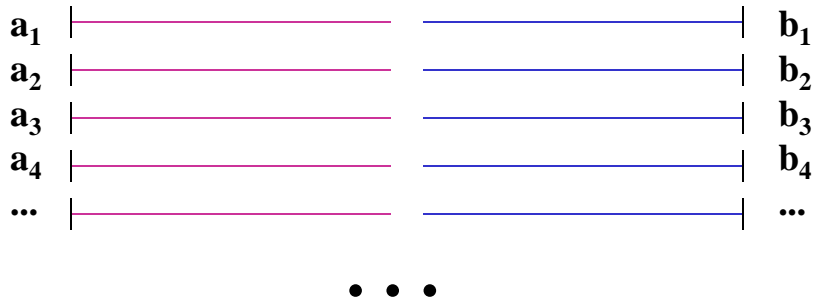
## STEP 1. AN INITIAL SOLUTION $x_0$



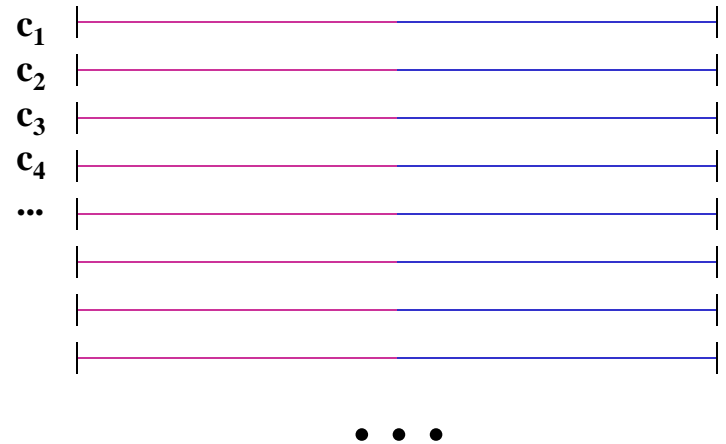
## STEP 2. DIVIDING OF $x_0$ INTO 2 PARTS



## STEP 3. MUTATION (GENERATION OF VERSIONS): \*exchange of elements, \*change of elements, etc.

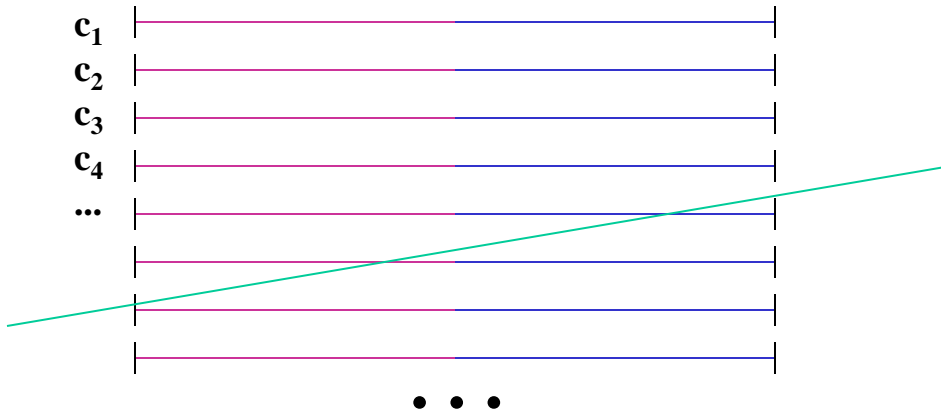


## STEP 4. GENERATION OF NEW SOLUTIONS BY PAIRS ( $a_i, b_j$ )

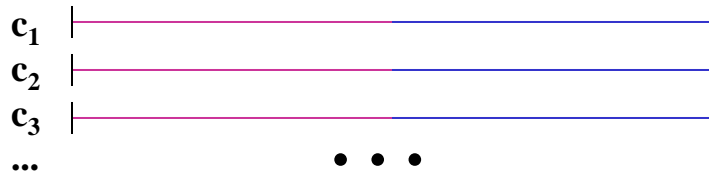


## Genetic algorithms (illustration for knapsack problem)

**STEP 5. Deletion of some solutions by resource constraint ( $\leq b$ )**



**STEP 6. Selection of the best solution(s)**



**Selection by two ways:**

**1. Selection by utility function**

**2. Selection of Pareto-effective solutions. This is**

**Multi-Objective Evolutionary Optimization**

**STEP 7. Repetition of steps 2, 3, 4, 5, and 6 for selected solutions**

**General optimization model  
with constraints corresponding to certain disciplines  
(e.g., weight, reliability, etc.):**

$$\begin{array}{ll} \max f(\mathbf{x}) & (\text{or } \text{extr } f(\mathbf{x})) \\ \text{subject to} & \\ \varphi_1(\mathbf{x}) \leq \mathbf{W} & \text{weight} \\ \mathbf{B}_1 \leq \varphi_2(\mathbf{x}) \leq \mathbf{B}_2 & \text{height} \\ \mathbf{C}_1 \leq \varphi_3(\mathbf{x}) \leq \mathbf{C}_2 & \text{temperature} \\ \mathbf{D}_1 \leq \varphi_4(\mathbf{x}) \leq \mathbf{D}_2 & \text{reliability} \\ & \dots \\ \varphi_k(\mathbf{x}) \leq \mathbf{0} & \end{array}$$

$\varphi_j(\mathbf{x})$  is a constraint function ( $1 \leq j \leq k$ )

Int. Society for Structural and Multidisciplinary Optimization  
(civil engineering, ship engineering, marine engineering, aerospace engineering)

[//www.issmo.org](http://www.issmo.org)

Optimal design of structures (including issues of fluids)

**Generalized optimization model that involves integer & continuous variables:**

*min*  $F(\mathbf{x}, \mathbf{y})$

*subject to*

$$\mathbf{h}(\mathbf{x}, \mathbf{y}) = \mathbf{0}$$

$$\mathbf{g}(\mathbf{x}, \mathbf{y}) \leq \mathbf{0}$$

*where*  $\mathbf{x}$  is vector of binary variables (selection of subsystems)

$\mathbf{y}$  is vector of continuous variables / parameters (e.g., size)

**\*Global optimization**

**\*Process Systems Engineering (chemical engineering, etc.)**

**\*Prof. C.A. Floudas (Princeton Univ, Chemical Engineering)**

**\*Prof. I.E. Grossmann (Carnegie Mellon Univ., Chemical Engineering)**

***1.Branch-And-Bound method***

***2.Combinatorial hybrid techniques***

***3.Gradient method***

***4.Interior point method***

**LECTURE 17-18. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

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**L.17. Basic models of combinatorial optimization III.**

**L.18. Basic models of combinatorial optimization IV.**

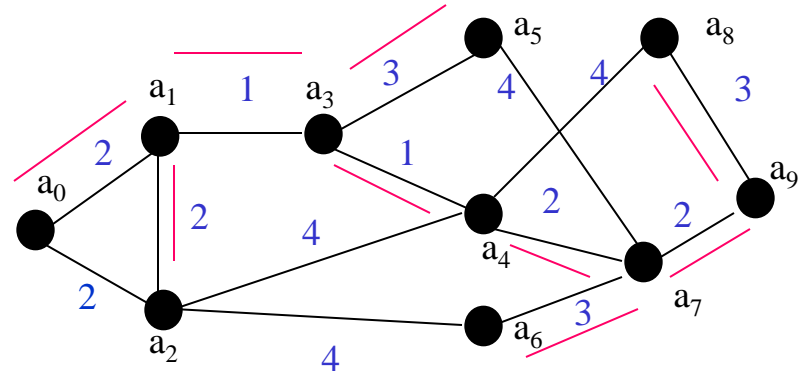
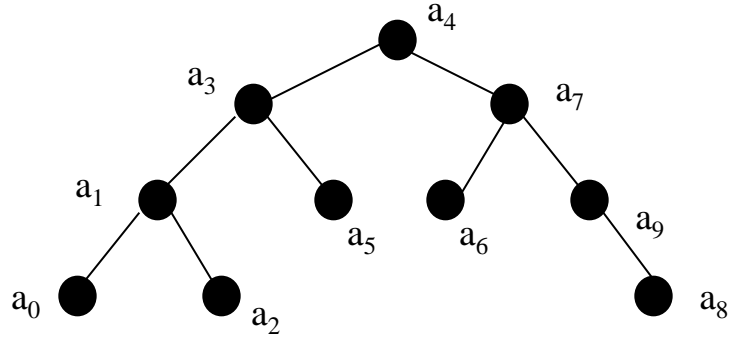
***PLAN:***

- 1.Spanning (illustration): \*spanning tree, \*minimal Steiner problem, \*2-connected graph
- 2.TSP, assignment (formulations)
- 3.Multiple matching (illustration) , usage in processing of experimental data
- 4.Graph coloring problem, covering problems (illustration and applications)
- 5.Alignment, maximal substructure, minimal superstructure (illustration, applications)
- 6.Timetabling

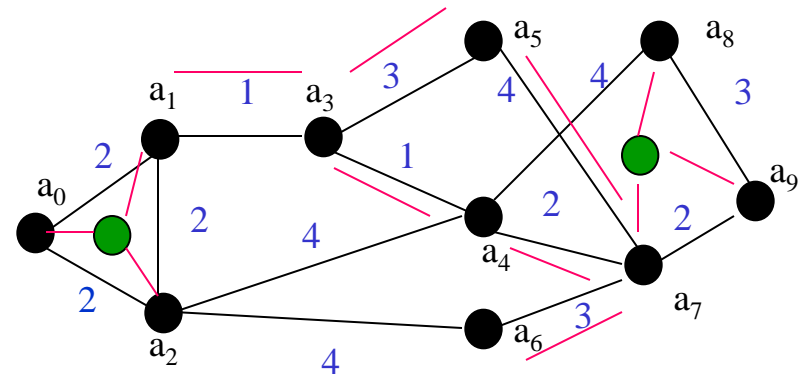
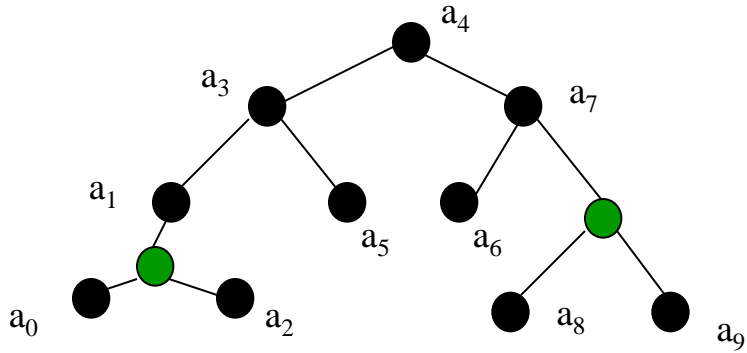
Oct. 9, 2004

## Spanning (illustration): 1-connected graph

**Spanning tree (length = 19):**

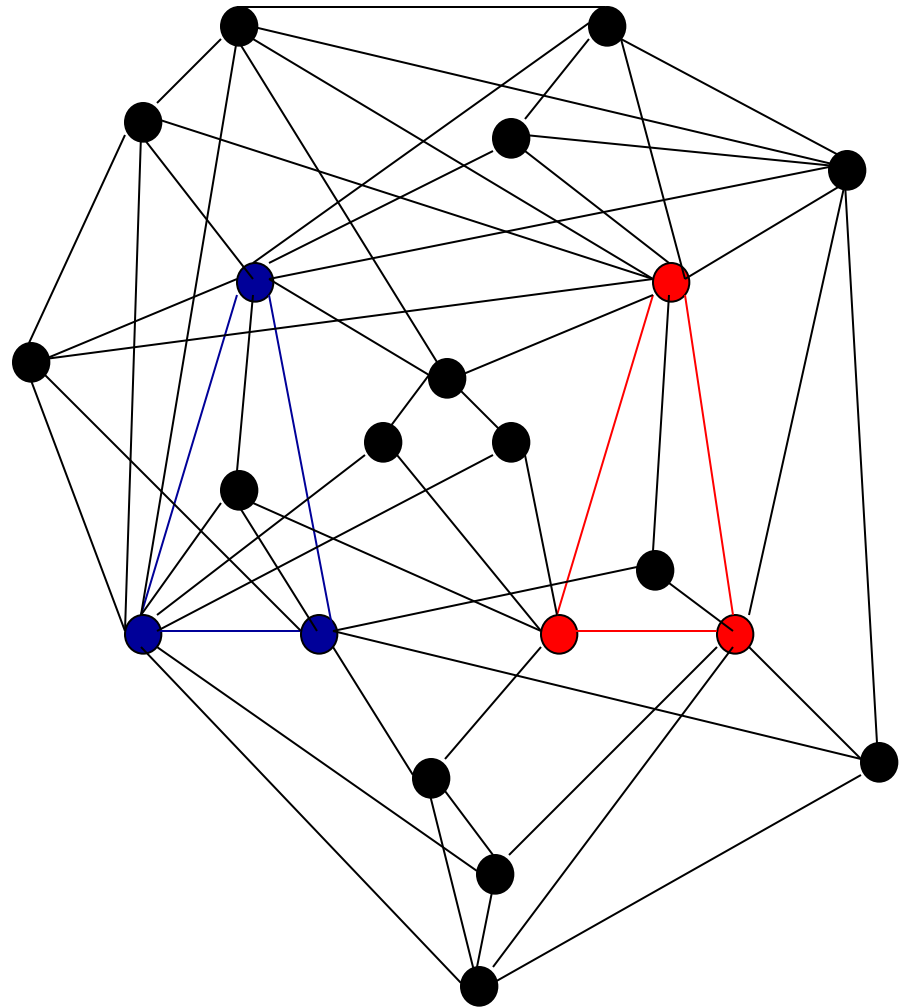


**Steiner tree (example):**



Spanning (illustration): 2-connected graph

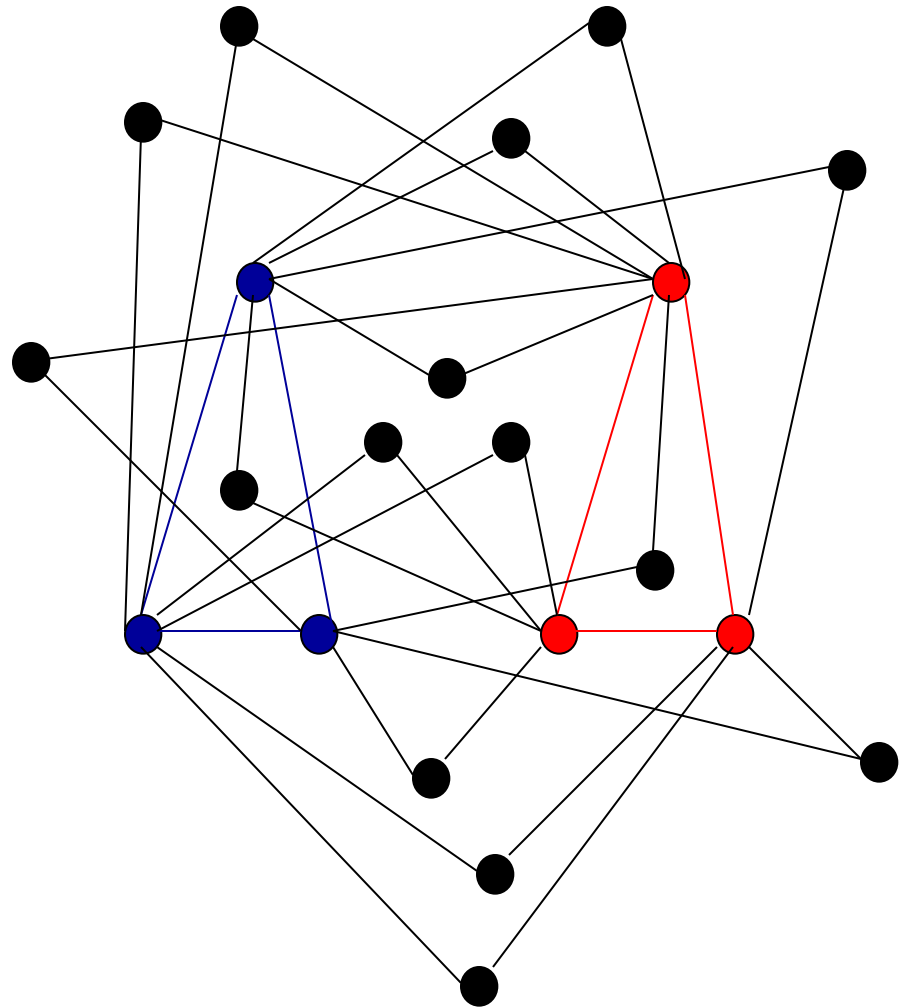
**Spanning by two-connected graph:**  
*Revelation of two 3-node cliques*  
*(centers)*



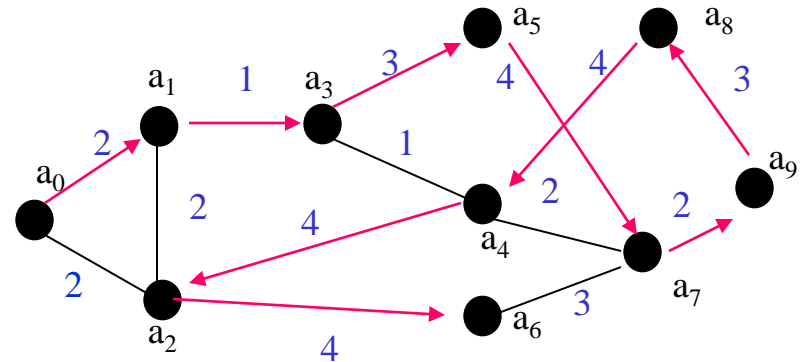
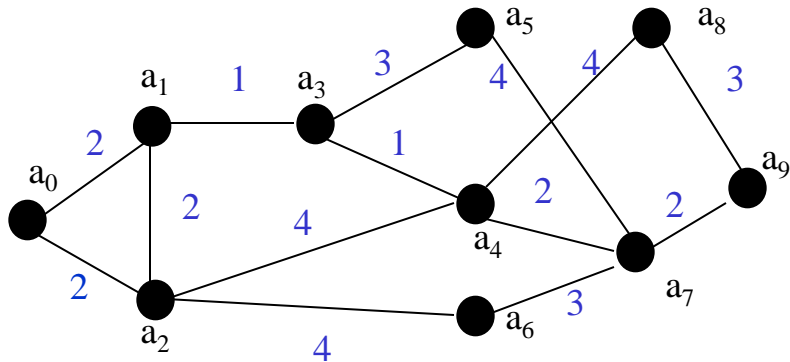


Spanning (illustration): 2-connected graph

**Spanning by two-connected graph:**  
*Connection of each other node  
with the two centers*



## Traveling salesman problem



$$L = \langle a_0, a_1, a_3, a_5, a_7, a_9, a_8, a_4, a_2, a_6 \rangle$$

$$2+1+3+4+2+2+3+4+4+4$$

### FORMULATION:

Set of cities:  $A = \{ a_1, \dots, a_i, \dots, a_n \}$

Distance between cities  $i$  and  $j$ :  $\rho(a_i, a_j)$

$\Pi$  is set of permutations of elements of  $A$ ,  
permutation

$$s^* = \langle a(s^*[1]), \dots, a(s^*[i]), \dots, a(s^*[n]) \rangle$$

$$\min_{(s \in \Pi)} f(s) = f(s^*)$$

$$f(s) = \sum_{i=1}^{n-1} \rho(a(s[i]), a(s[i+1])) + \rho(a(s[n]), a(s[1]))$$

## Traveling salesman problem

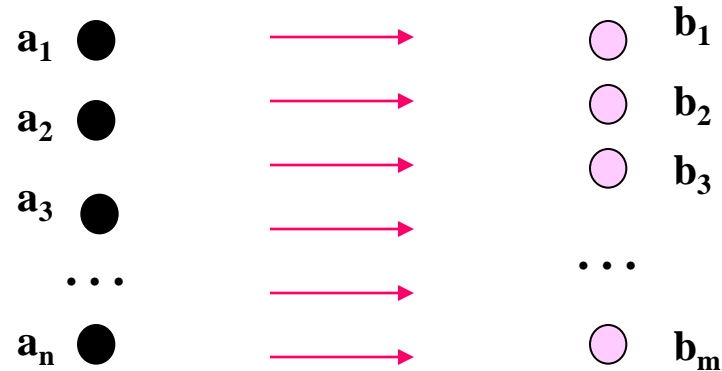
### **ALGORITHMS:**

- 1. Greedy algorithm**
  - 2. On the basis of minimal spanning tree**
  - 3. Branch-And-Bound**
- Etc.**

### **VERSIONS (many):**

- 1. Cycle or None**
  - 2. m-salesmen**
  - 3. asymmetric one (*i.e., distances*  
 $\rho(a_i, a_j)$  and  $\rho(a_j, a_i)$  are different ones)**
  - 4. Various spaces (metrical space, etc.)**
  - 5. Multicriteria problems**
- Etc.**

## Assignment problem



### FORMULATION:

Set of elements:  $A = \{ a_1, \dots, a_i, \dots, a_n \}$

Set of positions  $B = \{ b_1, \dots, b_j, \dots, b_m \}$  (now let  $n = m$ )

Effectiveness of pair  $i$  and  $j$  is:  $z(a_i, b_j)$

$\Pi = \{s\}$  is set of permutations (assignment) of elements of  $A$  into position set  $B$ :

$s^* = \langle (s^*[1]), \dots, (s^*[i]), \dots, (s^*[n]) \rangle$ , i.e., element  $a_i$  into position  $s[i]$  in  $B$

The goal is:

$$\max \sum_{i=1}^n z(i, s[i])$$

## Assignment problem

### **ALGORITHMS:**

**1. Polynomial algorithm (  $O(n^3)$  )**

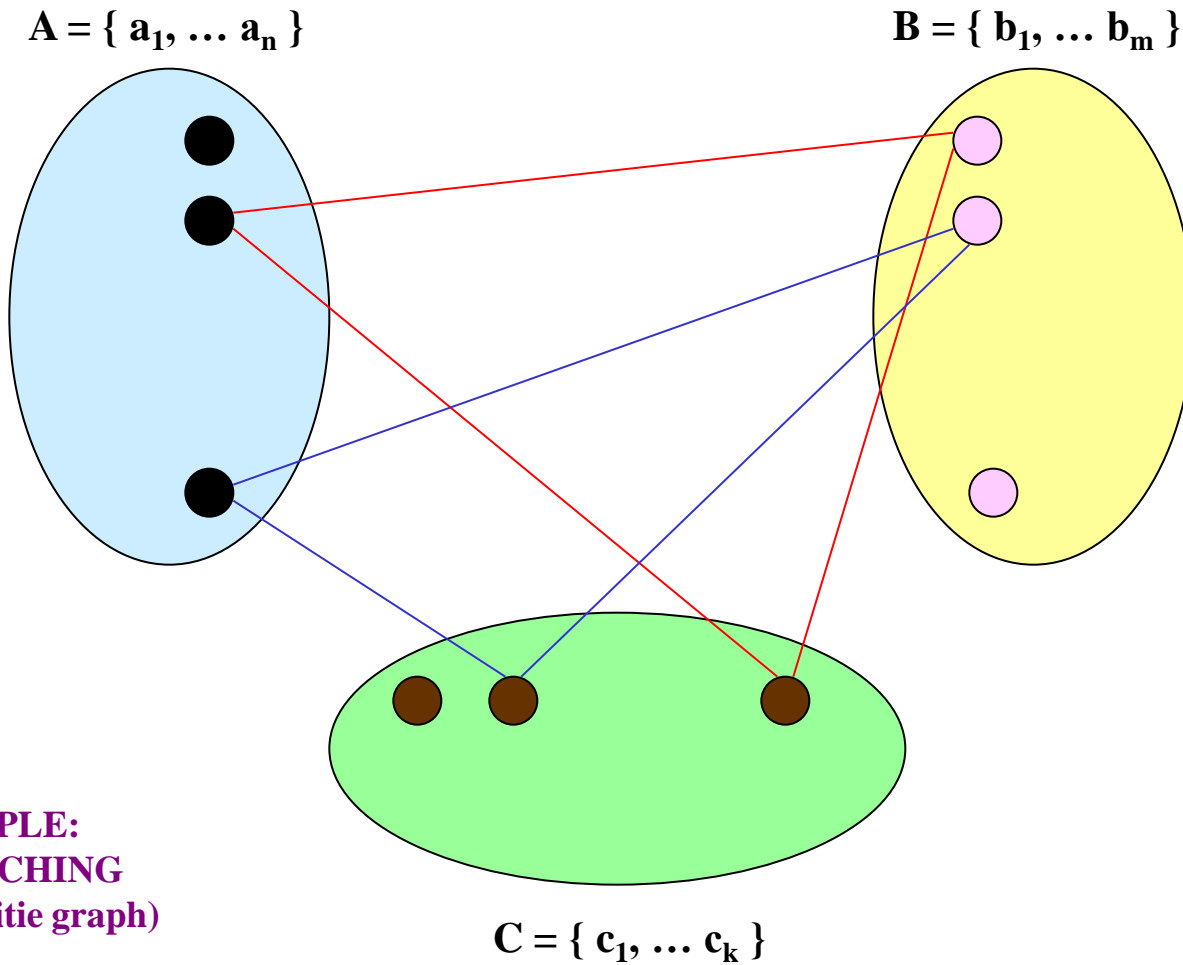
### **VERSIONS:**

**1. *Min max* problem**

**2. Multicriteria problems**

**Etc.**

# Multiple matching problem



**EXAMPLE:**  
**3-MATCHING**  
(3-partite graph)

## Multiple matching problem

### **ALGORITHMS:**

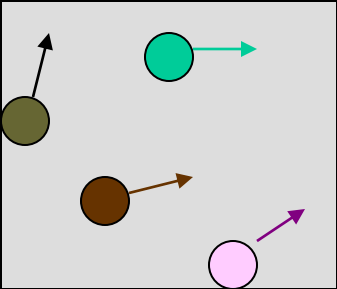
- 1. Heuristics**  
(e.g., greedy algorithms, local optimization, hybrid heuristics)
- 2. Enumerative algorithms** (e.g., Branch-And-Bound method)
- 3. Morphological approach**

### **VERSIONS:**

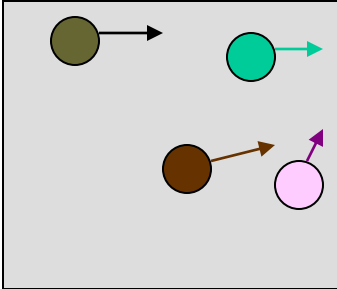
- 1. Dynamical problem** (multiple track assignment)
  - 2. Problem with errors**
  - 4. Problem with uncertainty** (probabilistic estimates, fuzzy sets)
- Etc.**

Recent applied example: usage of assignment problem(s) to define velocity of particles

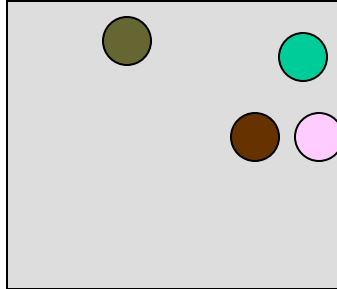
FRAME 1



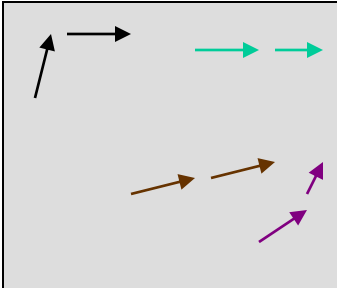
FRAME 2



FRAME 3



VELOCITY SPACE





Recent applied example: usage of assignment problem(s) to define velocity of particles

## **MODELS & ALGORITHMS:**

**1. Correlation functions (from radioengineering: signal processing)**

**2. Assignment problem between two neighbor frames**

**(algorithm schemes: genetic algorithms, other algorithms for assignment problems, hybrid schemes)**

**3. Multistage assignment problem (e.g., examination of 3 frames, etc.)**

**(algorithm schemes: genetic algorithms, other algorithms for assignment problems, hybrid schemes)**

## **VERSIONS :**

**1. Basic problem**

**2. With errors**

**3. Under uncertainty**

**Etc.**

Recent applied example: usage of assignment problem(s) to define velocity of particles

**APPLICATIONS (air/ water environments):**

- 1. Physical experiments**
- 2. Climat science**
- 3. Chemical processes**
- 4. Biotechnological processes**

**Contemporary sources:**

- 1. PIV systems (laser/optical systems)**
  - 2. Sattelite photos**
  - 3. Electronic microscope**
- Etc.**

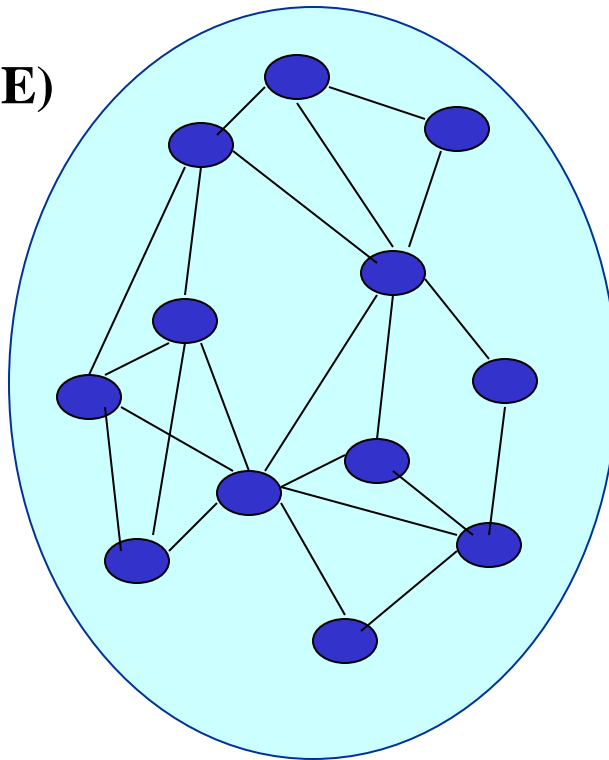
## Graph coloring problem (illustration)

**Initial graph  $G = (A, E)$ ,  $A$  is set of vertices,  $E$  is set of edges**

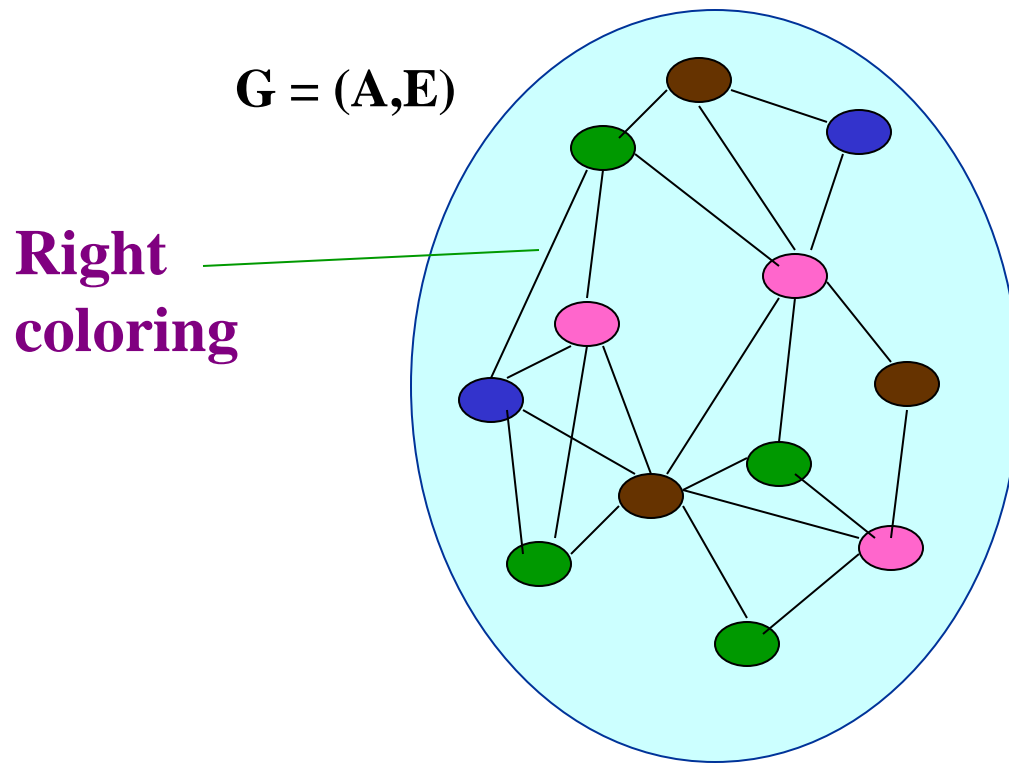
**Problem is:**

*Assign a color for each vertex with minimal number of colors under constraint: neighbor vertices have to have different colors*

**$G = (A, E)$**



Graph coloring problem (illustration)

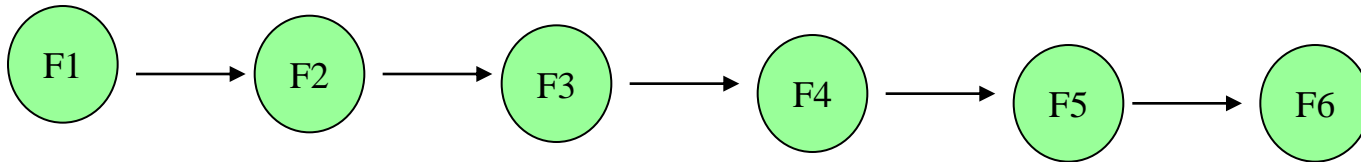
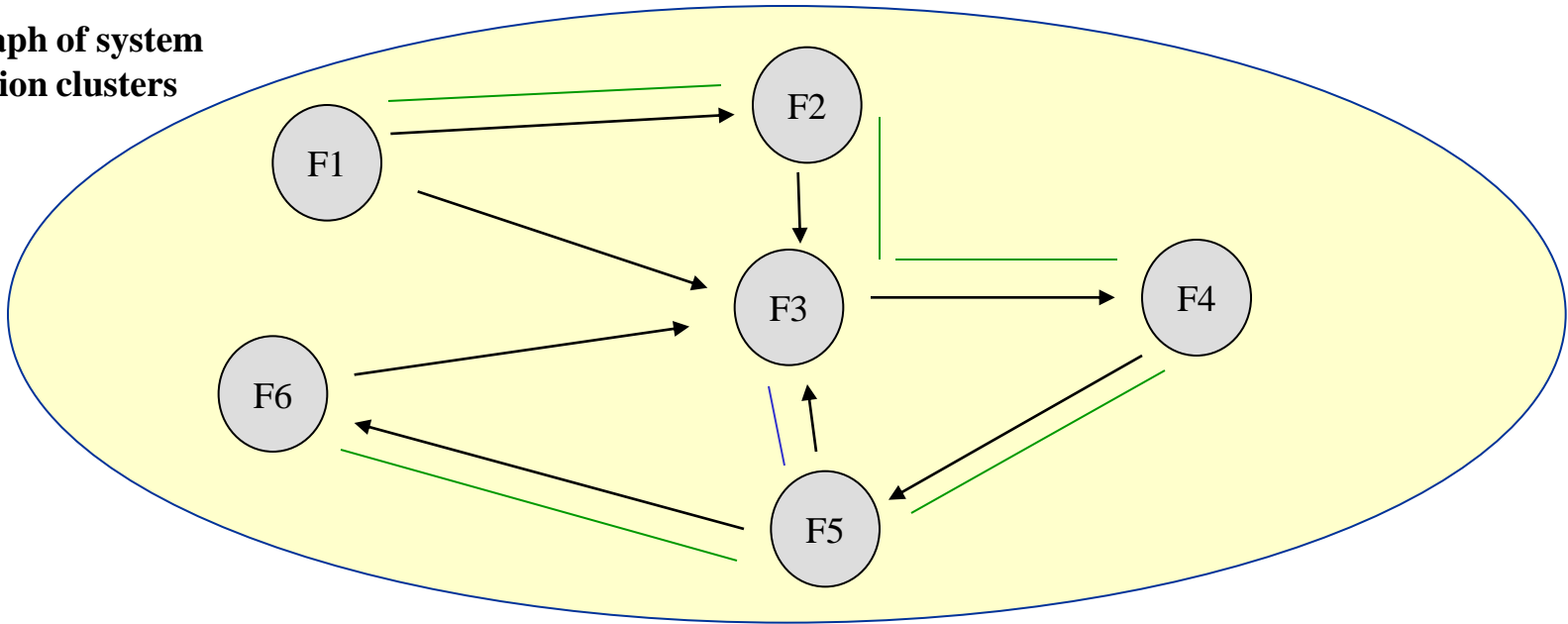


**APPLICATIONS:**

- 1. Assignment of registers in compilation process (A.P. Ershov, 1959)**
- 2. Frequency allocation / channel assignment**  
(static problem, dynamic problem, etc.)
- 3. VLSI design**  
etc.

Example: system function clusters and covering by chains (covering of vertices)

Digraph of system function clusters

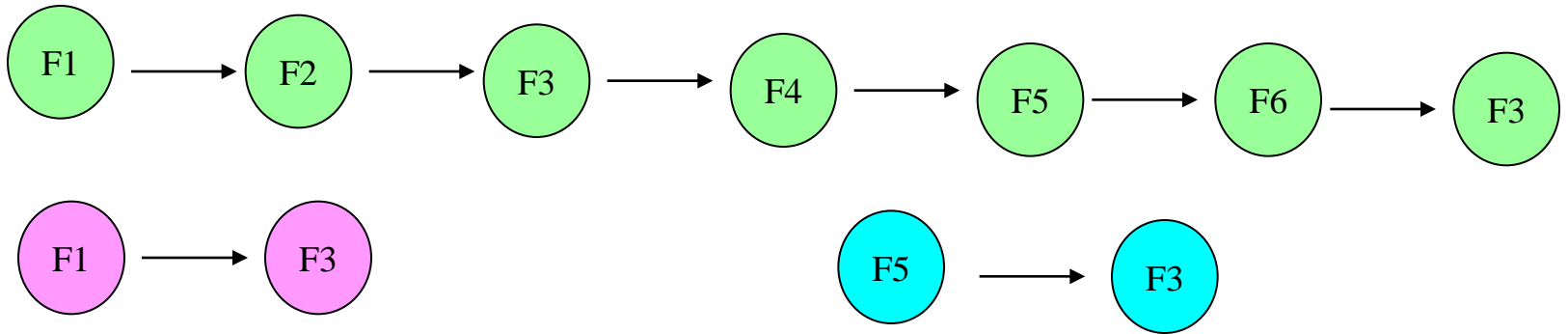
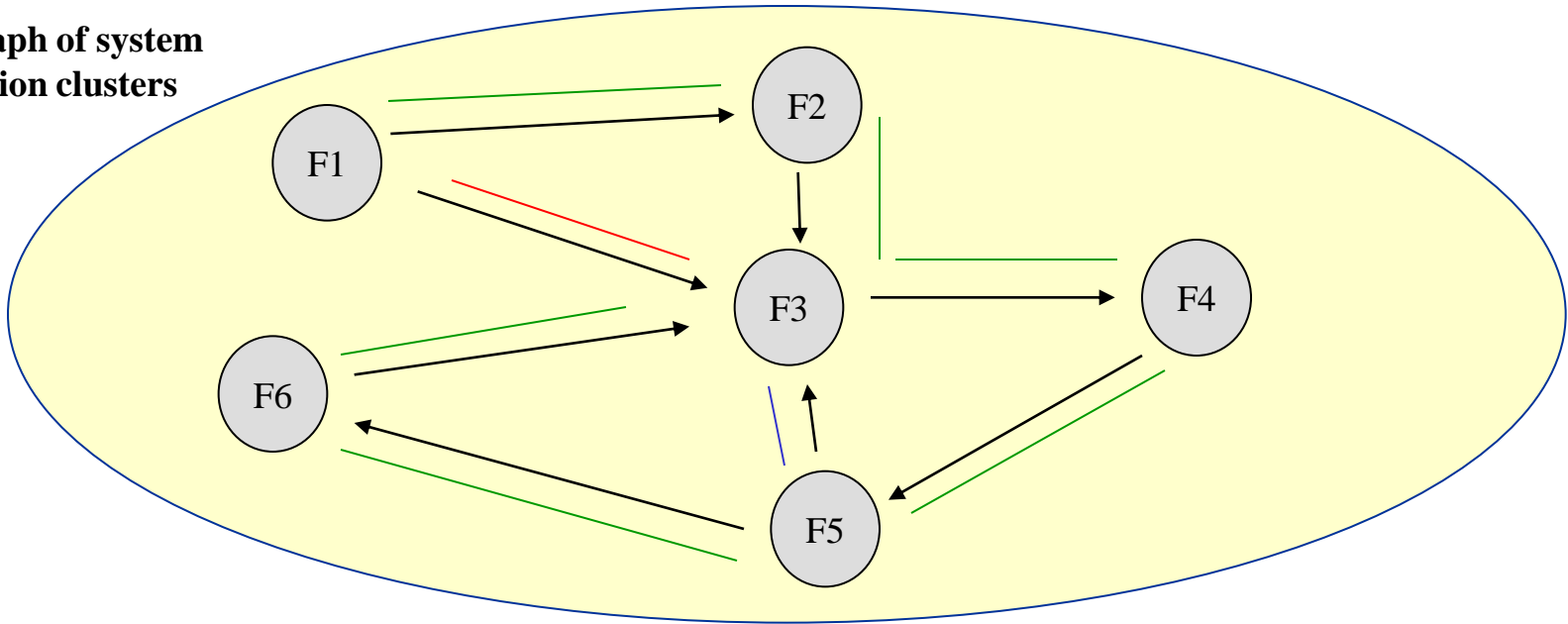


**THE LONGEST PATH**

**Application: system testing**

Example: system function clusters and covering by chains (covering of arcs)

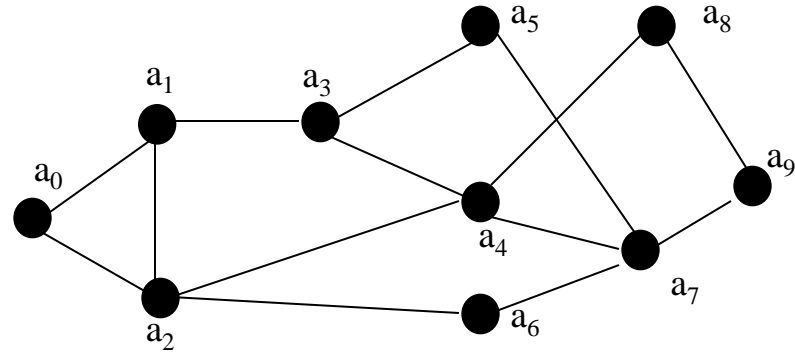
Digraph of system function clusters



APPLICATION: TESTING OF "CHANGES"

## Illustration: covering by cliques

### Basic graph



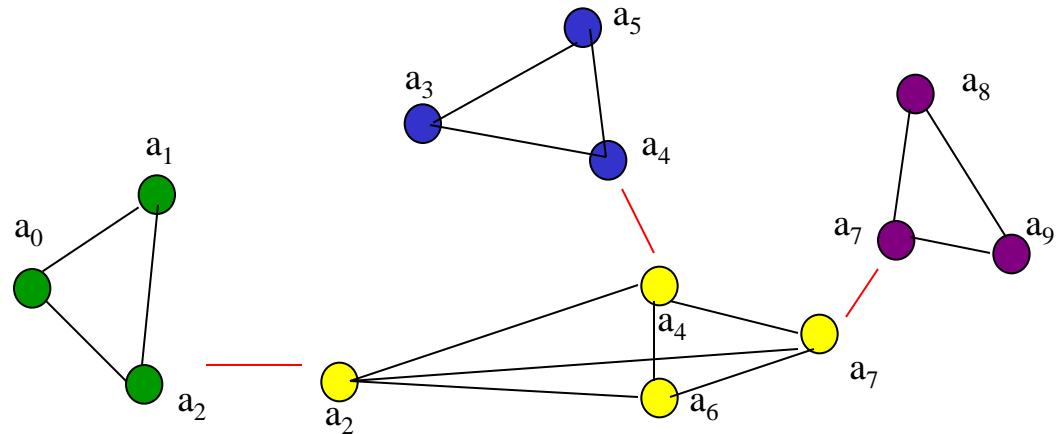
### Cliques (a version):

$$C_1 = \{ a_0, a_1, a_2 \}$$

$$C_2 = \{ a_3, a_5, a_4 \}$$

$$C_3 = \{ a_7, a_8, a_9 \}$$

$$C_4 = \{ a_2, a_4, a_6, a_7 \}$$



APPLICATION: ALLOCATION OF SERVICE (e.g., communication centers)



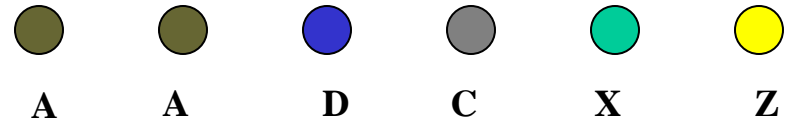
## Alignment (illustration)

**CASE OF 2 WORDS:**

**Word 1**



**Word 2**



**ALIGNMENT PROBLEM:** *minimal additional elements*

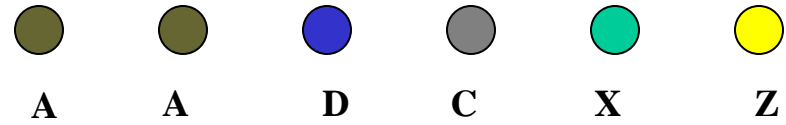
Alignment (illustration)

**CASE OF 2 WORDS:**

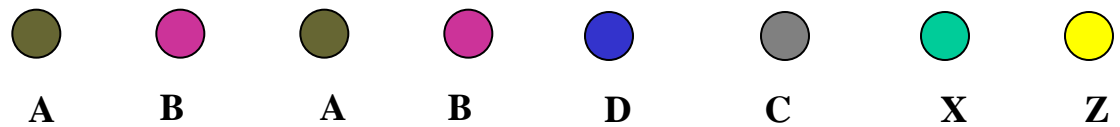
**Word 1**



**Word 2**



***Minimal Superstructure***



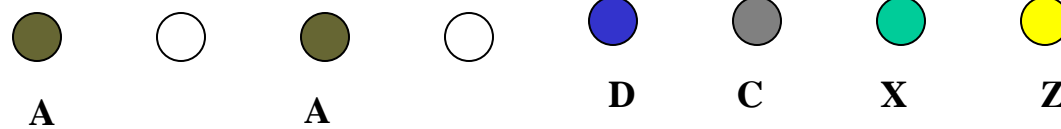
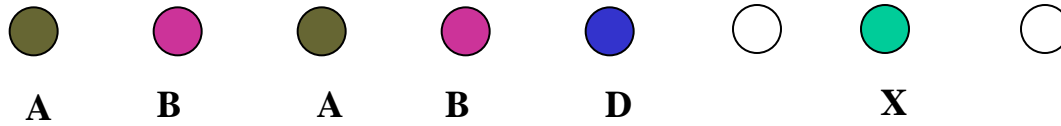
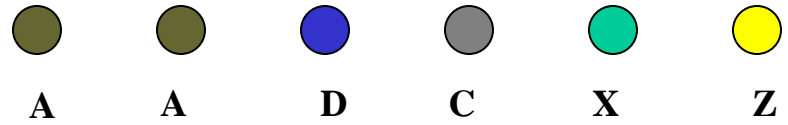
# Alignment (illustration)

**CASE OF 2 WORDS:**

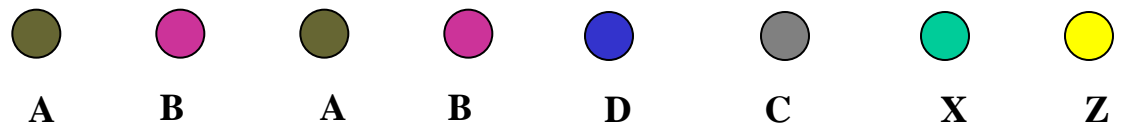
**Word 1**



**Word 2**



**Superstructure**



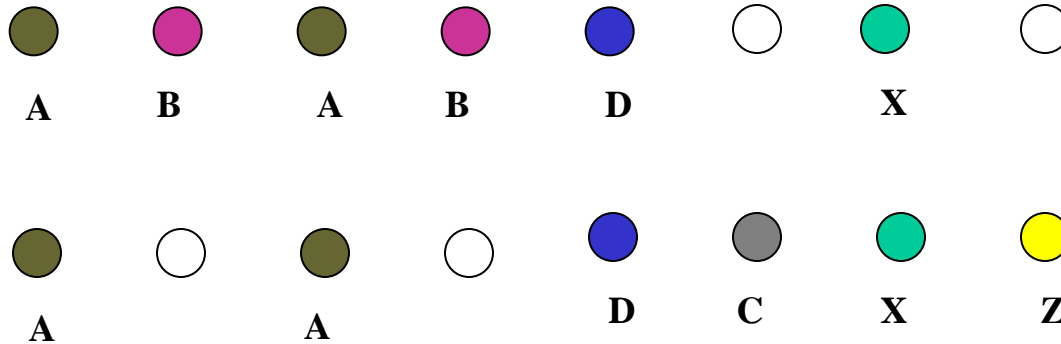
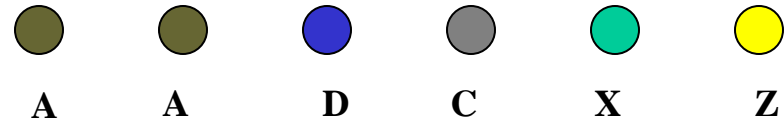
## Alignment (illustration)

**CASE OF 2 WORDS:**

**Word 1**



**Word 2**



- APPLICATIONS:**
1. Linguistics
  2. Bioinformatics (gene analysis, etc.)
  3. Processing of frame sequences (image processing)
  4. Modeling of conveyor-like manufacturing systems

## Alignment (illustration)

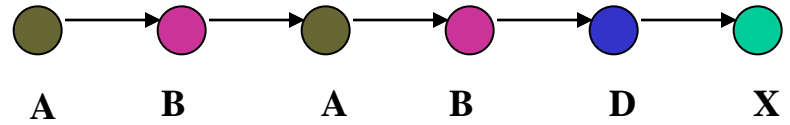
### **OTHER VERSIONS OF PROBLEM:**

- \*CASE OF N WORDS**
- \*CASE OF 2 ARRAYS**
- \*CASE OF N ARRAYS**
- \*M-DIMENSIONAL CASES**
- \*ETC.**

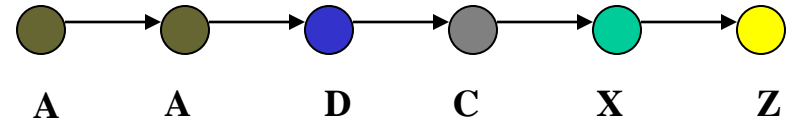
## Substructure and superstructure (illustration)

CASE OF 2 CHAINS:

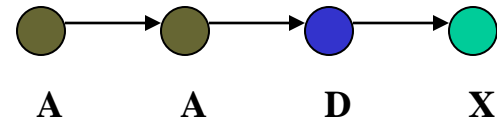
Chain 1



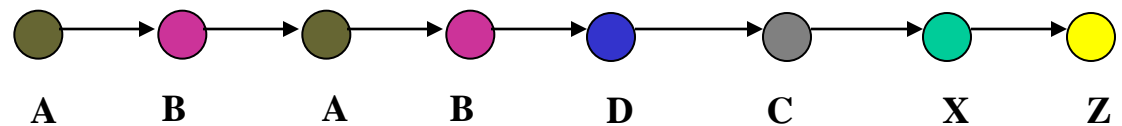
Chain 2



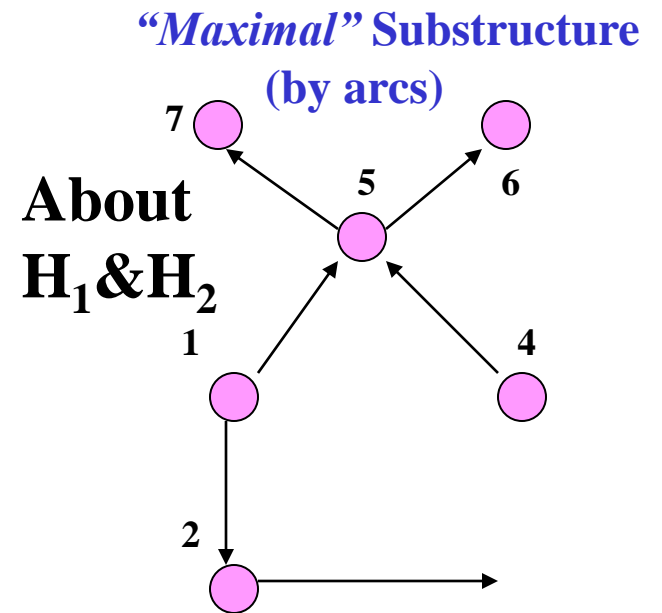
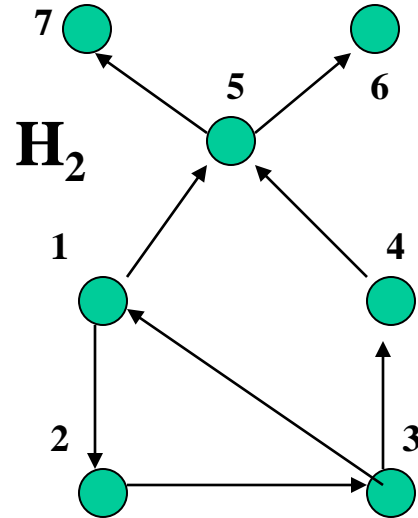
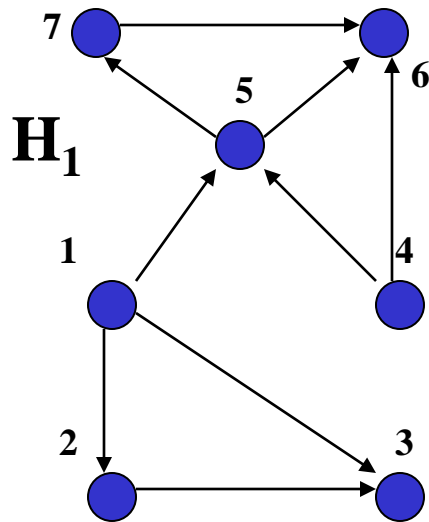
Problem 1: *Maximal Substructure*



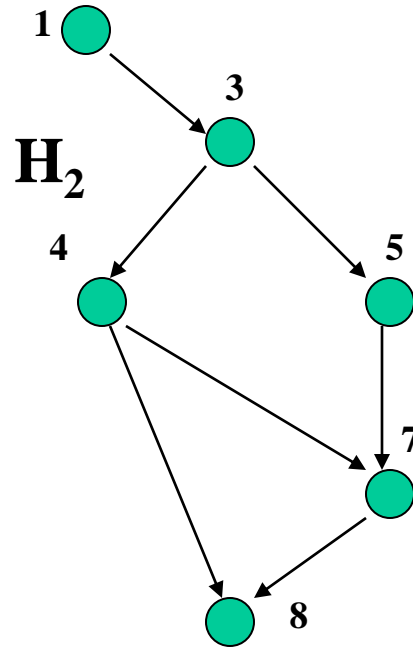
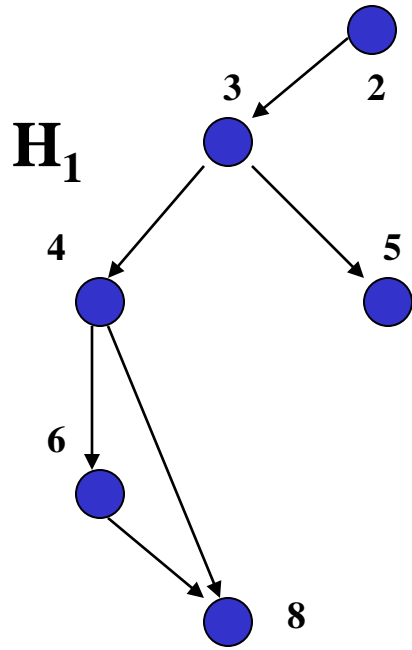
Problem 2: *Minimal Superstructure*



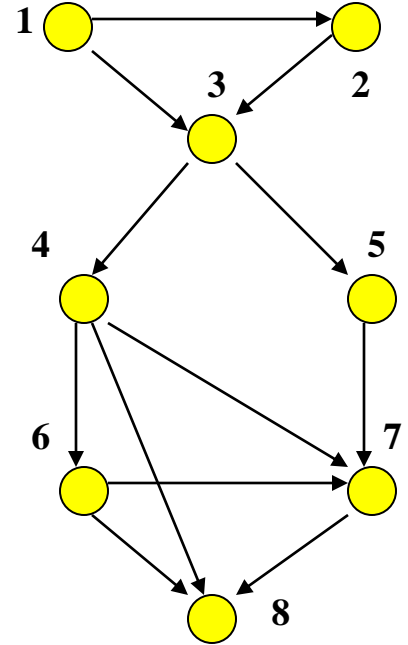
Substructure and superstructure (illustration): case of 2 orgraphs



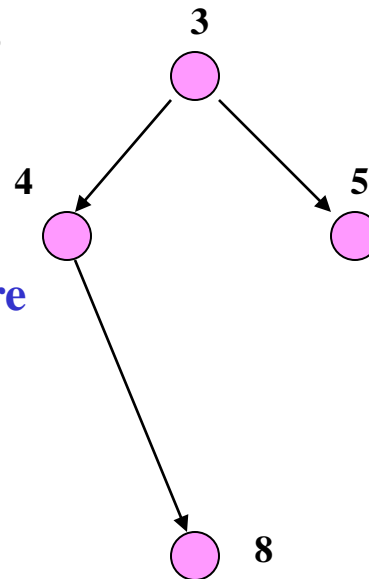
Substructure and superstructure (illustration): case of 2 orgraphs



*Minimal Superstructure*



*Maximal Substructure*





- APPLICATIONS:**
- 1. Decision making / Expert judgment  
(relation of dominance)**
  - 2. Information structures (data bases)**
  - 3. Information structures (knowledge bases)**
  - 4. Bioinformatics**
  - 5. Chemical structures**
  - 6. Network-like systems  
(e.g., social networks, software)**
  - 7. Graph-based patterns**
  - 8. Images (graph models of images)**
  - 9. Linguistics**
  - 10. Organizational structures**
  - 11. Engineering systems**
  - 12. Architecture**
  - 13. Information retrieval**
  - 14. Pattern recognition**
  - 15. Proximity for graph-like systems  
etc.**

## Substructures and superstructures (illustration)

### **OTHER VERSIONS OF PROBLEM:**

- \*CASE OF  $N$  GRAPHS (BINARY RELATIONS)**
- \*CASE OF WEIGHTED GRAPHS**
- \*CASES UNDER SPECIAL CONSTRAINTS**
- \*ETC.**

## Timetabling

**APPLICATIONS:**

- 1.Scheduling in educational institutions (universities, schools)**
- 2.Scheduling in hospitals**
- 3.Scheduling in sport (e.g., basketball)**

**ETC.**

**COMPOSITE ALGORITHM SCHEMES on the basis of model combination:**

- 1.Graph coloring**
  - 2.Assignment / Allocation**
  - 3.Combinatorial design**
  - 4.Basic scheduling**
- Etc.**

**LECTURE 19. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Institute of Physics and Technology (University)**

**Mark Sh. Levin**

**Inst. for Information Transmission Problems, RAS**

**Email: [mslevin@acm.org](mailto:mslevin@acm.org) / [mslevin@iitp.ru](mailto:mslevin@iitp.ru)**

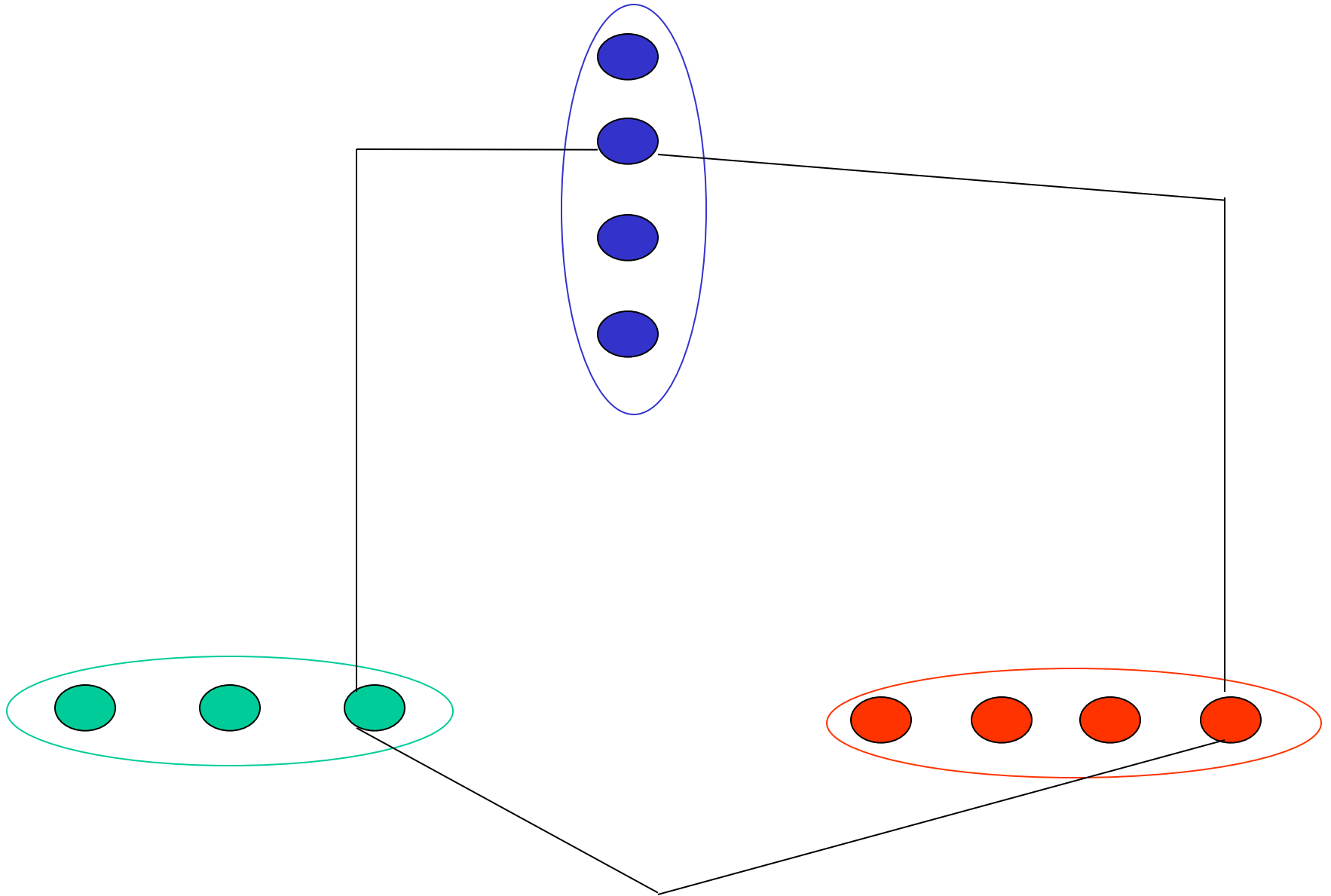
**L.19. Morphological syntheses.**

***PLAN:***

1. Morphological analysis
2. Hierarchical Morphological Multicriteria Design (HMMD)  
(morphological synthesis, combinatorial synthesis): Fundamentals
3. Five development phases of morphological analysis
4. Preliminary phases (1, 2, 3, 4)
5. HMMD: \*formulation, \*solving schemes, \*examples

Oct. 15, 2004

# Morphological clique



## Fundamentals of Hierarchical Morphological Design

1. Morphological analysis (F. Zwicky, 1943)

2. Paradigm of multicriteria decision making (H. Simon)

3. Dynamic programming (R. Bellman)

4. Engineering practice in hierarchical design of complex multidisciplinary systems

5. Combinatorial optimization

6. Knowledge engineering (multidisciplinary information):  
extraction, organization, usage

## Development Phases of Morphological Analysis

1. Morphological analysis [F. Zwicky]

2. Closeness of feasible combination to “IDEAL” [Ayres, 1969; Iakimets, Inst. for System Analysis, Russian Academy of Sci. (RAS), 1977]

3. Multicriteria evaluation of feasible combinations and selection of Pareto-effective decisions [Inst. for Control Problems (IPU), Inst. for System Analysis (ISA), Inst. for Machine-engineering (IMASH); RAS, 1972/82]

4. Hierarchical design (composition of local Pareto-effective solutions) [Comp. Center, RAS, Krasnoshekov et al., 1979...]

5. Hierarchical Morphological Multicriteria Design (HMMD) (combinatorial morphological synthesis) [Levin, 1994...]

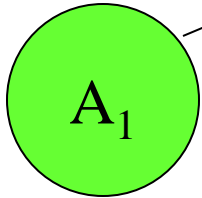
Morphological analysis

System

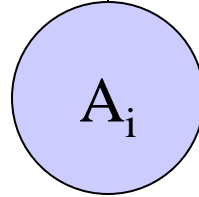
Subsystem 1

Subsystem i

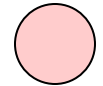
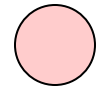
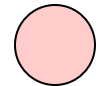
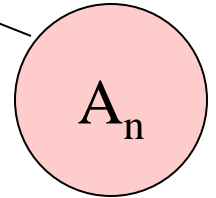
Subsystem n



...



...



Morphological class 1:

$$|A_1| = m(1)$$

Morphological class i:

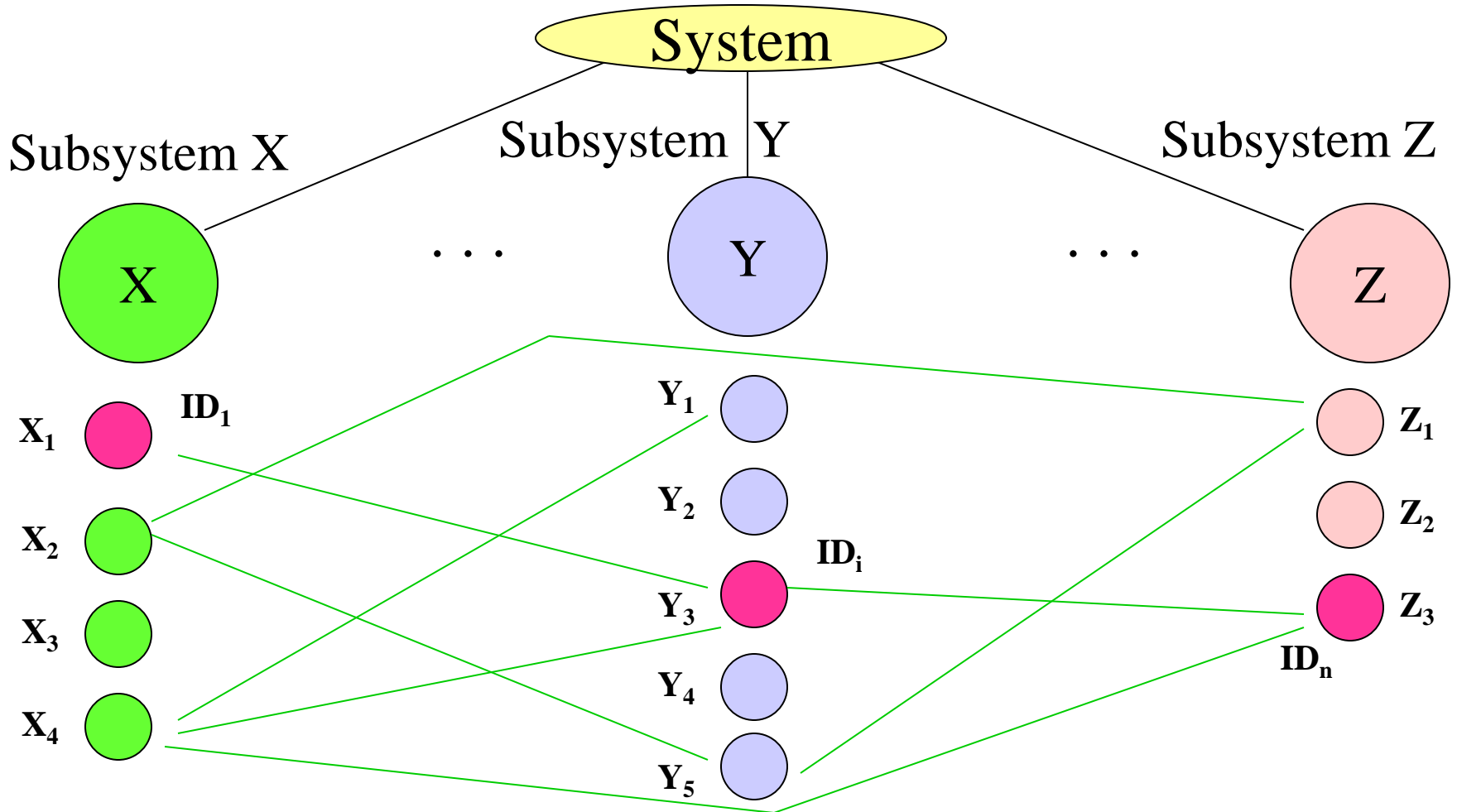
$$|A_i| = m(i)$$

Morphological class n:

$$|A_n| = m(n)$$



Morphological analysis: Closeness to "IDEAL" (Ayres, 1969)



**"IDEAL" :  $X_1 * Y_3 * Z_3$**   
**(infeasible combination)**

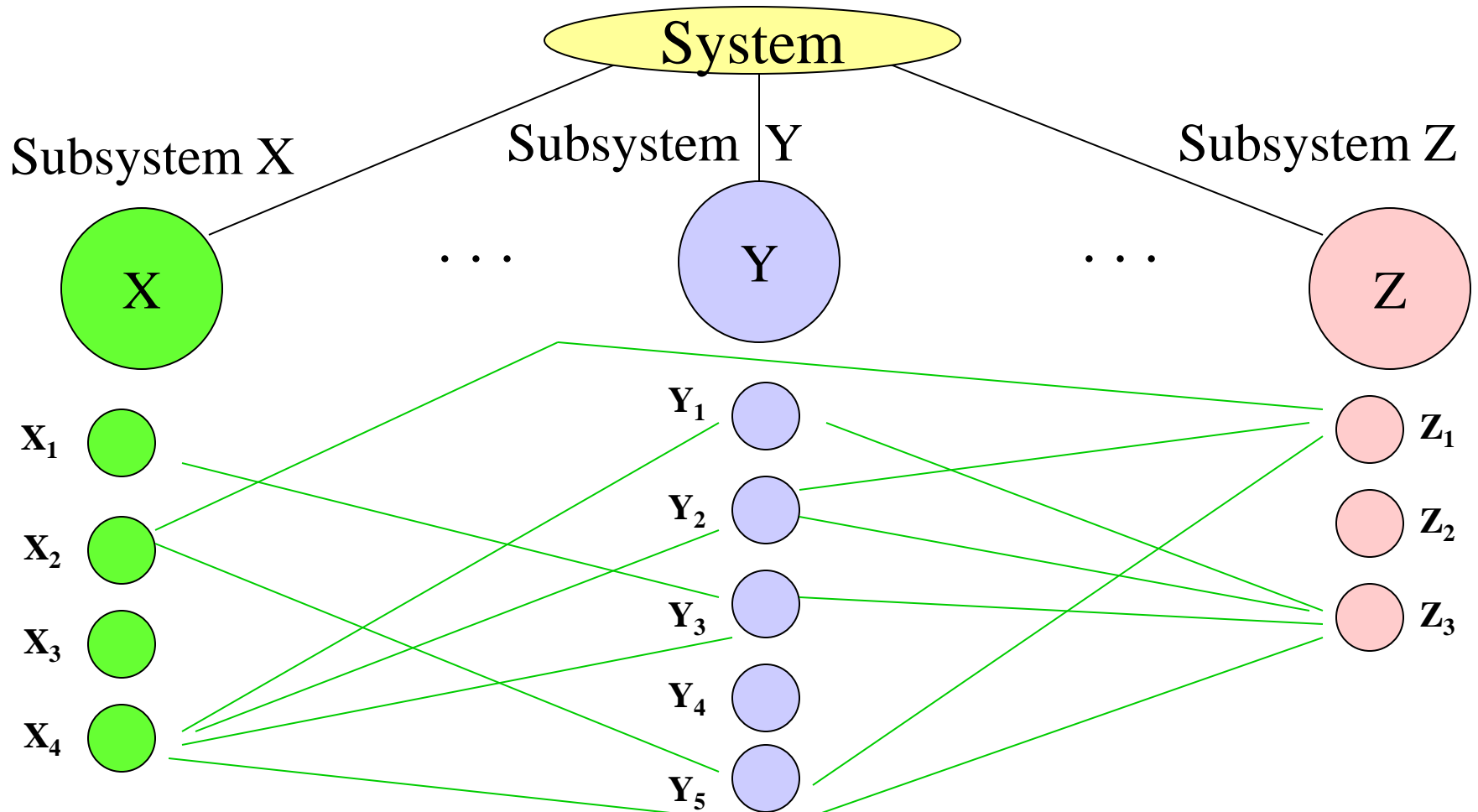
$$S_1 = X_4 * Y_3 * Z_3$$

$$S_2 = X_2 * Y_5 * Z_1$$

$$\rho(\text{"IDEAL"}, S_1) < \rho(\text{"IDEAL"}, S_2)$$

$\rho$  is a closeness (proximity)

Multicriteria evaluation of feasible combinations & selection of Pareto-effective points (1972..1982)



**STEP 1. Generation of feasible combinations:**

$$S_1 = X_4 * Y_3 * Z_3 \quad S_3 = X_4 * Y_2 * Z_3$$

$$S_2 = X_2 * Y_5 * Z_1 \quad S_4 = X_4 * Y_1 * Z_3$$

**STEP 2. Evaluation upon criteria**

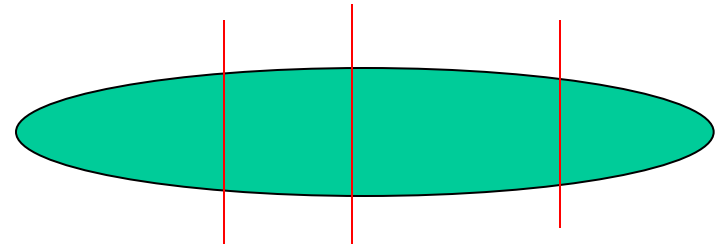
**STEP 3. Selection of Pareto-effective solutions**

## Morphological analysis

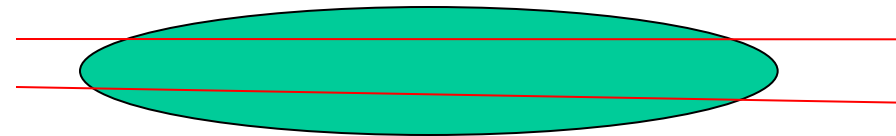
Complexity:  $m(1) * \dots * m(i) * \dots * m(n)$

Decreasing of complexity:

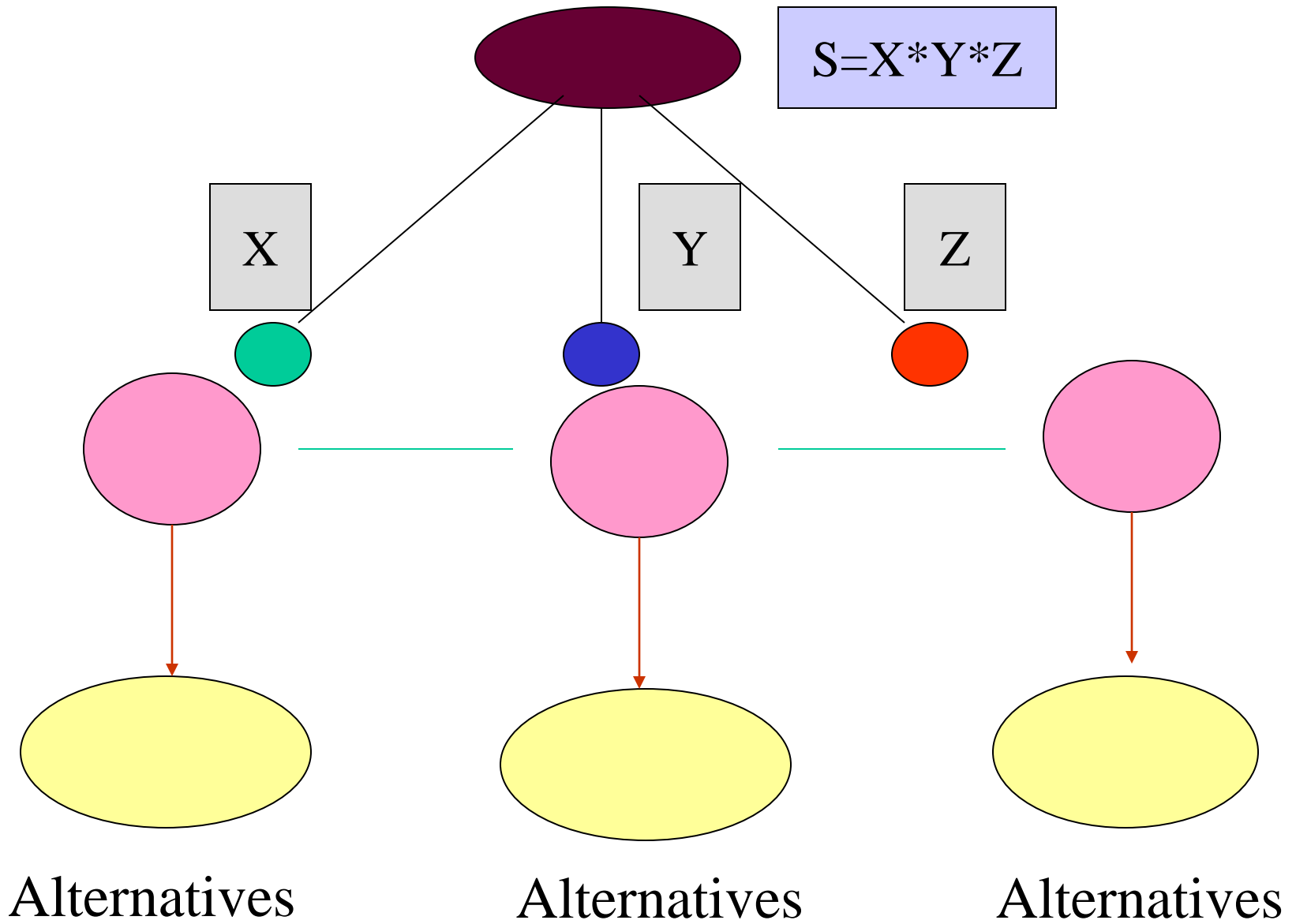
Horizontal partitioning



Vertical partitioning



Hierarchical morphological design (engineering experience, phases 3, 4)

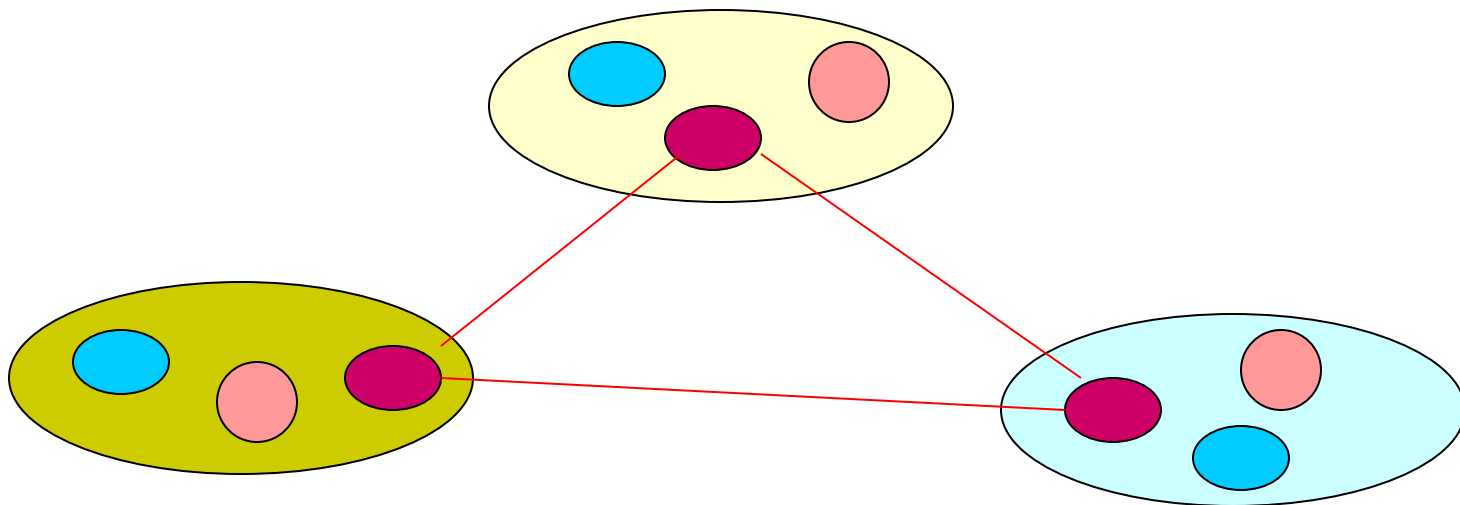
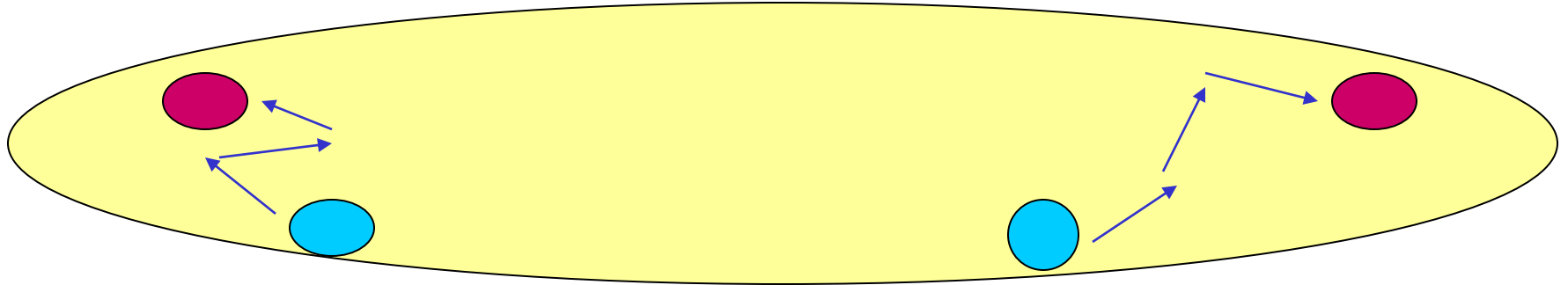
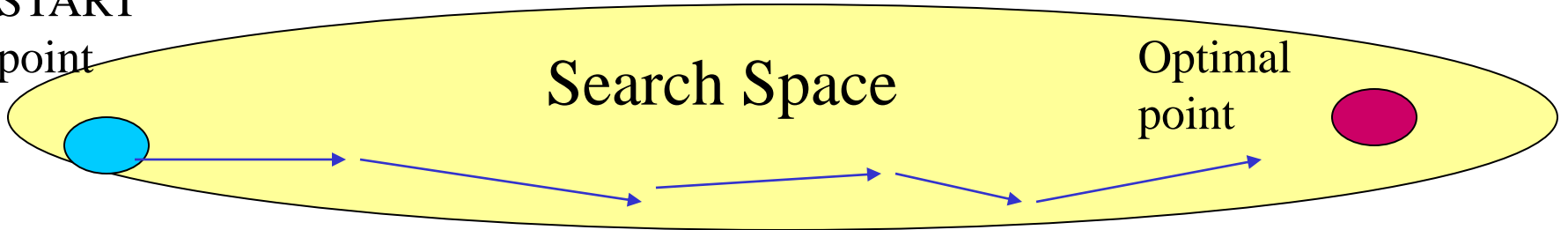


Search strategies

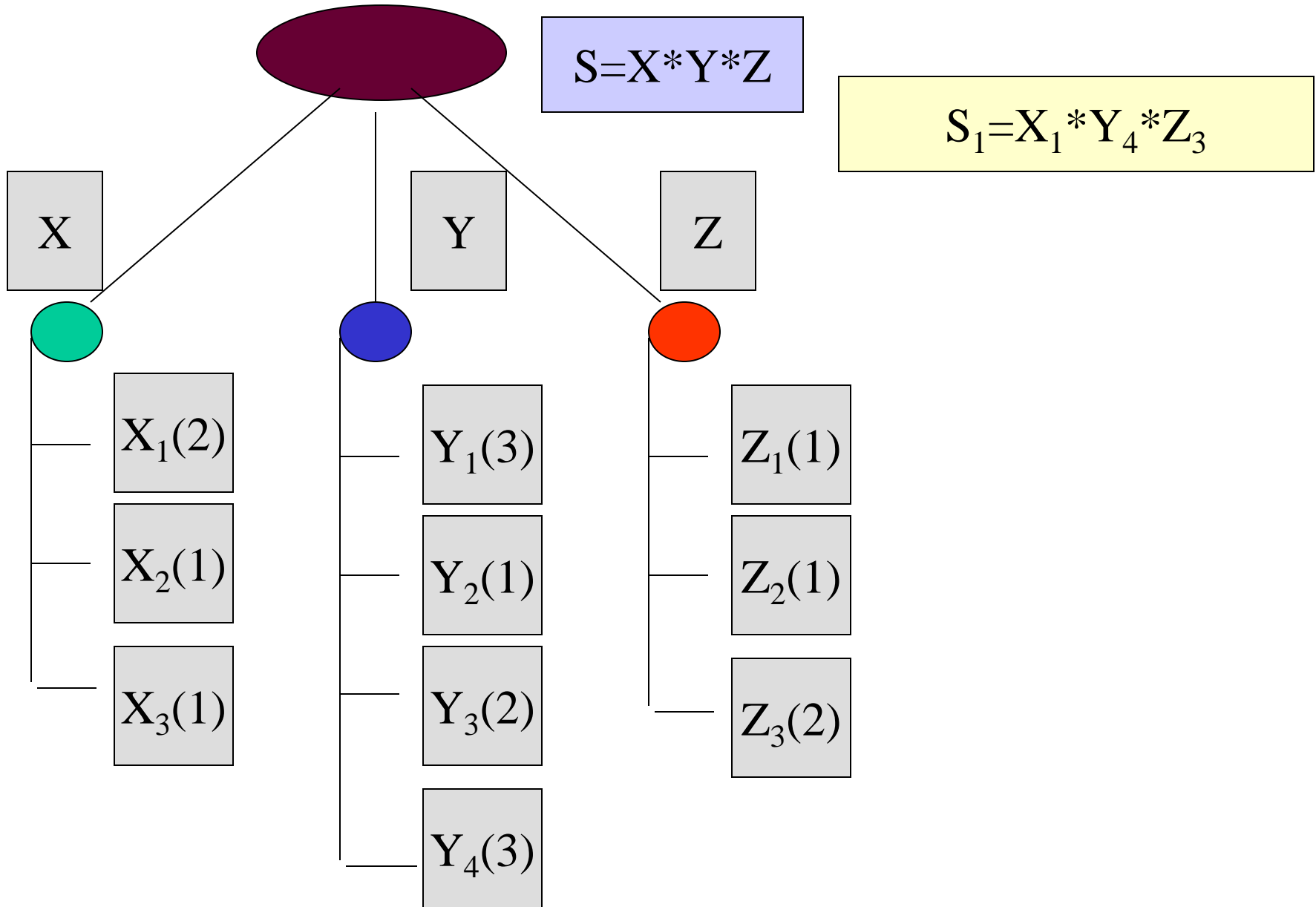
START  
point

Search Space

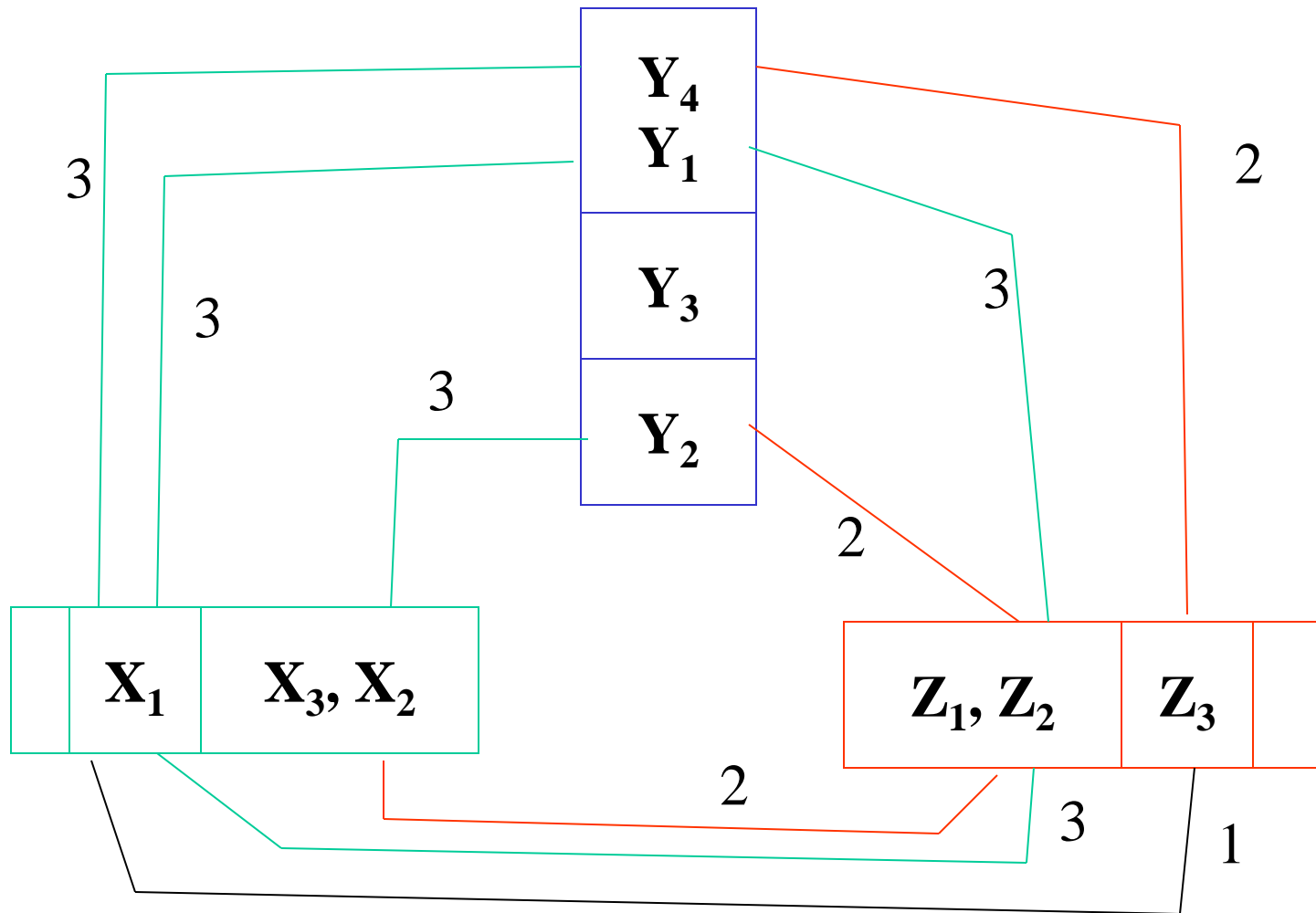
Optimal  
point



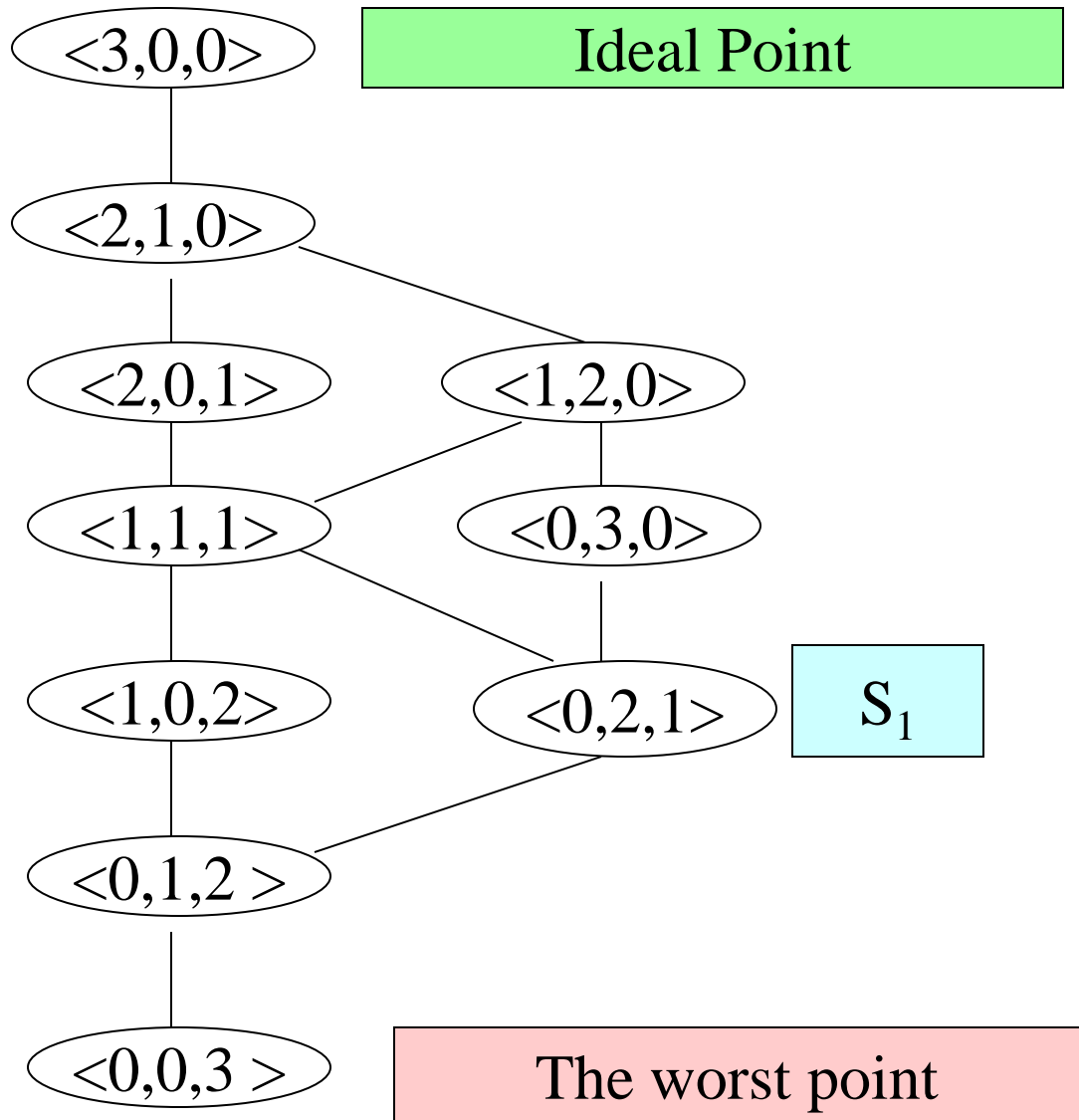
Morphological combinatorial synthesis: example



Concentric presentation of morphological clique with estimates of compatibility

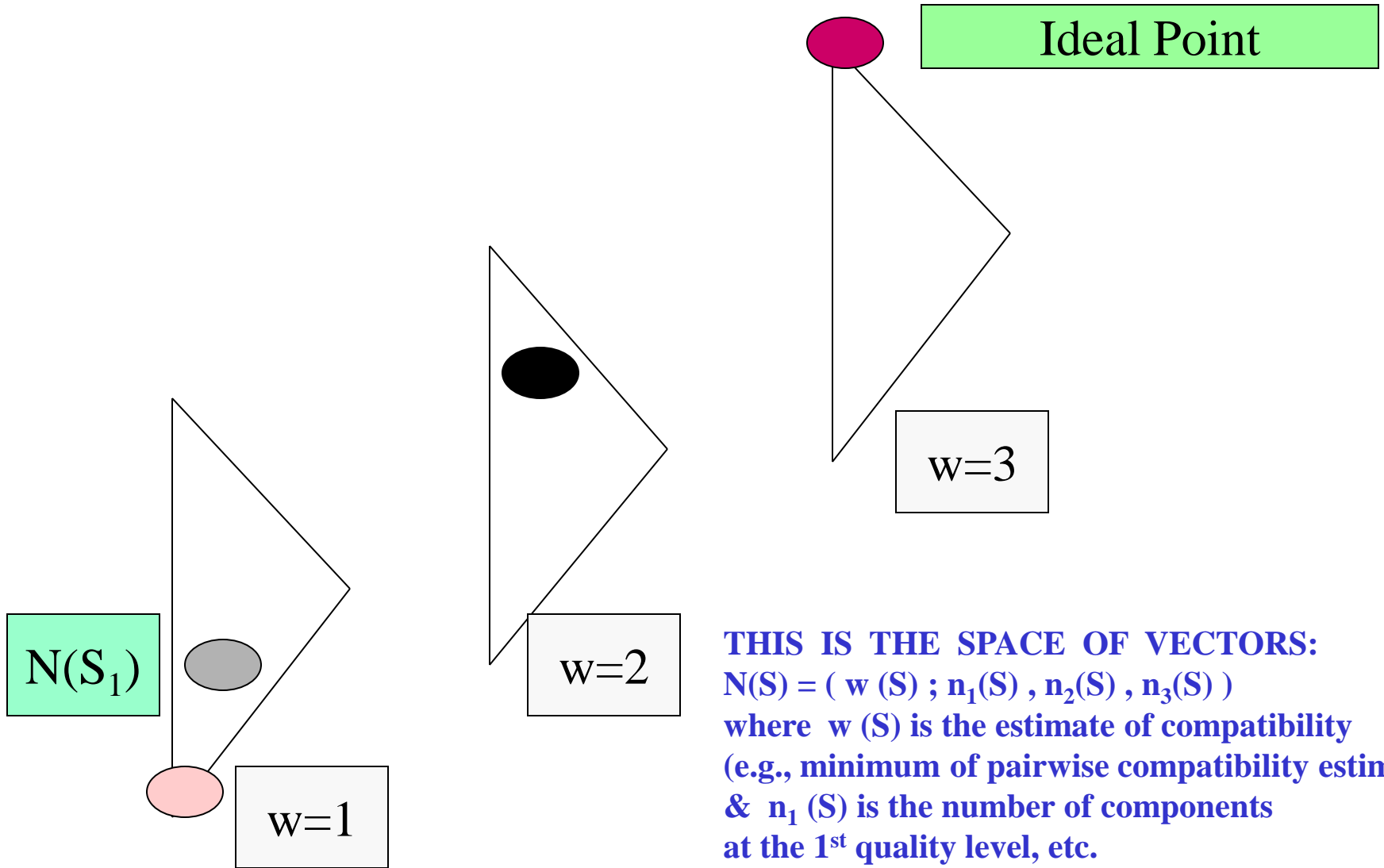


Discrete space of quality (by components)



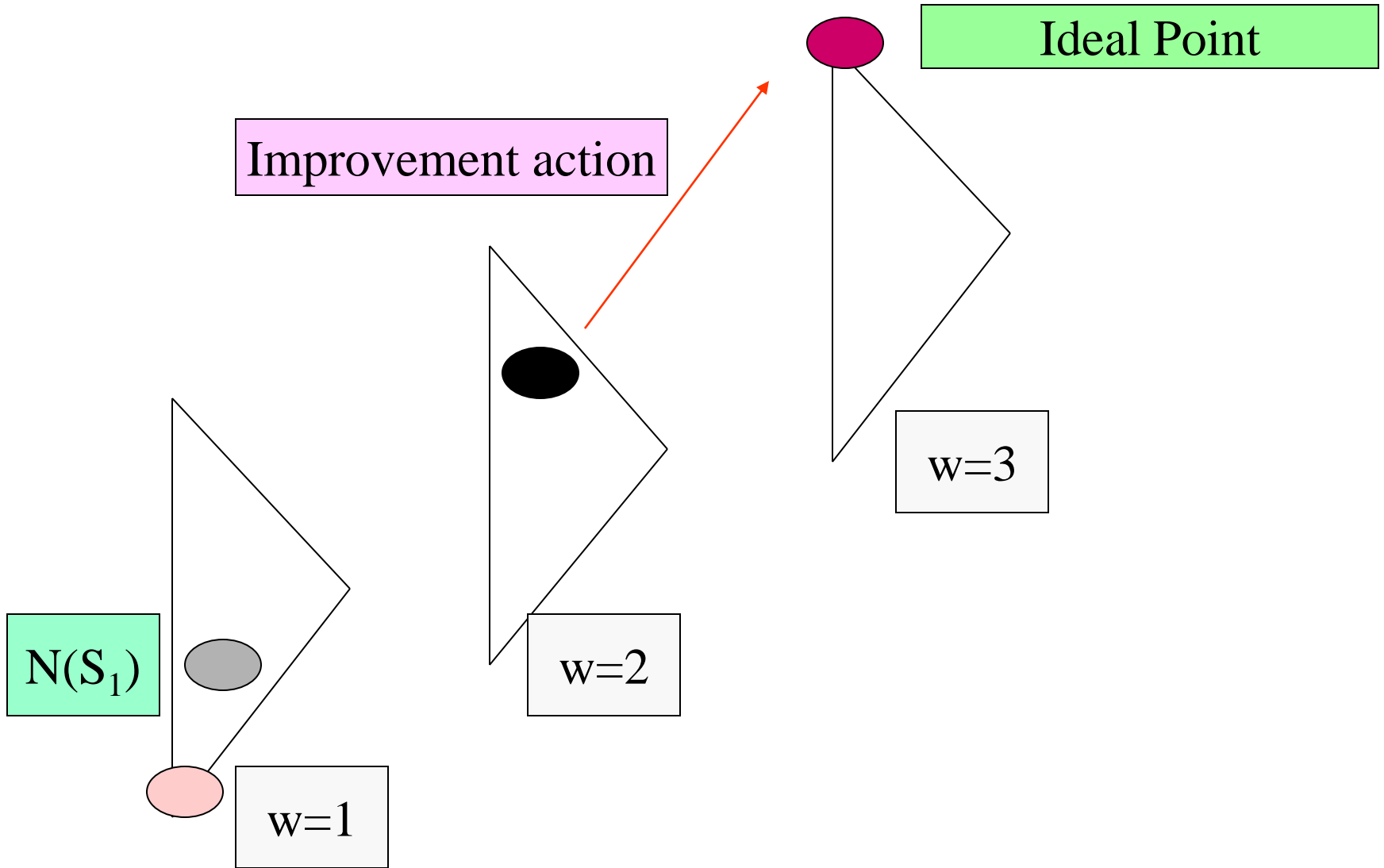


Discrete space of quality (by components, by compatibility)



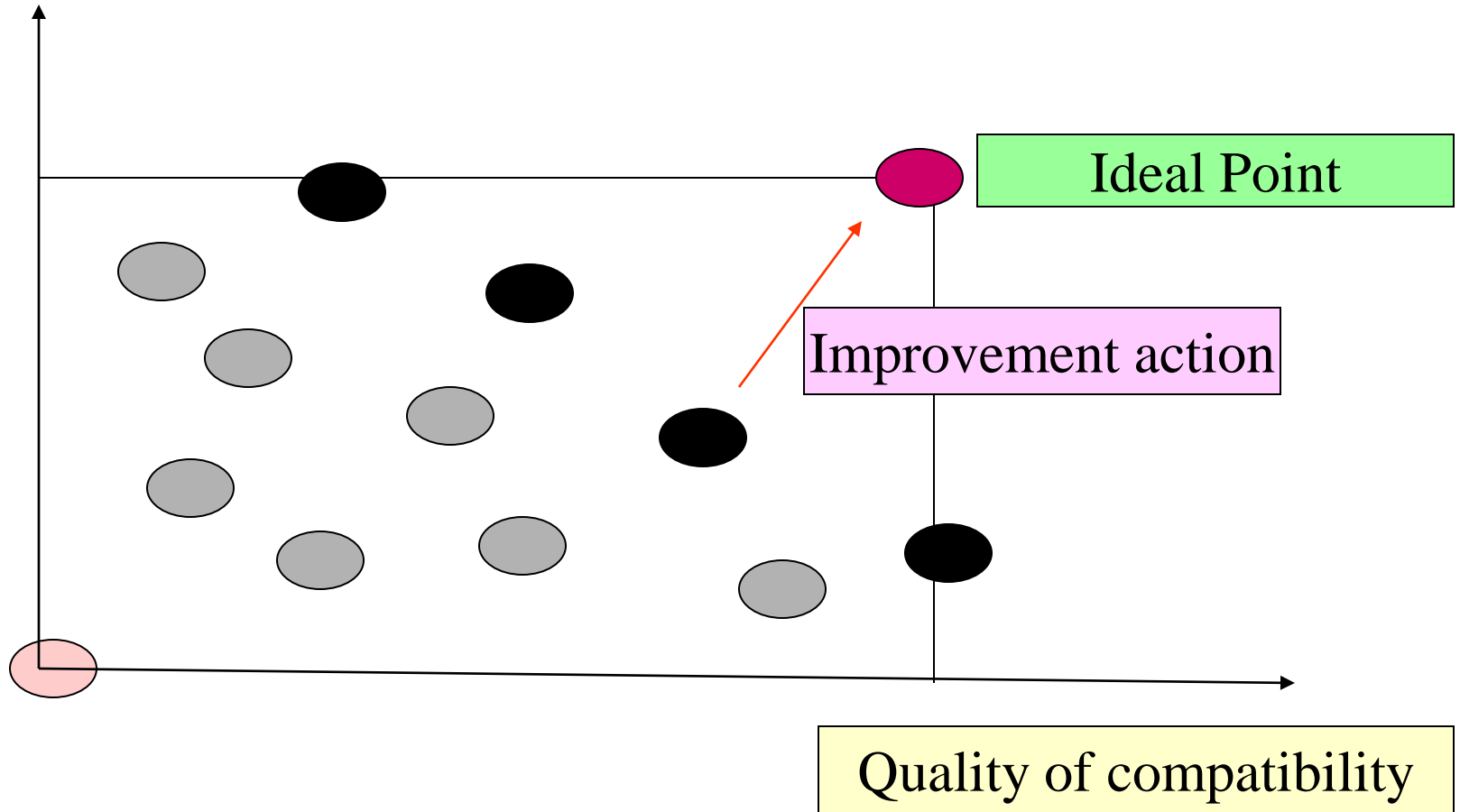
**THIS IS THE SPACE OF VECTORS:**  
 $N(S) = ( w (S) ; n_1(S) , n_2(S) , n_3(S) )$   
where  $w (S)$  is the estimate of compatibility  
(e.g., minimum of pairwise compatibility estimates)  
&  $n_1 (S)$  is the number of components  
at the 1<sup>st</sup> quality level, etc.

Discrete space of quality (by components, by compatibility) & improvement



Two-criteria space of quality & improvement action

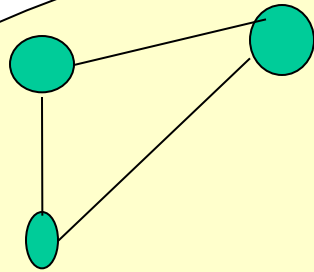
Quality of elements



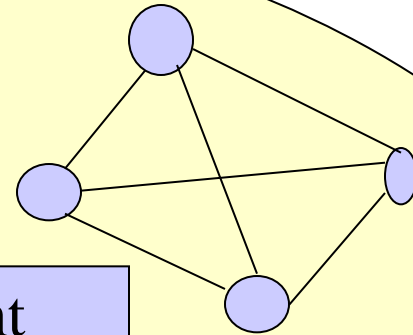
1.Enumerative directed heuristic:  
analysis and checking  
since the best point.

2.Dynamic programming methods:  
extension of the method for  
knapsack problem or  
multiple choice problem.

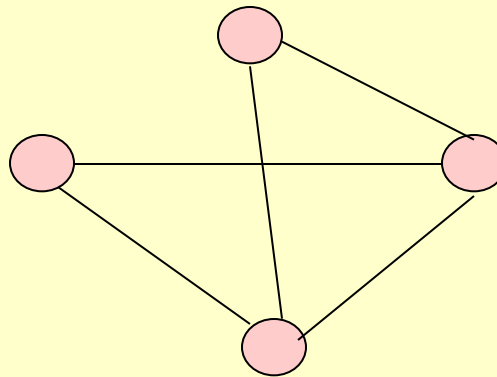
# Examples of clique & quasi-clique



3-element  
clique

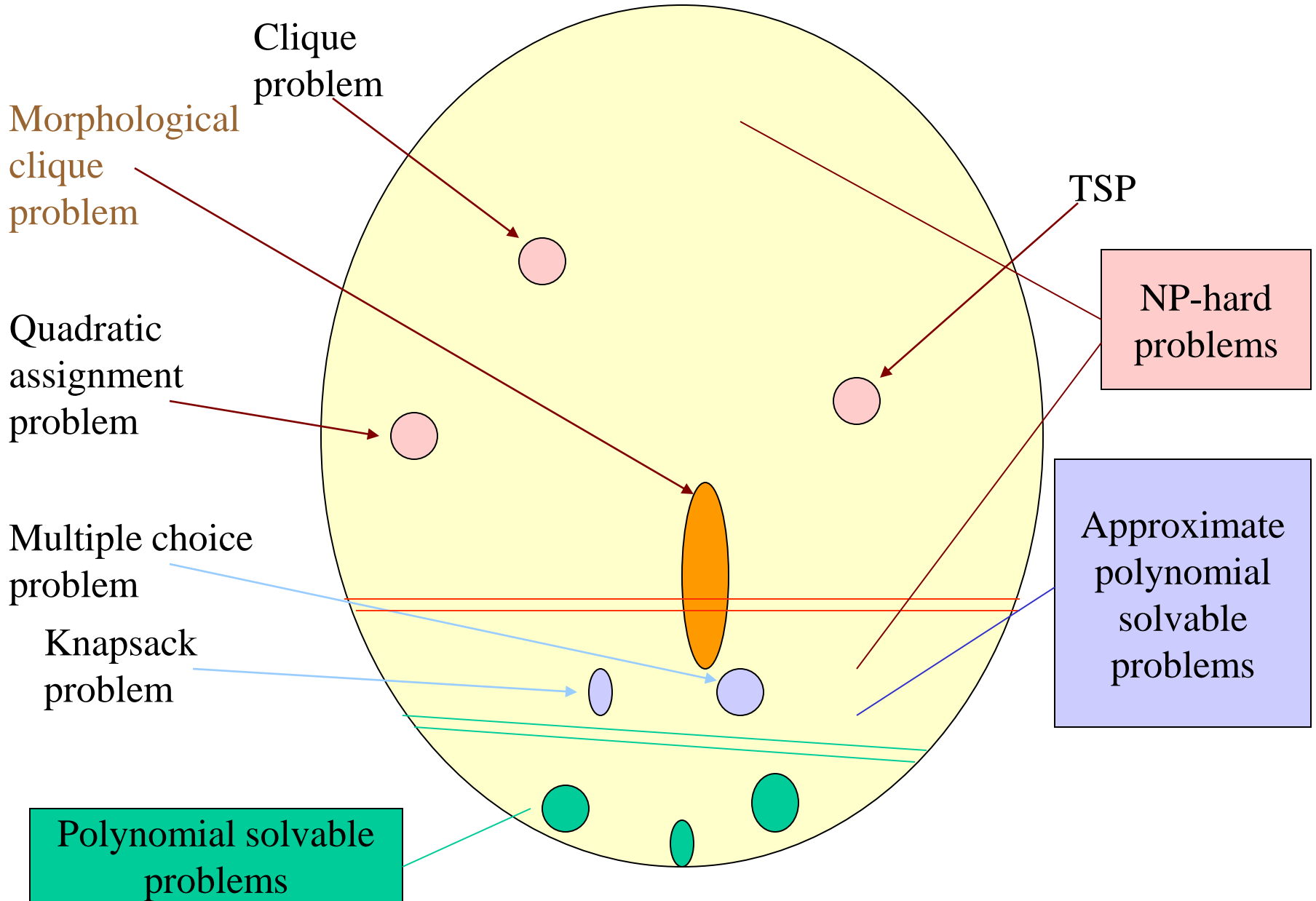


4-element  
clique



Quasi-clique

Space of combinatorial optimization problems



**1. Knapsack problem**

**2. Multiple choice problem**

**3. Quadratic assignment problem**

**4. Integer non-linear programming**

**5. Mixed non-linear programming**



**LECTURE 20-21. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Institute of Physics and Technology (University)**

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**L.20. Application of morphological synthesis**

**L.21. System bottlenecks, improvement, multistage design**

***PLAN:***

1 Hierarchical Morphological Multicriteria Design (HMMD).

Examples: \*design of team \*design of solving strategy \*

2. Approaches to revelation of bottlenecks

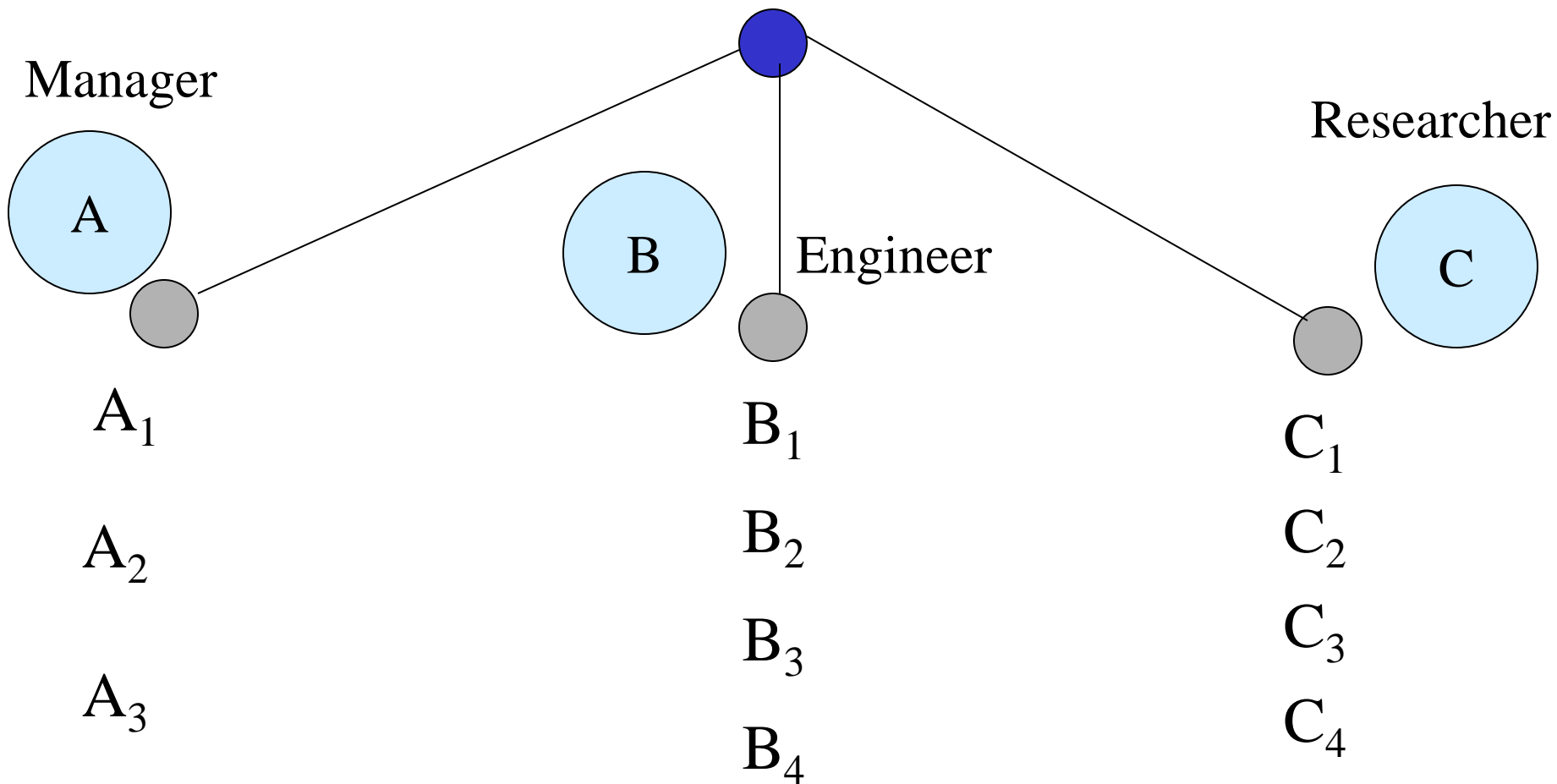
3. Multistage design

Oct. 16, 2004



Application example 1: Design of team

Team:  $S = A * B * C$



Application example 1: Design of team (assessment of alternatives for Manager)

	Experience	Market	Salary	Total rank
$A_1$	3	4	-1	3
$A_2$	5	3	10	1
$A_3$	15	5	20	2
$A_3$	10	4	15	2

Application example 1: Design of team (assessment of alternatives for Engineer)

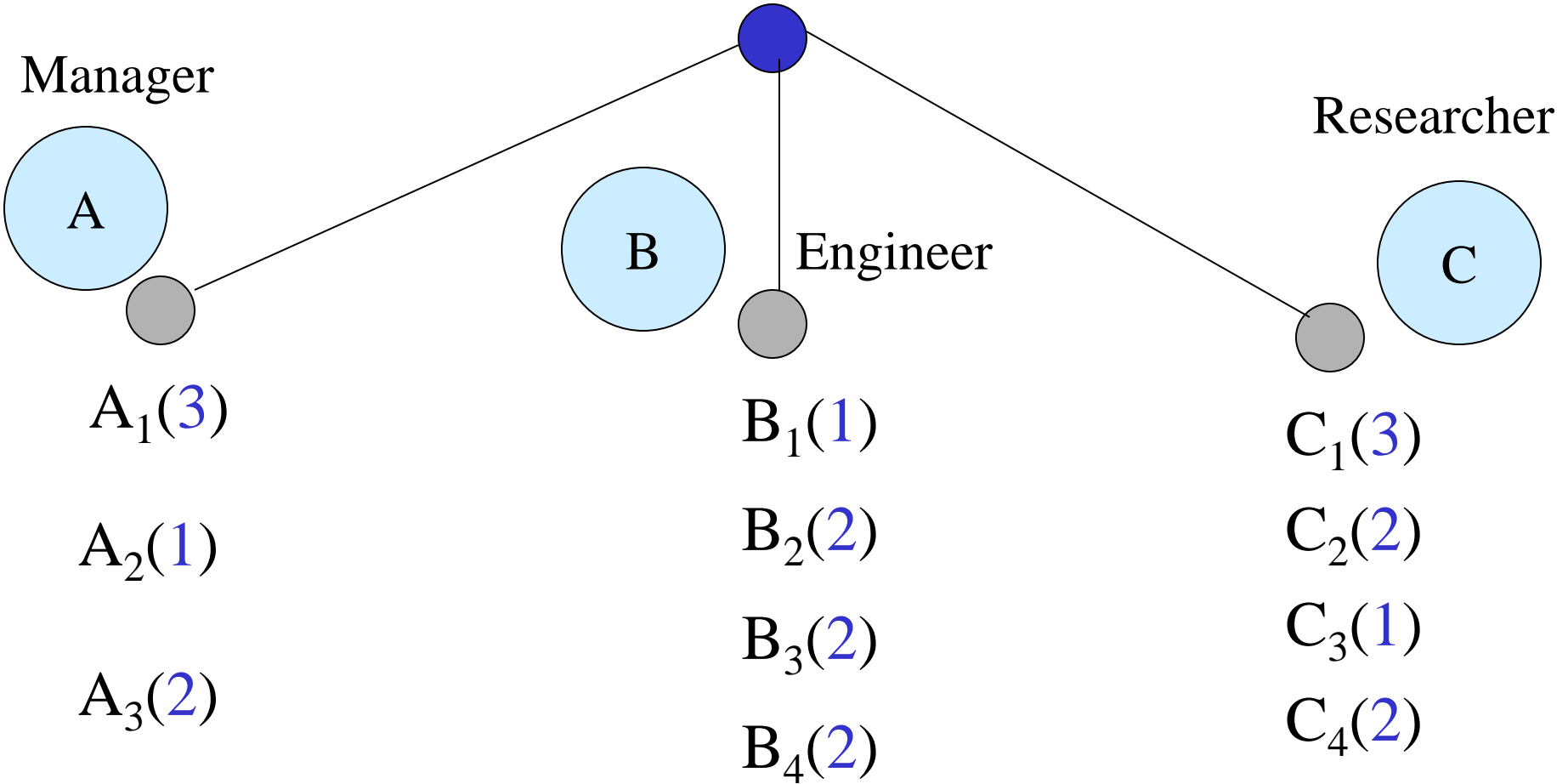
	Experience	West Experience	Salary	Total rank
$B_1$	3 15	2 5	-3 7	1
$B_2$	6	6	3	2
$B_3$	10	0	9	2
$B_4$	3	0	3	2

Application example 1: Design of team (assessment of alternatives for Researcher)

	Experience	Creativity	Salary	Total rank
$C_1$	2 15	4 3	-3 10	3
$C_2$	5	5	7	2
$C_3$	4	7	4	1
$C_4$	6	4	6	2

Application example 1: Design of team

Team:  $S = A * B * C$

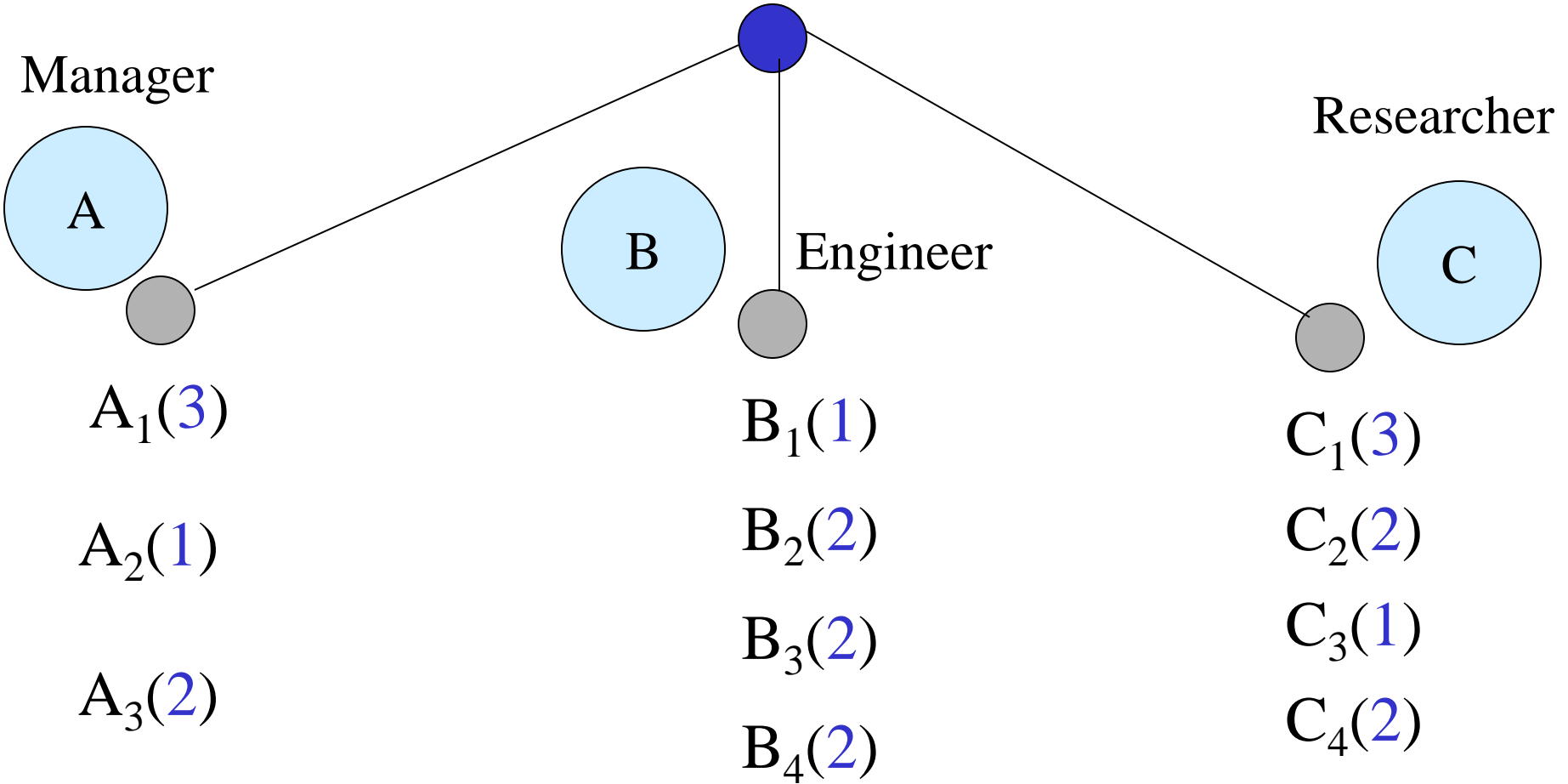


Application example 1: Design of team (estimates of compatibility)

	$B_1$	$B_2$	$B_3$	$B_4$	$C_1$	$C_2$	$C_3$	$C_4$
$A_1$	2	3	3	1	2	0	0	3
$A_2$	3	0	0	1	0	3	0	0
$A_3$	3	0	1	1	1	0	3	0
$B_1$					0	3	3	0
$B_2$					1	1	0	3
$B_3$					0	0	2	0
$B_4$					0	0	0	3

Application example 1: Design of team

Team:  $S = A * B * C$



## Application Example 1: Design of Team (resultant decisions)

$$S_1 = A_2 * B_1 * C_2 \quad N(S_1) = (3; 2, 1, 0)$$

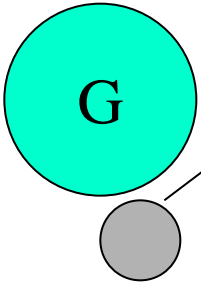
$$S_2 = A_3 * B_1 * C_3 \quad N(S_2) = (3; 2, 1, 0)$$



Application Example 2: Strategy for multicriteria ranking

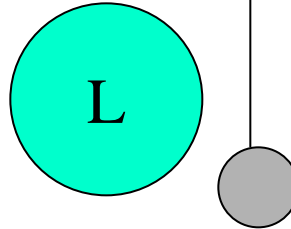
$$\text{Strategy: } S = G * L * R$$

Stage 1:  
Forming  
preference  
relations



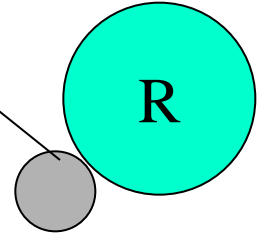
- $G_1$
- $G_2$
- $G_3$
- $G_4$

Stage 2:  
Forming  
a linear  
ordering



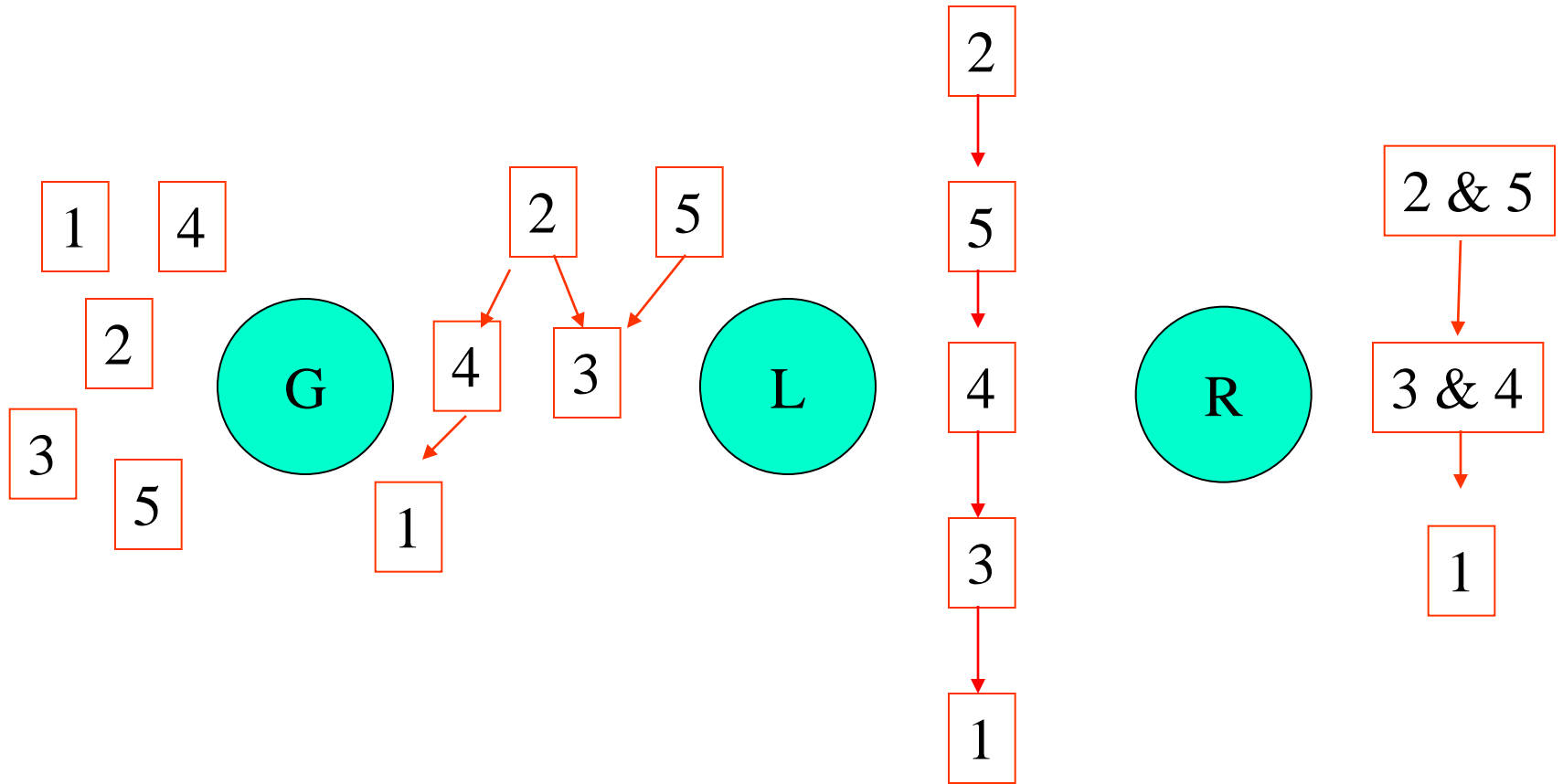
- $L_1$
- $L_2$
- $L_3$
- $L_4$
- $L_5$

Stage 3:  
Ranking



- $R_1$
- $R_2$
- $R_3$
- $R_4$

Application Example 2: Strategy for multicriteria ranking

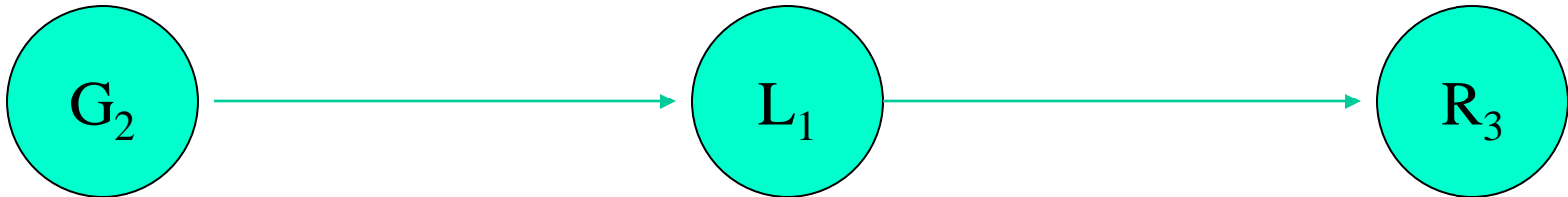


## **LOCAL ALTERNATIVES (COMBI-PC, 1989...):**

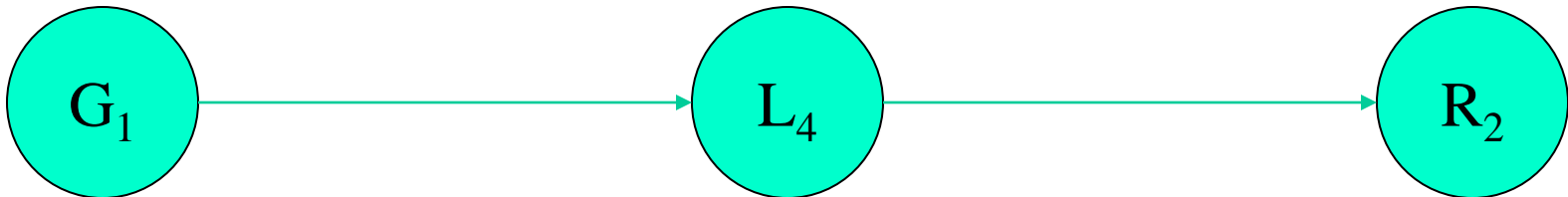
- G<sub>1</sub> Pairwise comparison**
- G<sub>2</sub> ELECTRE-like technique**
- G<sub>3</sub> Additive utility function method**
- G<sub>4</sub> Expert stratification**
- L<sub>1</sub> Line element sum of preference matrix**
- L<sub>2</sub> Additive function**
- L<sub>3</sub> Series revelation of “max” element**
- L<sub>4</sub> Series revelation of Pareto-elements**
- L<sub>5</sub> Expert stratification**
- R<sub>1</sub> Series revelation of “max” element**
- R<sub>2</sub> Series revelation of Pareto elements**
- R<sub>3</sub> Dividing the linear ordering**
- R<sub>4</sub> Expert stratification**

## EXAMPLES OF STRATEGIES:

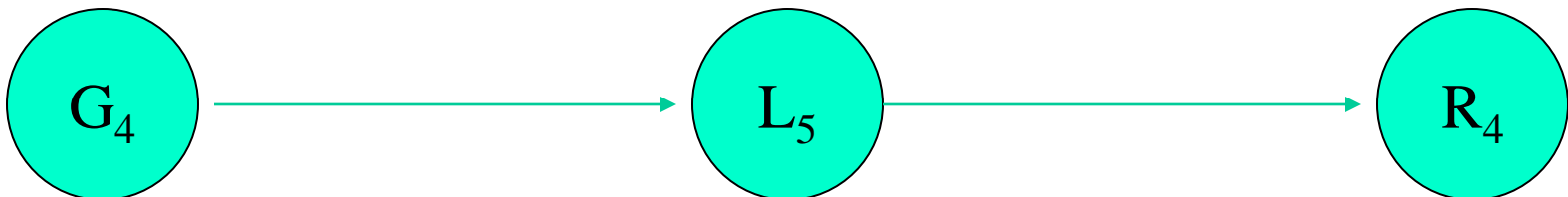
**Strategy 1**



**Strategy 2**

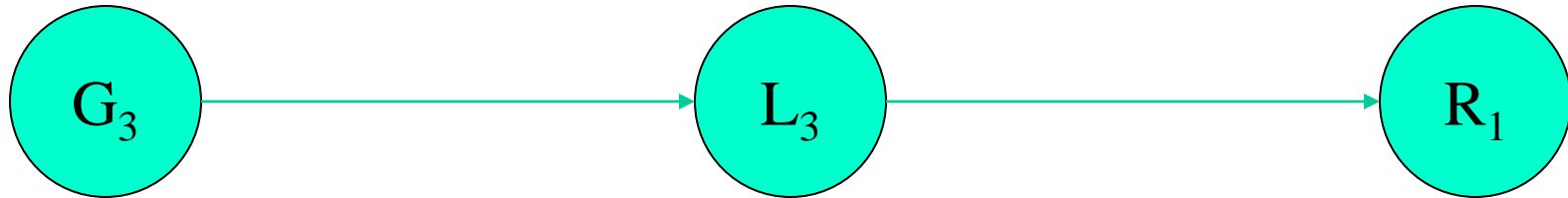


**Strategy 3**

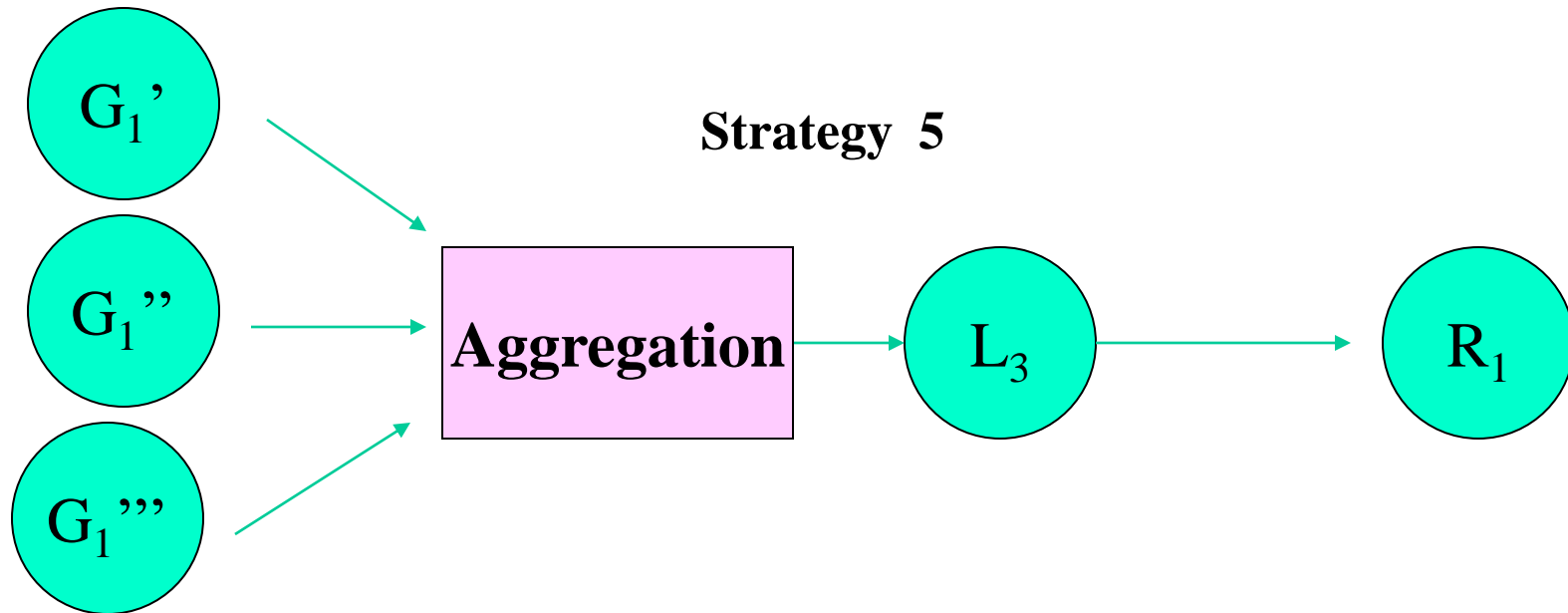


## EXAMPLES OF STRATEGIES:

Strategy 4



Strategy 5



**Approach 1.**

**Engineering analysis (expert judgment)**

**Approach 2.**

**Design of system structure, assessment of reliability of system components, and selection of the most non-reliable ones (“Pareto approach” from Japanese system of quality management )**

**Approach 3.**

**Design of system structure, assessment of system components / parts, and multicriteria ranking of the system part (to reveal the most important parts)**

**Approach 4.**

**Analysis of the total vector estimate of the composable system decision S:  $N(S) = ( w(S); n_1(S), n_2(S), \dots )$**

Approaches to revelation of bottlenecks



1.1  
1.2  
1.3  
1.4

2.1

5.1  
5.2  
5.3

6.1  
6.2  
6.3  
6.4  
6.5  
6.6  
6.7  
6.8  
6.9

7.1  
7.2  
7.3  
7.4  
7.5  
7.6  
7.7  
7.8  
7.9  
7.10  
7.11

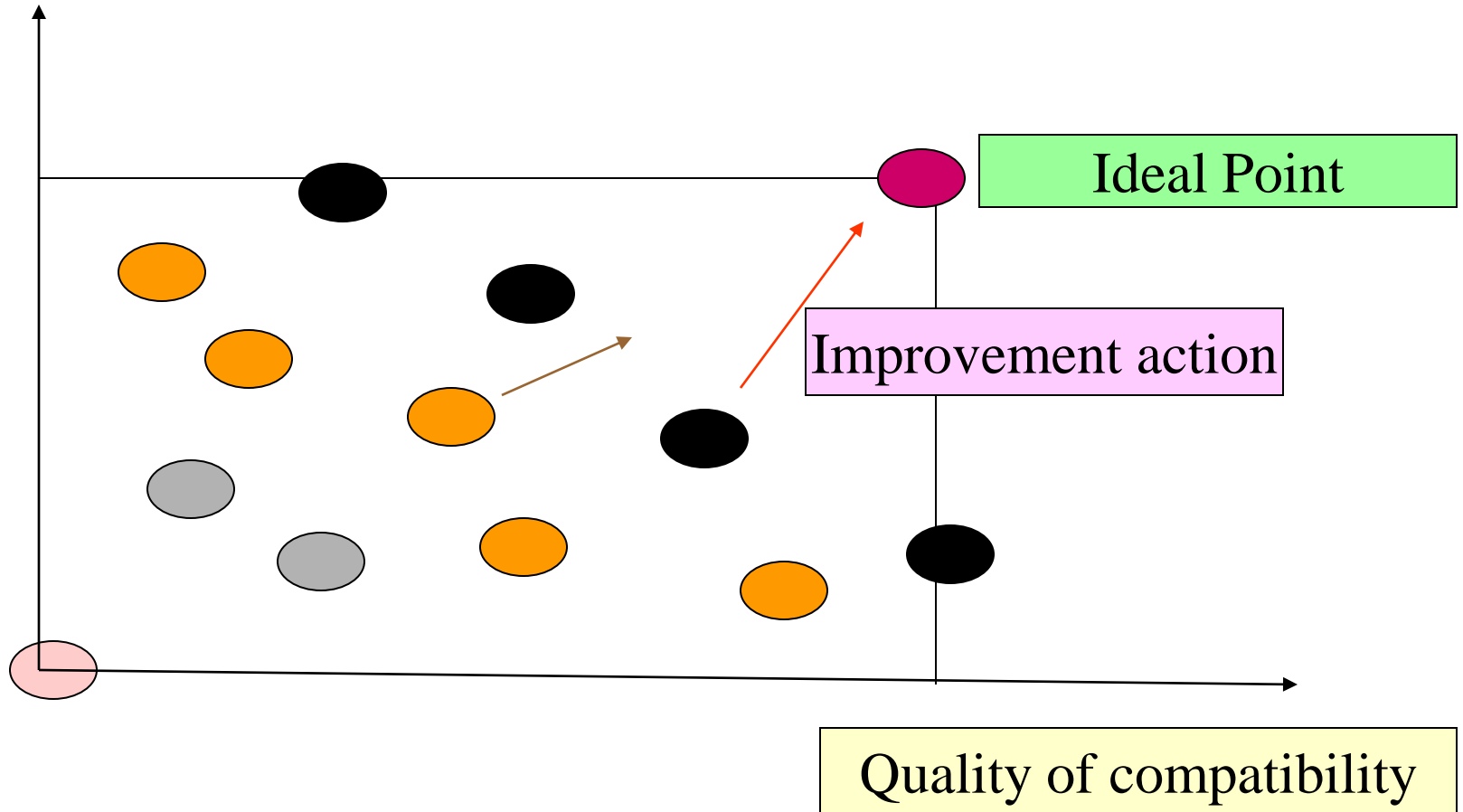
8.1  
8.2  
8.3  
8.4

**CRITERIA:**

- C<sub>1</sub>
- C<sub>2</sub>
- C<sub>3</sub>
- C<sub>4</sub>
- C<sub>5</sub>
- C<sub>6</sub>

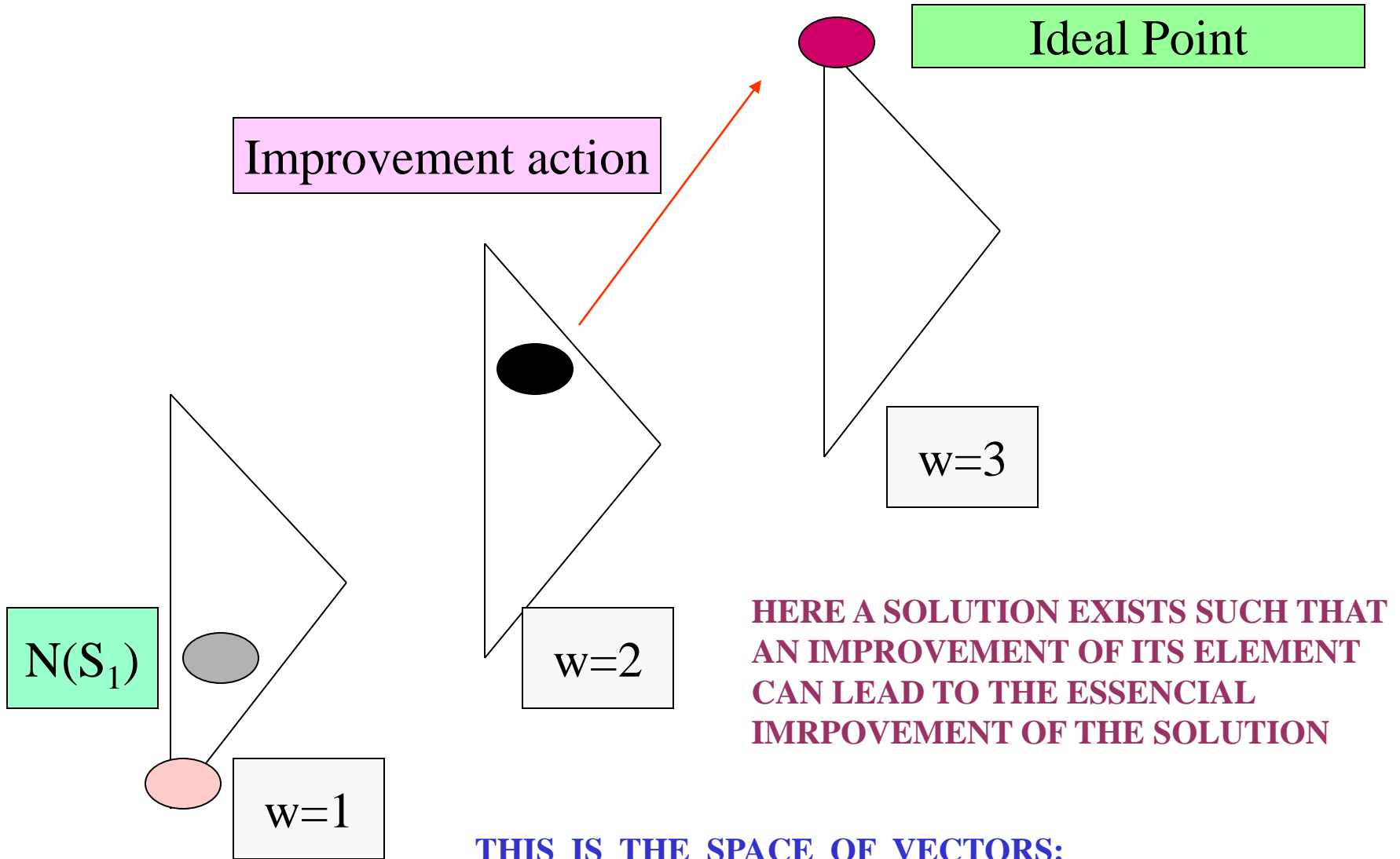
Pareto-effective & near Pareto-effective solutions

Quality of elements





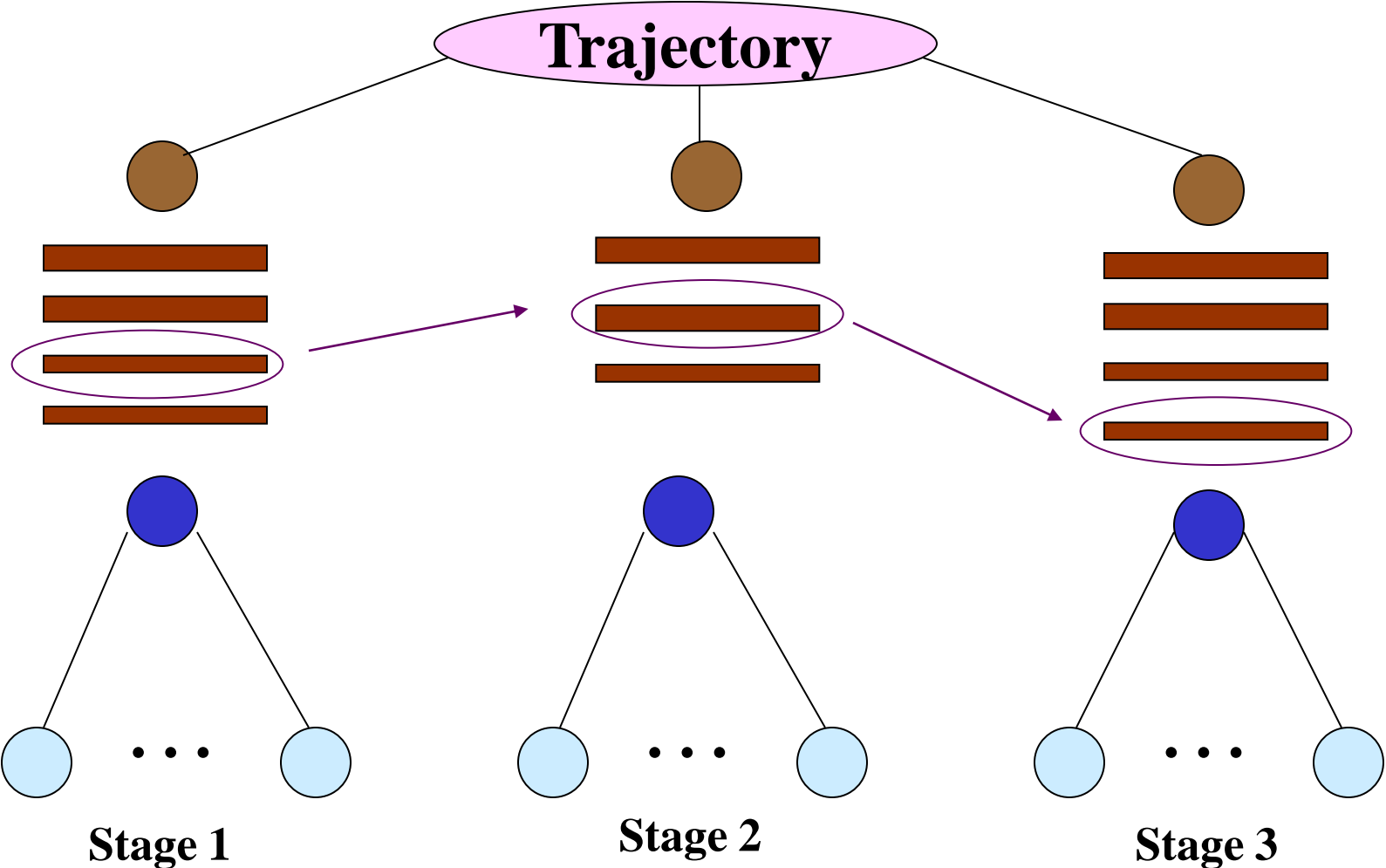
Revelation of bottlenecks via analysis of  $N(S)$



**HERE A SOLUTION EXISTS SUCH THAT AN IMPROVEMENT OF ITS ELEMENT CAN LEAD TO THE ESSENCIAL IMPROVEMENT OF THE SOLUTION**

**THIS IS THE SPACE OF VECTORS:  
 $N(S) = ( w(S); n1(S) , n2(S) , n3(S) )$**

Application Example 3: Multistage design



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T

## LECTURE 22. Course: “Design of Systems: Structural Approach”

Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics

Moscow Inst. of Physics and Technology (University)

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Inst. for Information Transmission Problems, RAS

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### L.22. System improvement (examples), system evolution (example)

#### *PLAN:*

1 Hierarchical Morphological Multicriteria Design (HMMD):

Application to system improvement (transformation, upgrade, adaptation)

2. System evolution / development:

Example for several generations of software DSS COMBI-PC

3. Example for complex system analysis and design

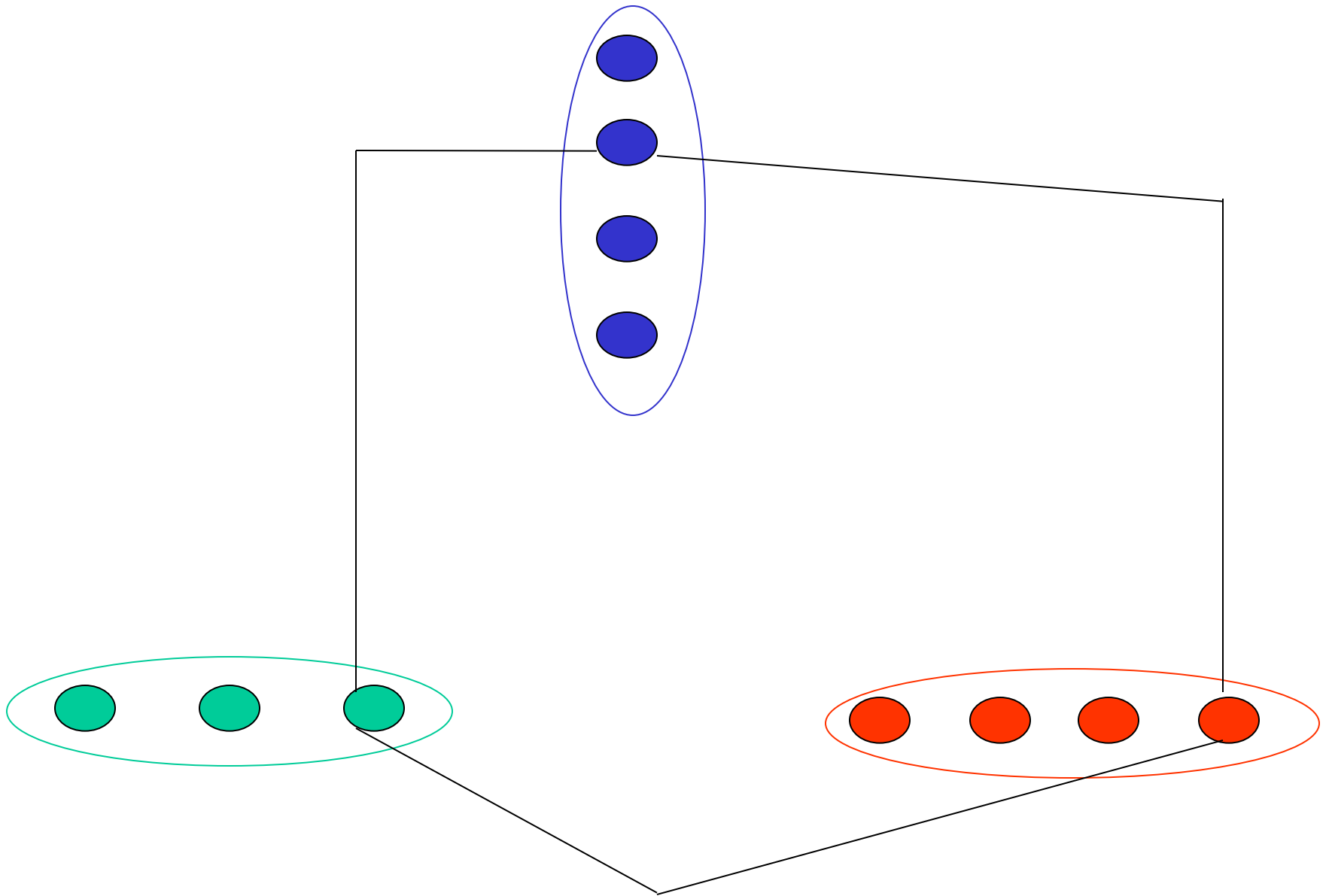
(modeling, analysis, & design of modular system):

(a) system modeling, (b) system comparison, (c) revelation of bottlenecks,

(d) system design, (e) system upgrade, (f) modeling of system generations.

Oct. 22, 2004

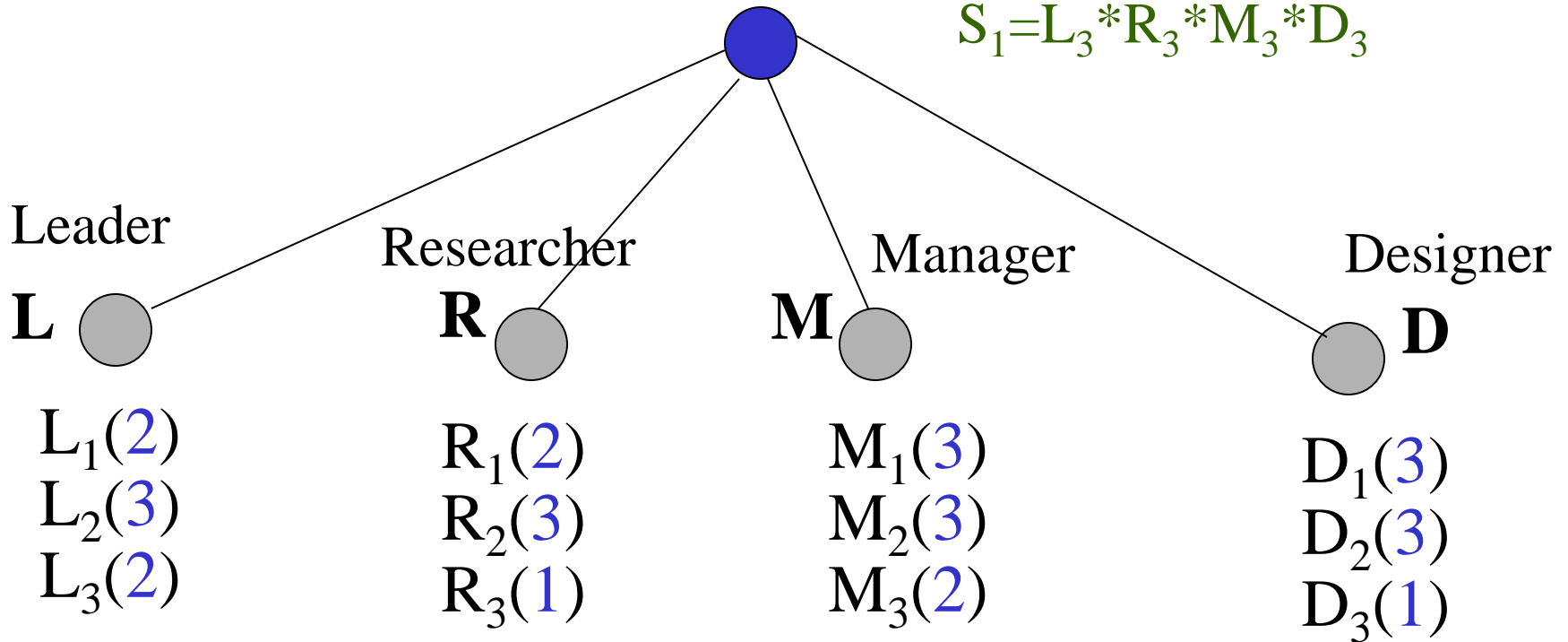
# Morphological clique



Application example 1: Design of team

Team:  $S = L * R * M * D$

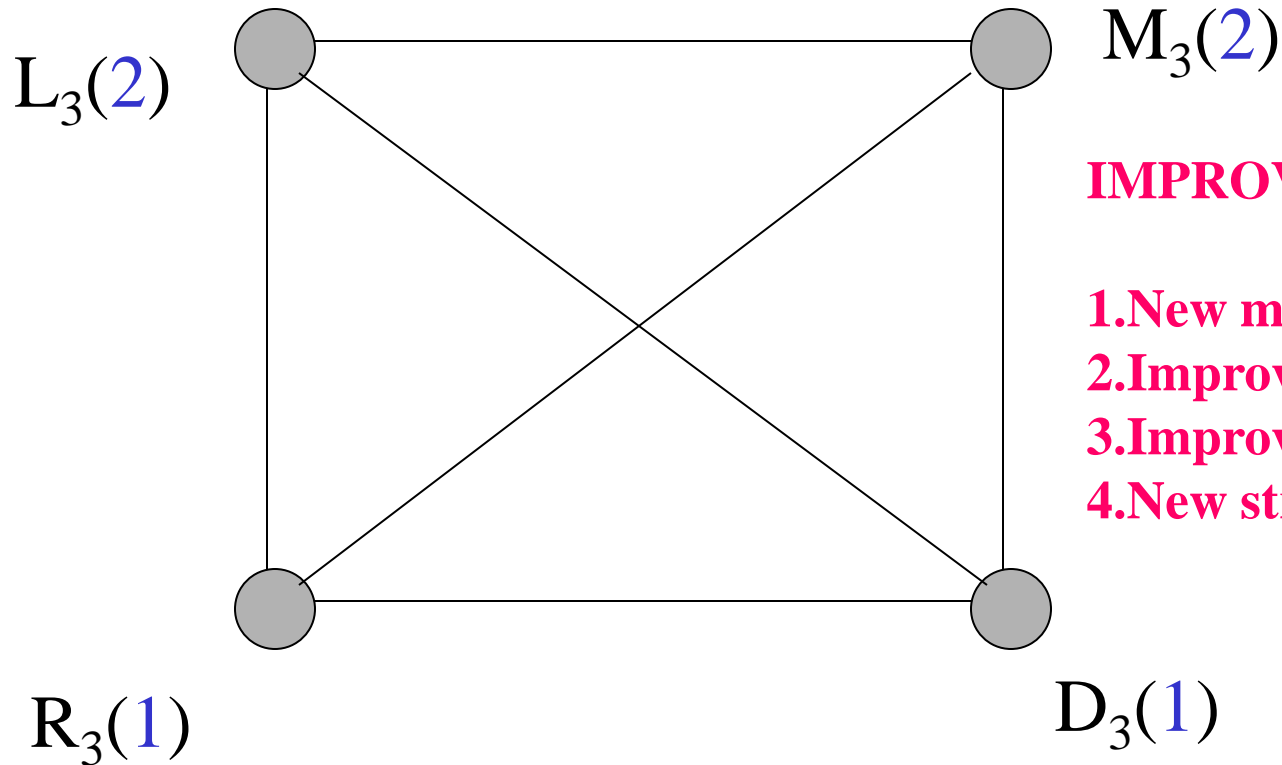
$$S_1 = L_3 * R_3 * M_3 * D_3$$



Application example 1: Design of team

Team:  $S = L * R * M * D$

$$S_1 = L_3 * R_3 * M_3 * D_3$$



**IMPROVEMENT ACTIONS:**

- 1. New member**
- 2. Improvement of member**
- 3. Improvement of compatibility**
- 4. New structure**

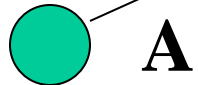
# Plan for improvement

$$S = A * B * C$$

$$S_1 = A_3 * B_1 * C_2$$

$$S_2 = A_3 * B_1 * C_3$$

**New members**



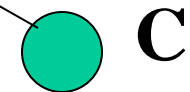
- $A_1(2)$
- $A_2(2)$
- $A_3 = A_1 \& A_2(2)$

**Professional courses**



- $B_1(1)$
- $B_2(2)$
- $B_3(1)$
- $B_4(1)$
- $B_5 = B_3 \& B_4(2)$
- $B_6 = B_1 \& B_4(3)$
- $B_7 = B_1 \& B_2 \& B_4(3)$

**Trips & communication**



- $C_1(2)$
- $C_2(1)$
- $C_3(1)$
- $C_4 = C_1 \& C_3(3)$

**A<sub>1</sub> new leader**

**A<sub>2</sub> new manager**

**A<sub>3</sub> = A<sub>1</sub> & A<sub>2</sub>**

**B<sub>1</sub> course on advances in science & engineering**

**B<sub>2</sub> course on foreign language**

**B<sub>3</sub> course on system analysis**

**B<sub>4</sub> course on creativity**

**B<sub>5</sub> = B<sub>3</sub> & B<sub>4</sub>**

**B<sub>6</sub> = B<sub>1</sub> & B<sub>4</sub>**

**B<sub>7</sub> = B<sub>1</sub> & B<sub>2</sub> & B<sub>4</sub>**

**C<sub>1</sub> course on human relation**

**C<sub>2</sub> joint trip to rest-home**

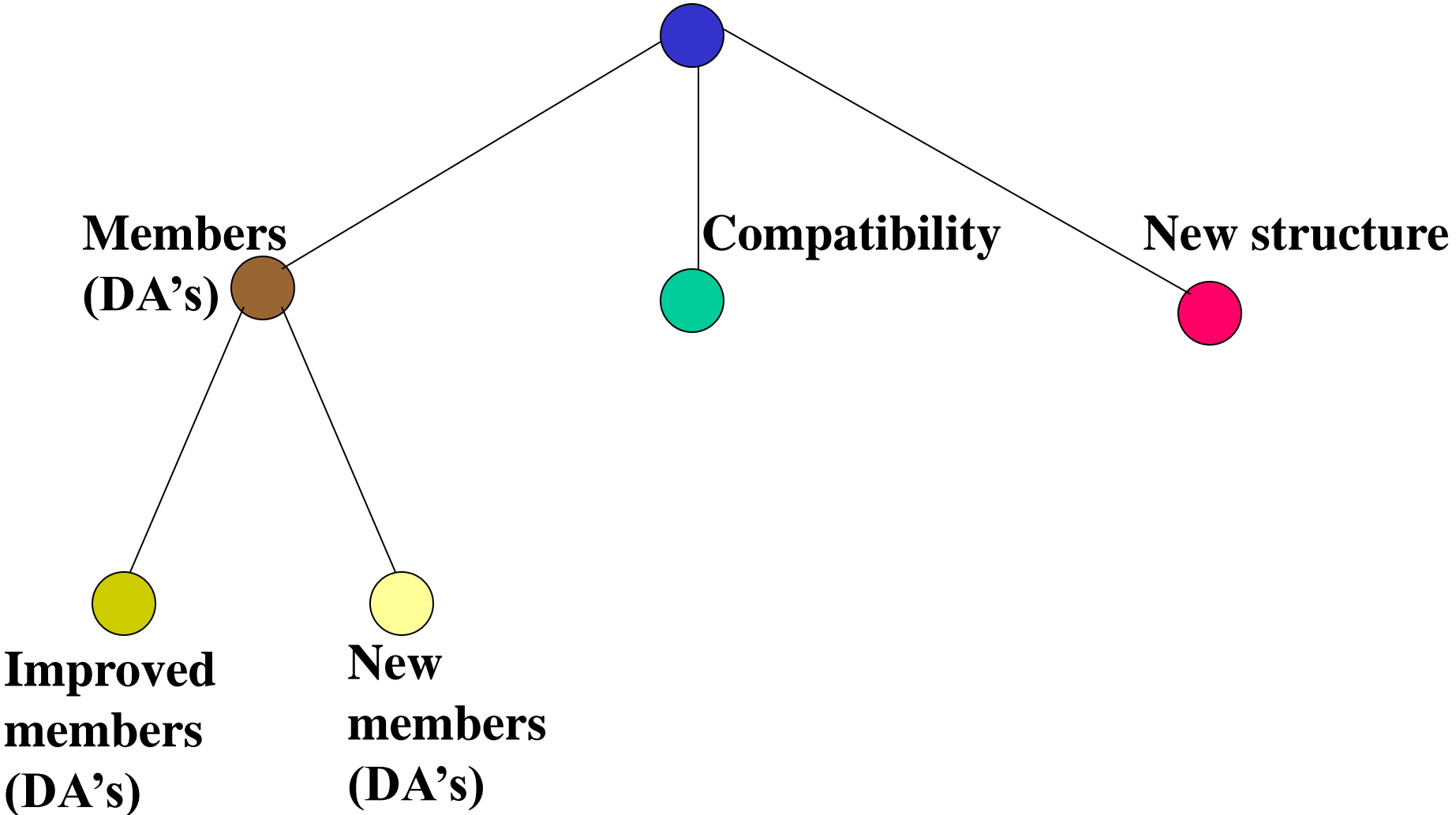
**C<sub>3</sub> joint participation in professional conference**

**C<sub>4</sub> = C<sub>1</sub> & C<sub>2</sub>**



Improvement of system

**Improvement (change) system**



## **SYSTEM IMPROVEMENT PROCESS:**

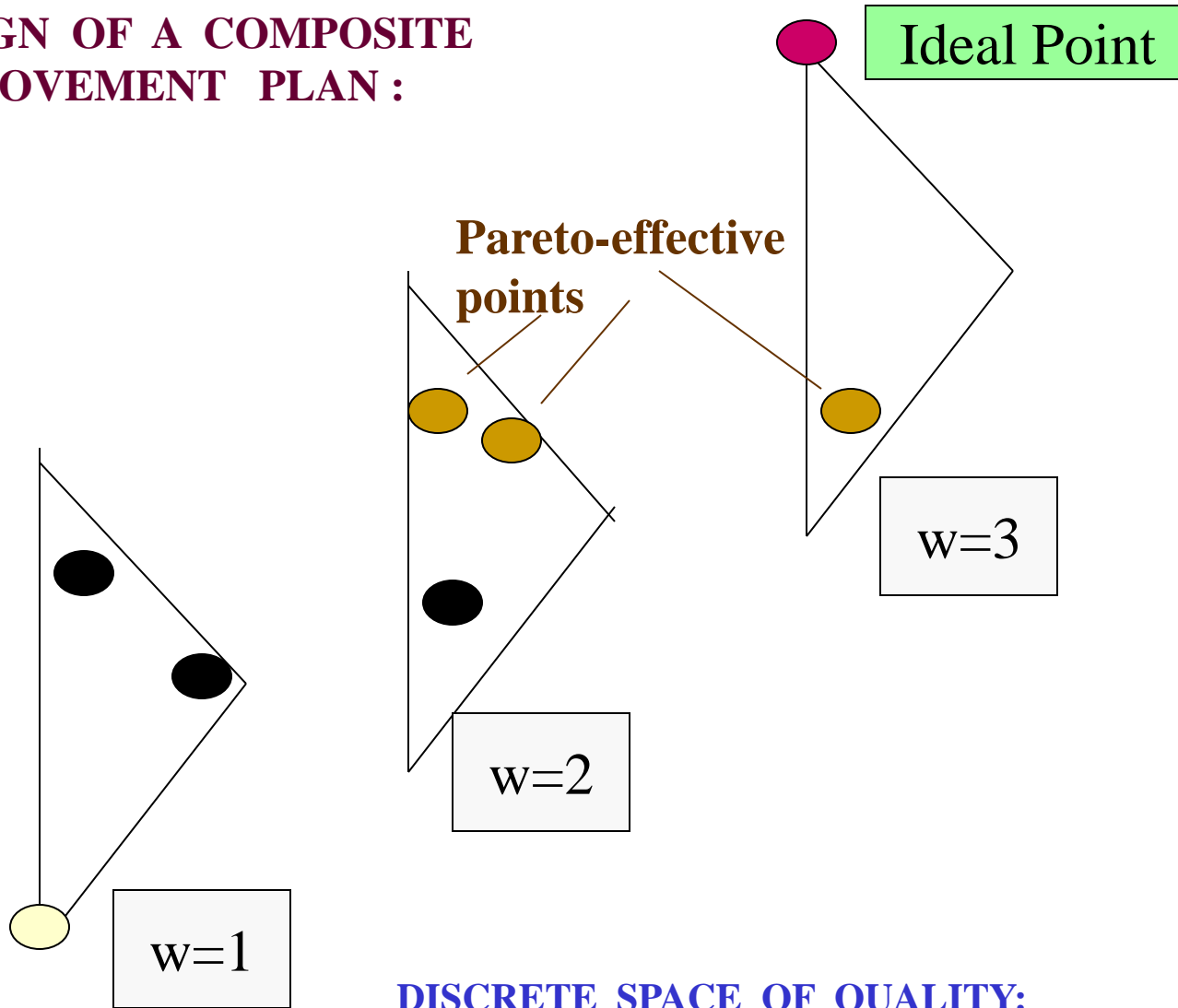
**1.Improvement / Upgrade**

**2.System change (e.g., development / evolution)**

**3.System transformation**

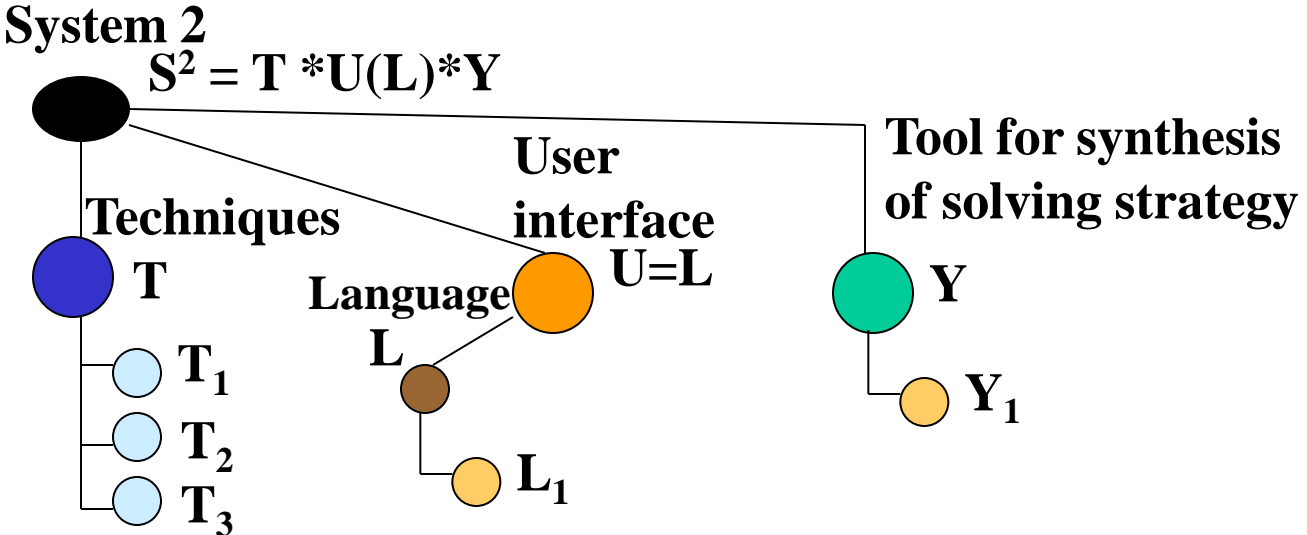
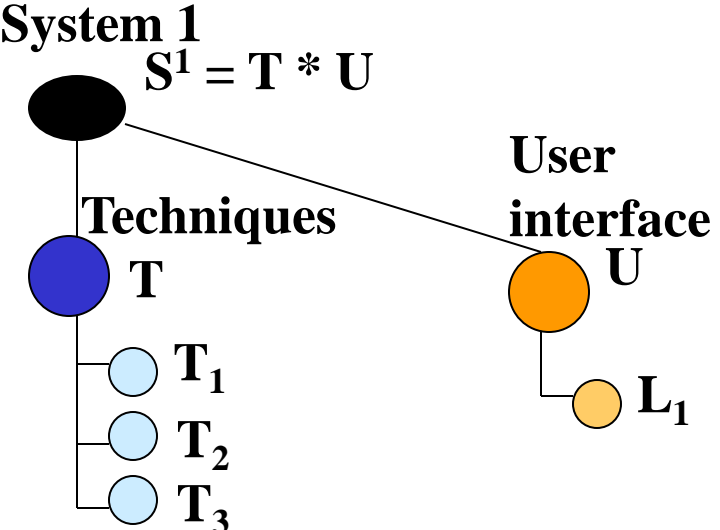
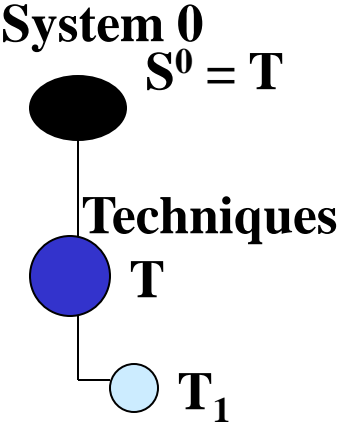
**4.Adaptation (including on-line adaptation)**

**DESIGN OF A COMPOSITE  
IMPROVEMENT PLAN :**



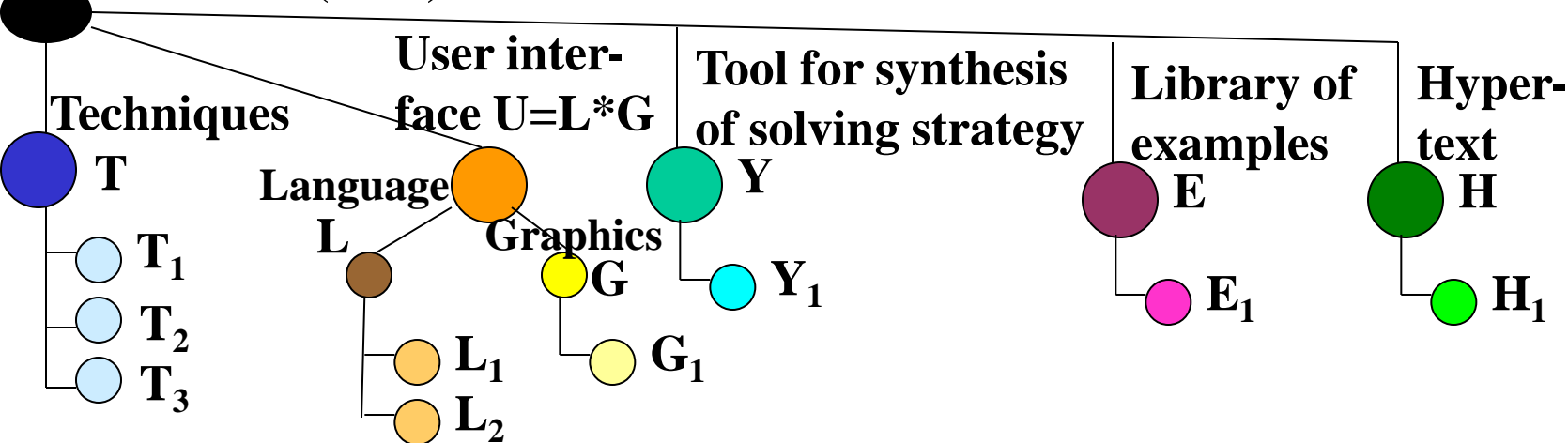
**DISCRETE SPACE OF QUALITY:**  
 $N(S) = ( w(S); n1(S) , n2(S) , n3(S) )$

Generations of software DSS COMBI-PC



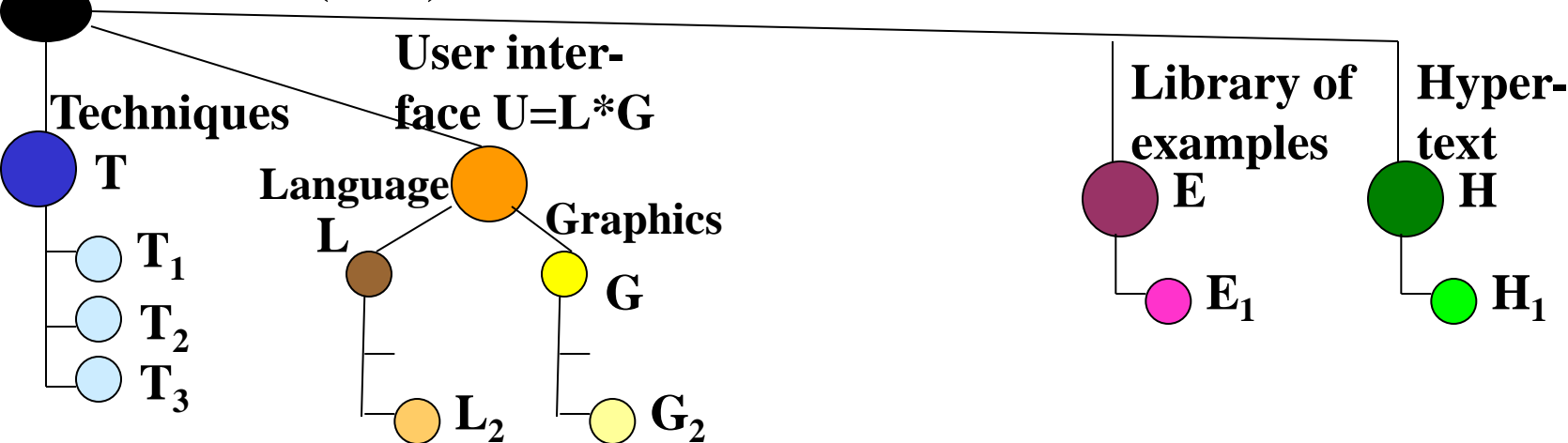
**System 3**

$$S^3 = T * U(L * G) * Y * E * H$$



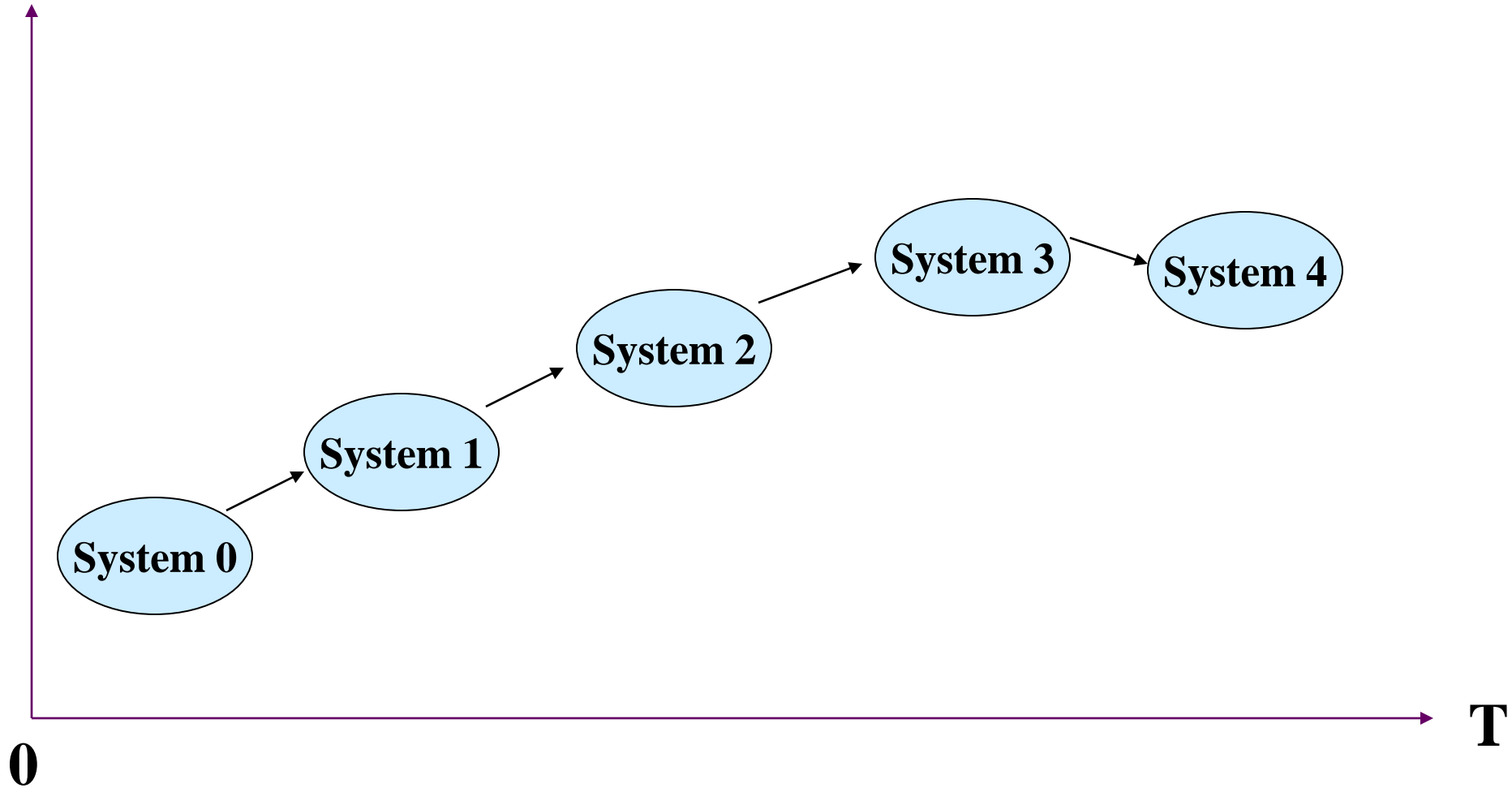
**System 4**

$$S^4 = T * U(L * G) * E * H$$



Generations of software DSS COMBI

Improvement



**LECTURE 23-24 (compressed version). Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Inst. of Physics and Technology (University)**

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**L.23. Basic systems problems. Example for notebook.**

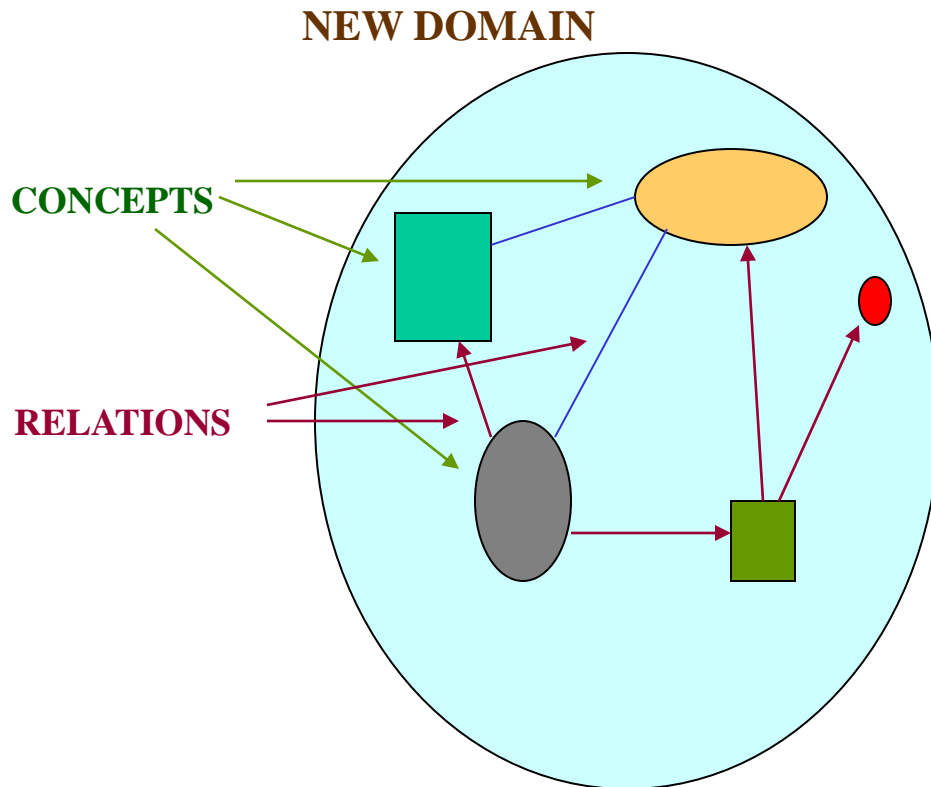
**L.24. Systems for signal processing, system change process.**

***PLAN:***

1. Analysis of a new domain: construction of a new world
2. Hierarchical Morphological Multicriteria Design (HMMD): framework of analysis & design problems.
3. Illustrative example for note book
4. Layers of “systems”: \*system, \* requirements, \*standards
5. Development / evolution of modular systems: illustrative examples: \*notebook, \*device for signal processing
6. Standard “system change” operations
7. Basic combinatorial optimization problems for system improvement / adaptation / upgrade / etc.:
  - \*knapsack, \*multicriteria ranking, \*multiple choice problem, \*multicriteria knapsack,
  - \*multicriteria multiple choice problem, \*scheduling, \*combinatorial synthesis, \*multi-stage design

Oct. 23, 2004

**TWO SITUATIONS:**  
**1. Principally New Domain**  
**2. New Domain for Researcher**



**ALGORITHM SCHEME:**

- 1. Revelation of basic concepts (objects, resources, goals, participants)**
- 2. Revelation of basic relations over the above-mentioned concepts**
- 3. Formulation of main problems (e.g., resource assignment, planning/scheduling)**
- 4. Design of solving schemes**
- 5. Solving of numerical examples**
- 6. Study of real (realistic) applications**
- 7. Etc.**



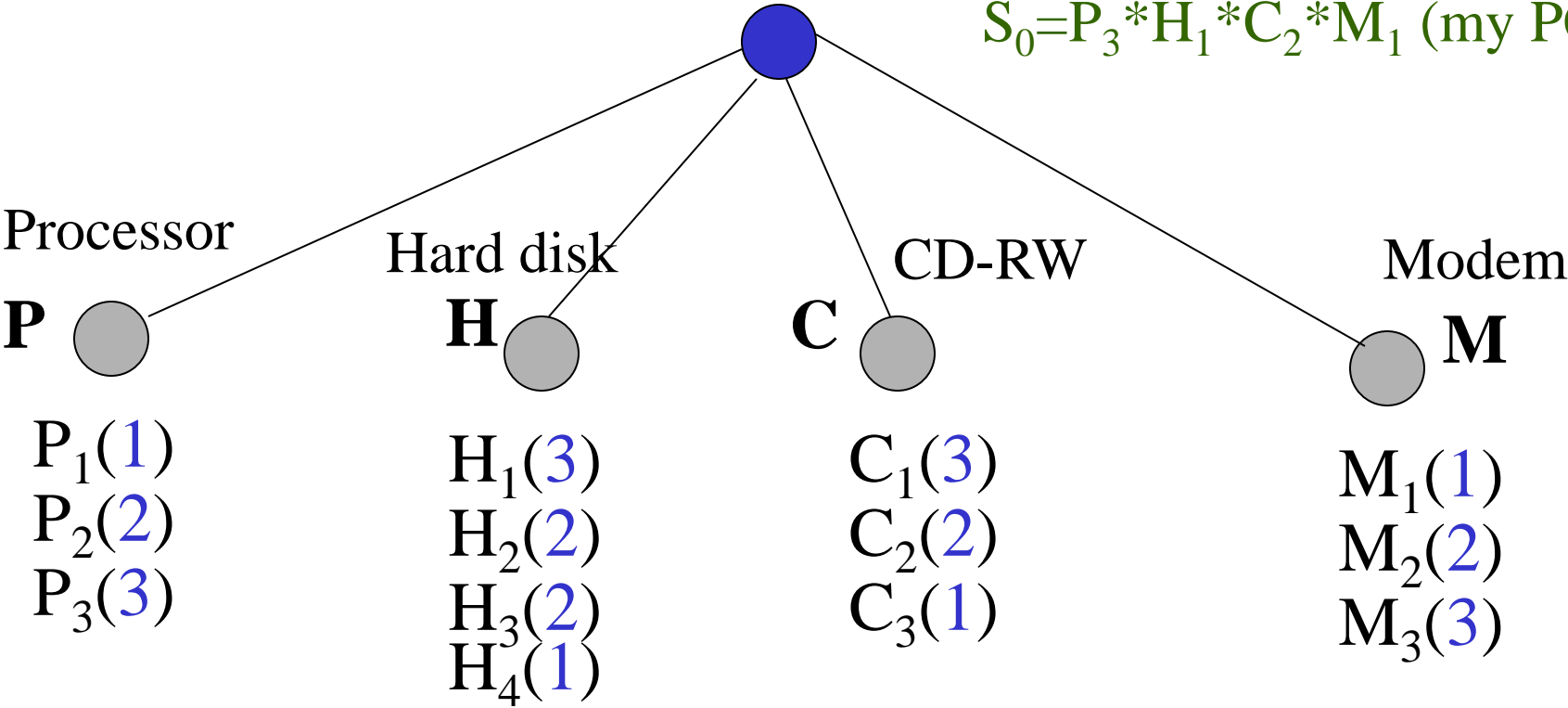
## **SYSTEM ANALYSIS & DESIGN PROBLEMS**

- 1. Modeling of system (structural model, e.g., and-or graph)** ●
- 2. Multicriteria comparison** ●
- 3. Revelation of bottlenecks** ●
- 4. Hierarchical modular design** ●
- 5. Upgrade (improvement, adaptation)** ●
- 6. Multi-stage design**
- 7. Modeling of development process (flow of system generations)** ●
- 8. System forecasting**

Example: Notebook

My notebook:  $S = P * H * C * M$

$S_0 = P_3 * H_1 * C_2 * M_1$  (my PC)



**ALTERNATIVES:**

**P<sub>1</sub> Intel-4**

**P<sub>2</sub> Intel-3**

**P<sub>3</sub> Celeron**

**H<sub>1</sub> 20Gb**

**H<sub>2</sub> 40Gb**

**H<sub>3</sub> 60Gb**

**H<sub>4</sub> 80Gb**

**C<sub>1</sub> None**

**C<sub>2</sub> Read**

**C<sub>3</sub> Read & Write**

**M<sub>1</sub> 56 Kbit / sec**

**M<sub>2</sub> 48 Kbit / sec**

**M<sub>3</sub> 32 Kbit / sec**

**COMPARISON of note books:**

	<b>Cost (-)</b>	<b>Reliability (+)</b>	<b>Maintenance- ability(+)</b>	<b>Upgrade- ability(+)</b>	<b>Total</b>
<b>1.Alternative 1</b>	<b>1300 (6)</b>	<b>5</b>	<b>3</b>	<b>5</b>	<b>2 (1)</b>
<b>2.Alternative 2</b>	<b>1250 (5)</b>	<b>4</b>	<b>3</b>	<b>4</b>	<b>3 (3)</b>
<b>3.My note book</b>	<b>900 (2)</b>	<b>4</b>	<b>4</b>	<b>5</b>	<b>1 (1)</b>
<b>4.Alternative 3</b>	<b>1200 (4)</b>	<b>5</b>	<b>3</b>	<b>4</b>	<b>3 (2)</b>
<b>5.Alternative 4</b>	<b>1200 (4)</b>	<b>5</b>	<b>3</b>	<b>3</b>	<b>3 (3)</b>
<b>6.Alternative 5</b>	<b>1100 (3)</b>	<b>4</b>	<b>4</b>	<b>4</b>	<b>2 (2)</b>
<b>7.Moscow's PC</b>	<b>700 (1)</b>	<b>3</b>	<b>5</b>	<b>5</b>	<b>1 (1)</b>
<b>8.Alternative 6</b>	<b>1200 (4)</b>	<b>4</b>	<b>3</b>	<b>3</b>	<b>4 (4)</b>

**Weights of criteria:**

**2 2 3 2**

**2 5 4 5**

Example: Notebook

$$\text{NOW: } S_0 = P_3 * H_1 * C_2 * M_1$$

**BOTTLENECKS:**

	<b>Cost of upgrade (-)</b>	<b>Reliability (-)</b>	<b>Damage (+)</b>	<b>Total</b>
<b>1.P<sub>3</sub></b>	<b>100</b>	<b>5</b>	<b>2</b>	<b>3 (3)</b>
<b>2.H<sub>1</sub></b>	<b>80</b>	<b>3</b>	<b>5</b>	<b>1 (1)</b>
<b>3.C<sub>2</sub></b>	<b>200</b>	<b>4</b>	<b>1</b>	<b>4 (4)</b>
<b>4.M<sub>1</sub></b>	<b>50</b>	<b>5</b>	<b>4</b>	<b>2 (2)</b>

**Weights of criteria:**

**1 1 1**  
**2 4 5**

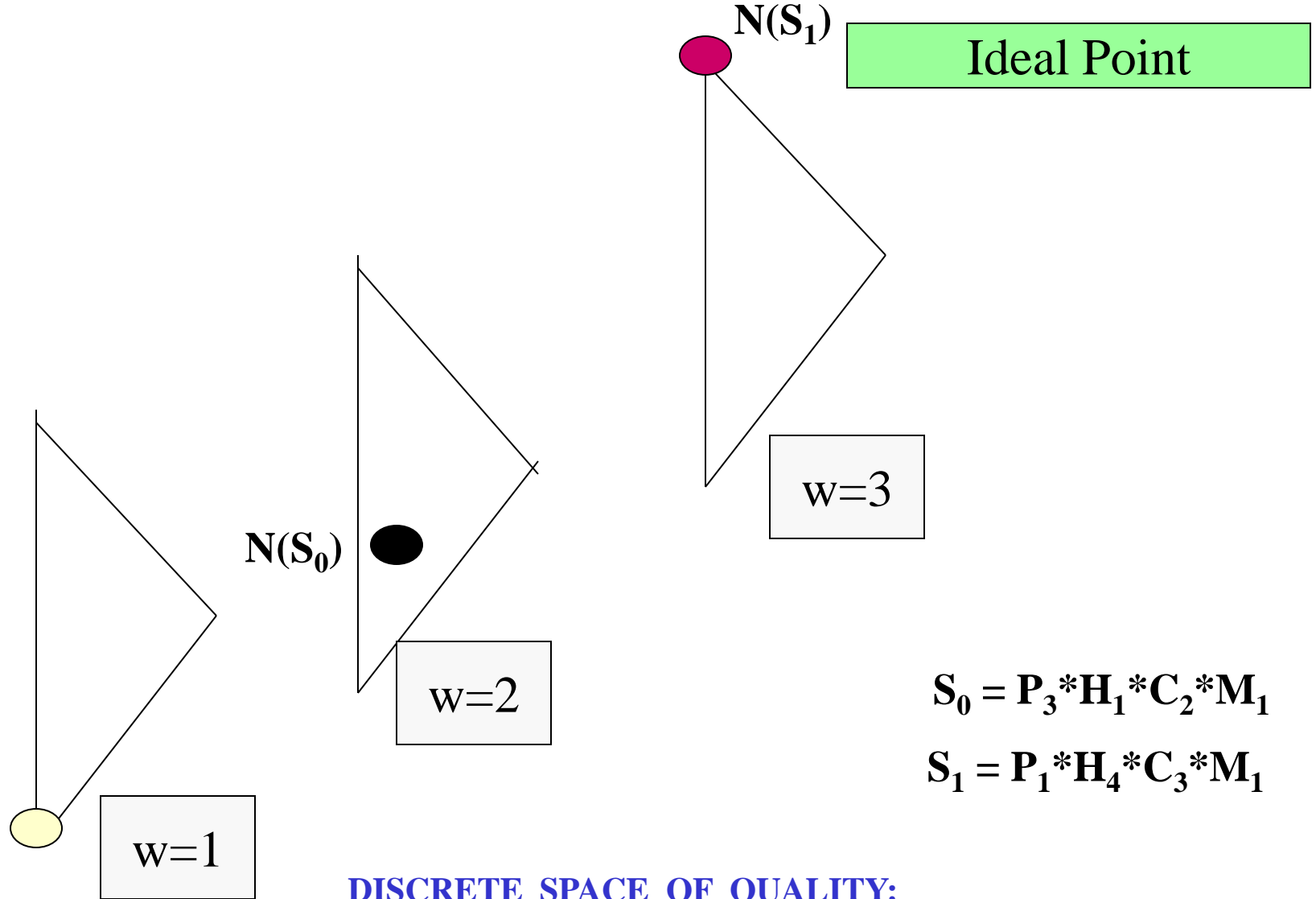
Assessment of compatibility between alternatives (example for notebook)

	H <sub>1</sub>	H <sub>2</sub>	H <sub>3</sub>	H <sub>4</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>
P <sub>1</sub>	1	2	3	3	0	2	3	3	1	1
P <sub>2</sub>	3	2	1	1	1	3	3	2	3	1
P <sub>3</sub>	2	3	1	2	0	3	2	3	2	1
H <sub>1</sub>					1	3	1	3	3	2
H <sub>2</sub>					1	3	2	3	3	1
H <sub>3</sub>					1	2	3	3	2	1
H <sub>4</sub>					1	2	3	3	2	1
C <sub>1</sub>								3	2	2
C <sub>2</sub>								3	3	1
C <sub>3</sub>								3	3	1

**NOTE: 3 corresponds to the best level of compatibility**  
**0 corresponds to incompatibility**

The best combination is :  $S_1 = P_1 * H_4 * C_3 * M_1$   $N(S_1) = (3 ; 4,0,0)$

Upgrade (example for notebook)



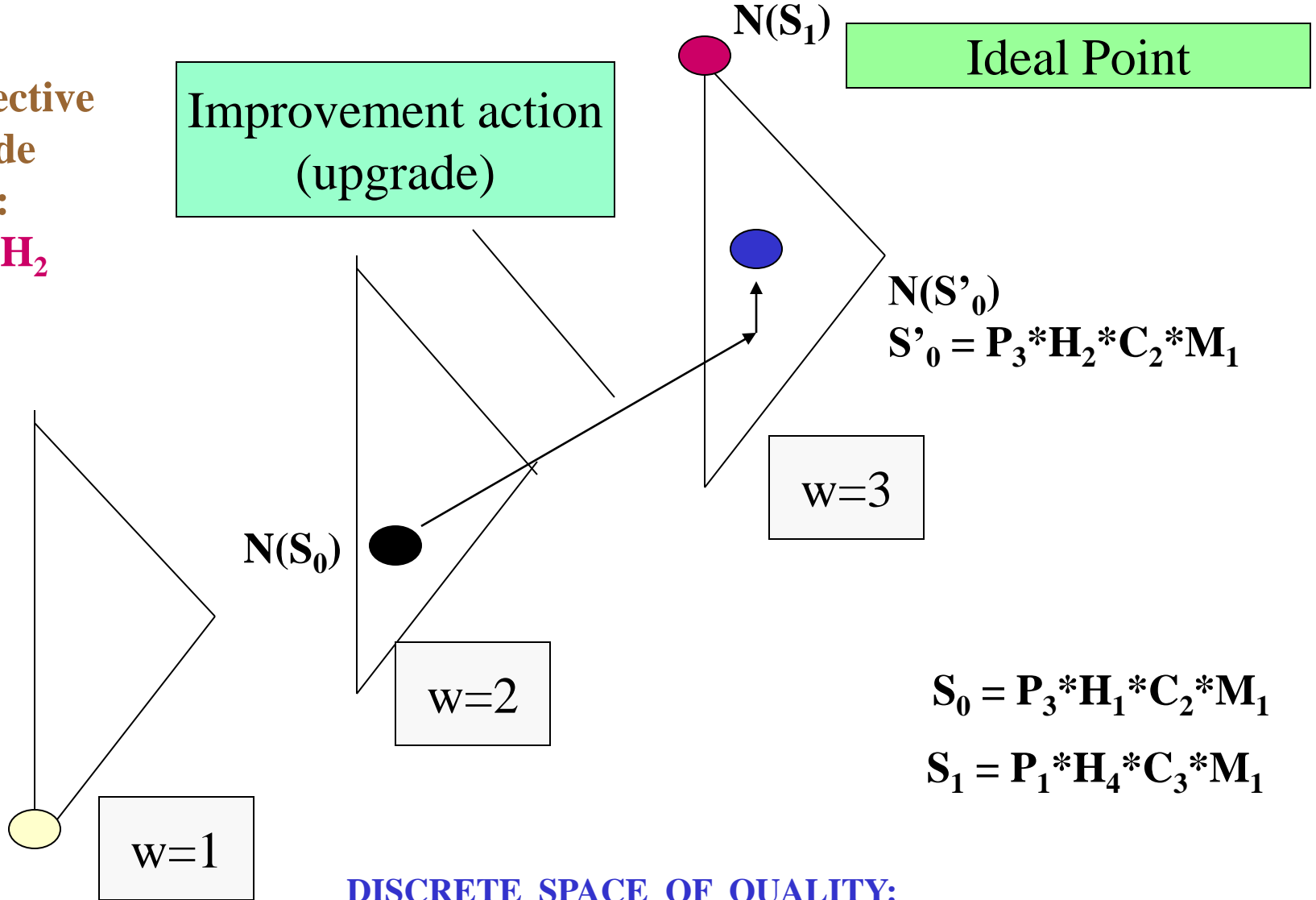
$$S_0 = P_3 * H_1 * C_2 * M_1$$

$$S_1 = P_1 * H_4 * C_3 * M_1$$

**DISCRETE SPACE OF QUALITY:**  
 $N(S) = ( w(S); n1(S) , n2(S) , n3(S) )$

Upgrade (example for notebook)

Prospective  
upgrade  
action:  
 $H_1 \Rightarrow H_2$



Ideal Point

$$N(S'_0)$$

$$S'_0 = P_3 * H_2 * C_2 * M_1$$

w=3

$N(S_0)$

w=2

$$S_0 = P_3 * H_1 * C_2 * M_1$$

$$S_1 = P_1 * H_4 * C_3 * M_1$$

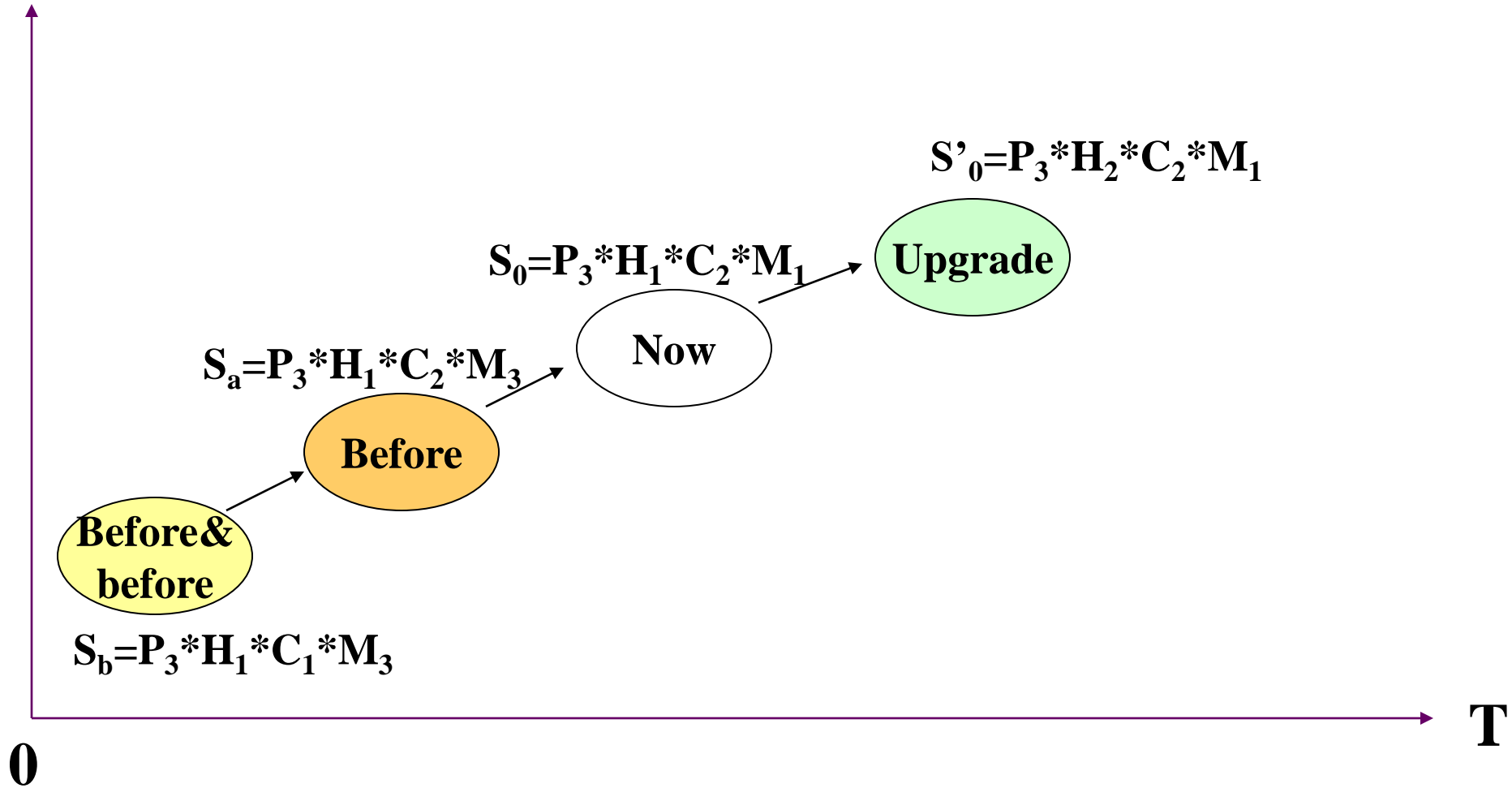
w=1

DISCRETE SPACE OF QUALITY:  
 $N(S) = ( w(S); n1(S) , n2(S) , n3(S) )$



Example for notebook

Improvement



**STANDARDS to system**

**REQUIREMENTS / CRITERIA to system**

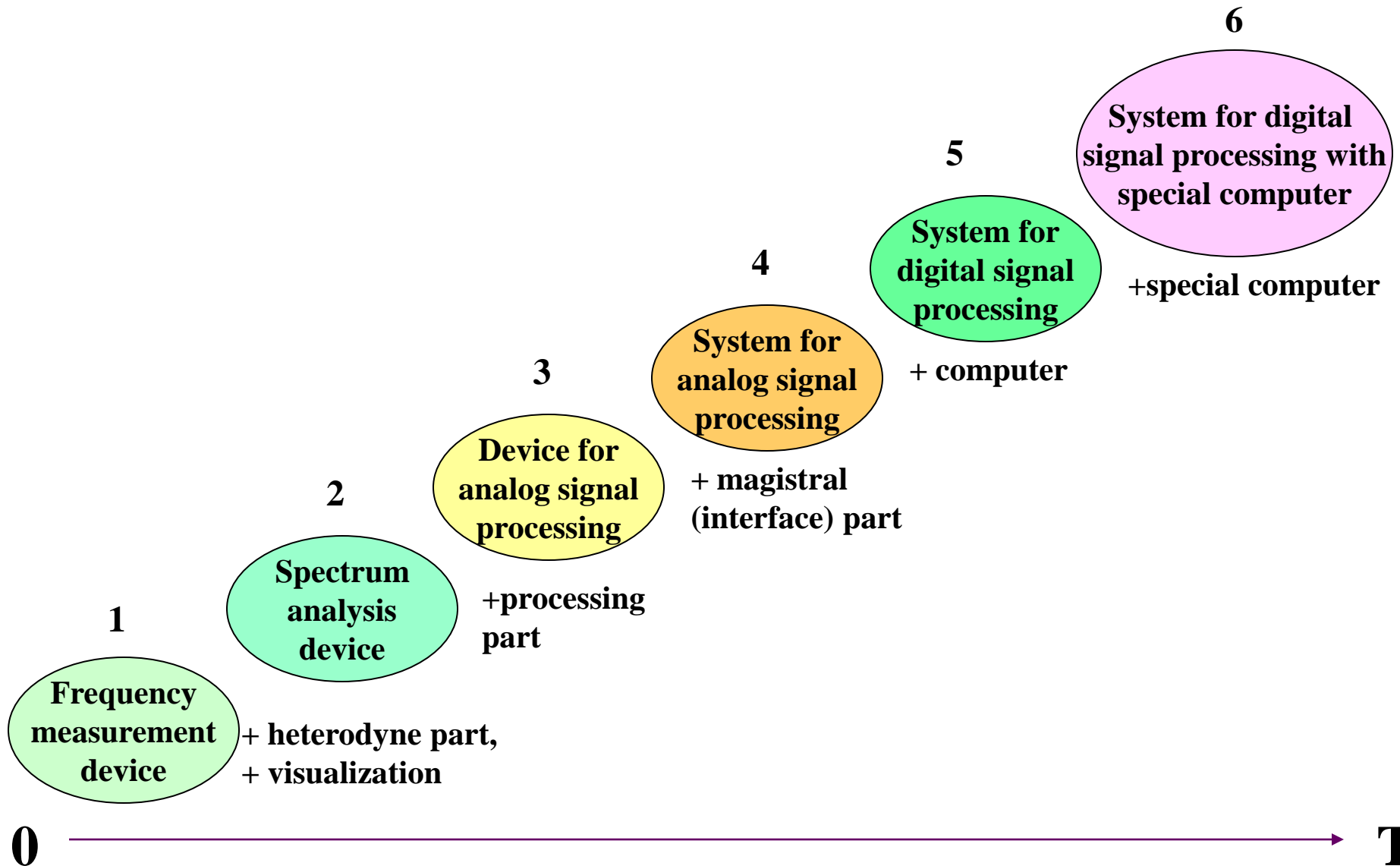
**SYSTEM**

**0**



**T**

Macro-evolution process for signal processing



## CHANGE OPERATIONS:

### I. Operations for DA's:

1.1. Change / improvement of DA's  $O_1: A_i \Rightarrow A'_i$

1.2. Deletion of DA  $O_2$

1.3. Addition of DA  $O_3$

1.4. Aggregation of DA's  $O_4: \{ A_i \} \Rightarrow A^a = A_1 \& A_2 \& \dots$

1.5. Standardization of DA's  $O_5: \{ A_i \} \Rightarrow A^s$

### II. Operations for subsystems (parts, components):

2.1. Change / improvement of a system part  $O_6$

2.2. Deletion of system part  $O_7$

2.3. Addition of system part  $O_8$

2.4. Aggregation of system part  $O_9$

## **I.Characteristics over change operations:**

- 1.Required resource**
- 2.Possible profit**
- 3.Etc.**

## **II.Binary relations over change operations:**

- 1.Precedence constraints (  $O_i \Rightarrow O_j$  )**
- 2.Equivalence**
- 3.Complementarity**

## **POSSIBLE COMBINATORIAL PROBLEMS:**

- 1.Multicriteria ranking**
- 2.Knapsack problem**
- 3.Multiple choice problem**
- 4.Multicriteria knapsack problem**
- 5.Multicriteria multiple choice problem**
- 6.Scheduling**
- 7.Combinatorial synthesis (modular design)**
- 8.Multi-stage design**

**LECTURE 25. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Inst. of Physics and Technology (University)**

**Mark Sh. Levin**

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Email: [mslevin@acm.org](mailto:mslevin@acm.org) / [mslevin@iitp.ru](mailto:mslevin@iitp.ru)

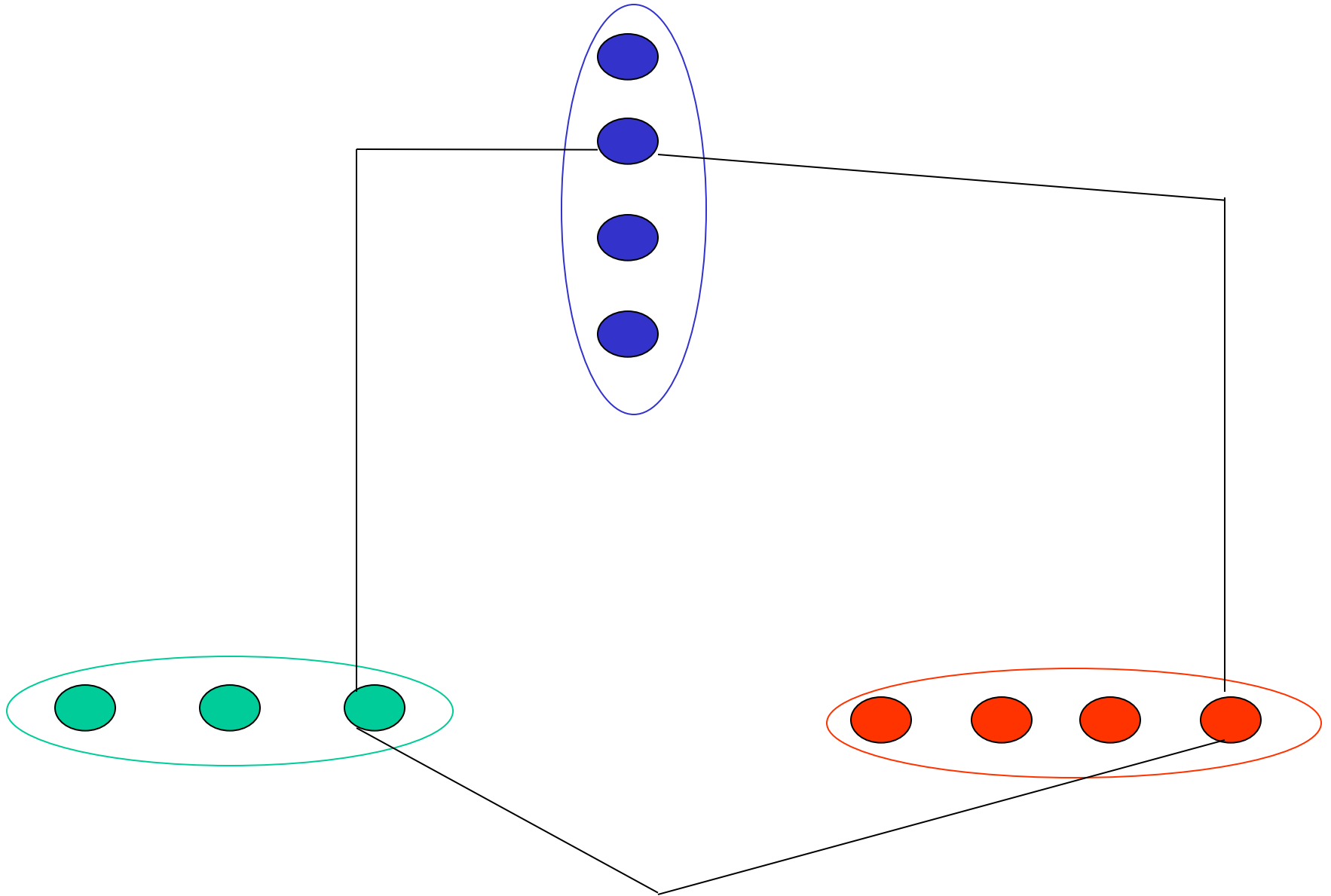
**L.25. Design of life cycle. Systems with common modules.**

***PLAN:***

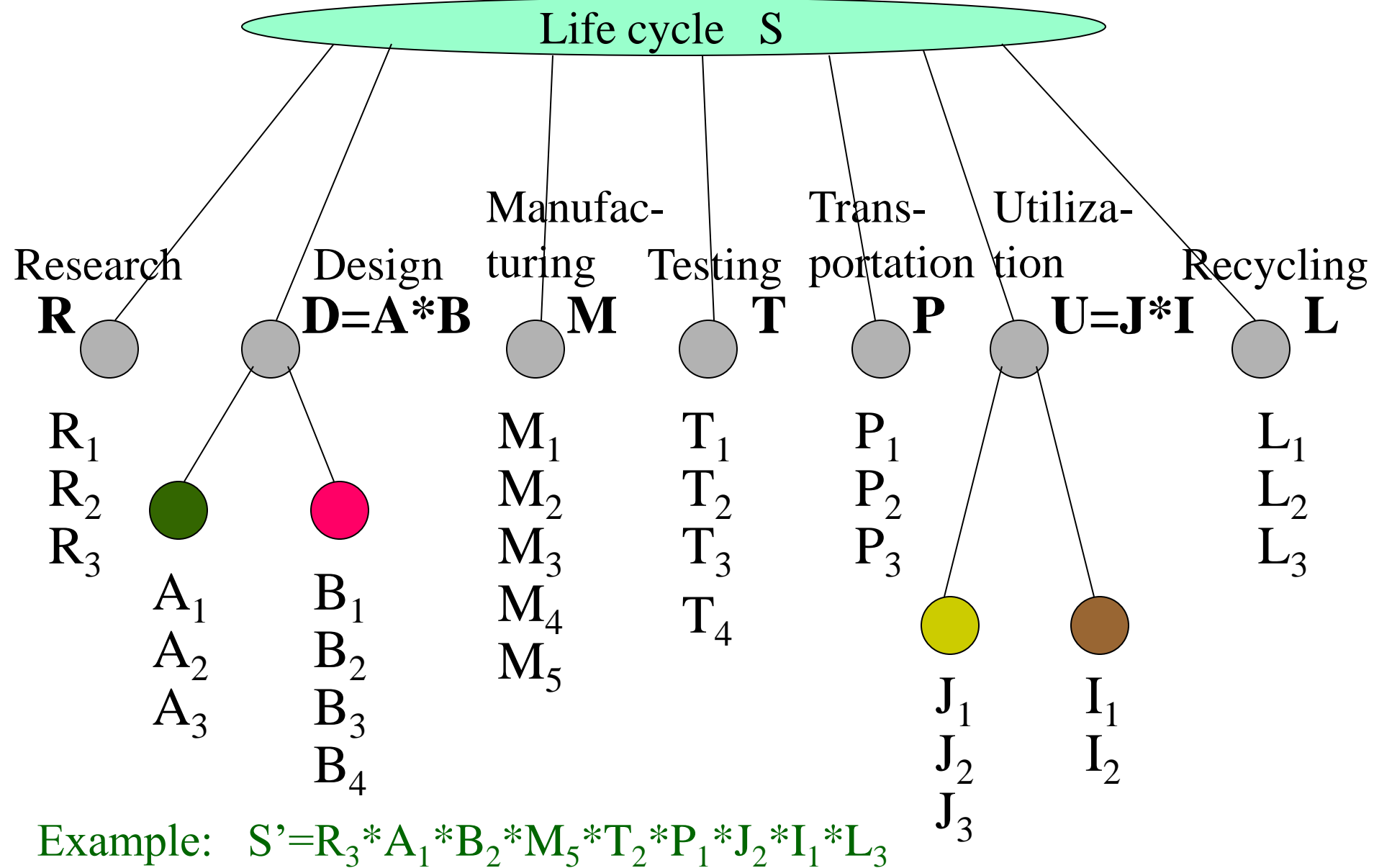
- 1.Design of life cycle: illustrative example (morphological combinatorial approach)
- 2.Morphological combinatorial approach to multi-product system: common modules:
  - \*2-product system (one common module, k common modules)
  - \*m-product system (one common module, k common modules)
- 3.Recent references

Oct. 29, 2004

# Morphological clique



Design (planning) of life cycle





Life cycle S

Example 1: Design & Manufacturing  $S_1 = D^1(A_2 * B_1) * M_3$

Example 2: Design & Testing  $S_2 = D^2(A_1 * B_3) * T_3$

Example 3: Research & Design & Manufacturing

$S_3 = R_2 * D'(A_3 * B_1) * M_4$

Example 4: Design & Manufacturing & Transportation

$S_1 = D''(A_3 * B_3) * M_4 * T_3$

Life cycle S

**Design (Planning) of Life Cycle**

**Life Cycle Management**

**Life Cycle Engineering**

**Support of Life Cycle**

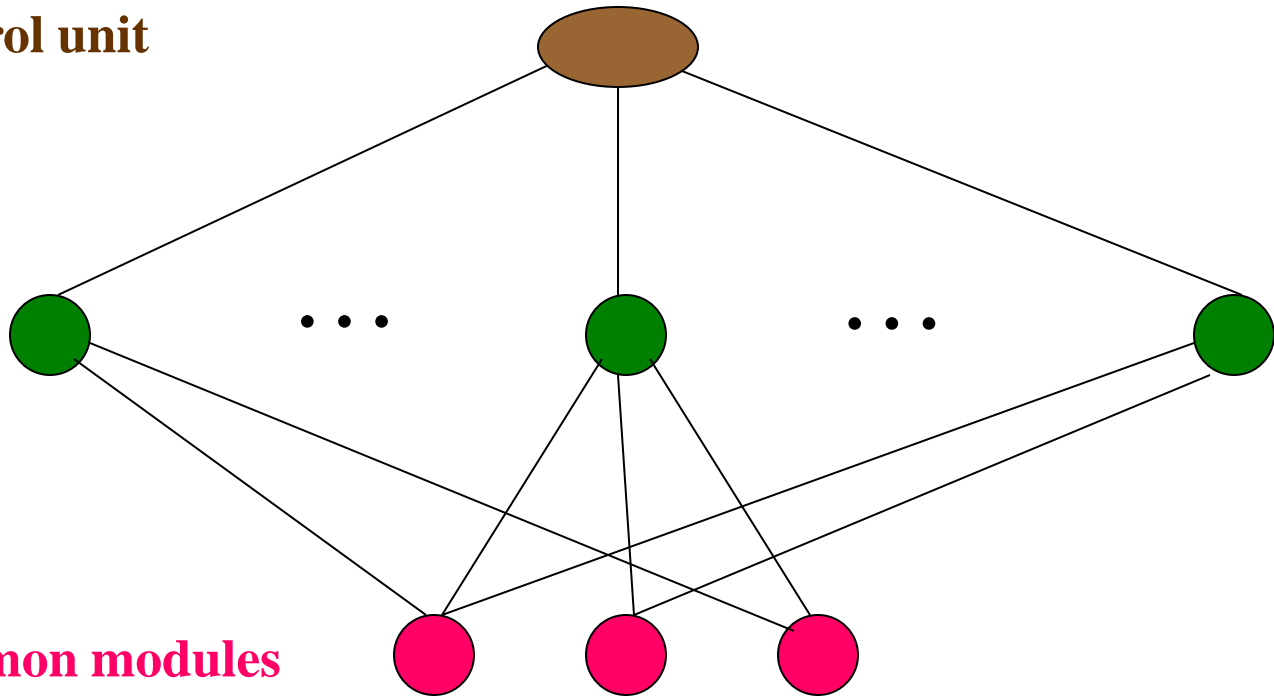
**Maintenance of Life Cycle**

## Structure for Modular Software Package: 3 layers

**Layer 1: Control unit**

**Layer 2:  
Function  
units**

**Layer 3: Common modules**



# General Structure for Modular Software Package

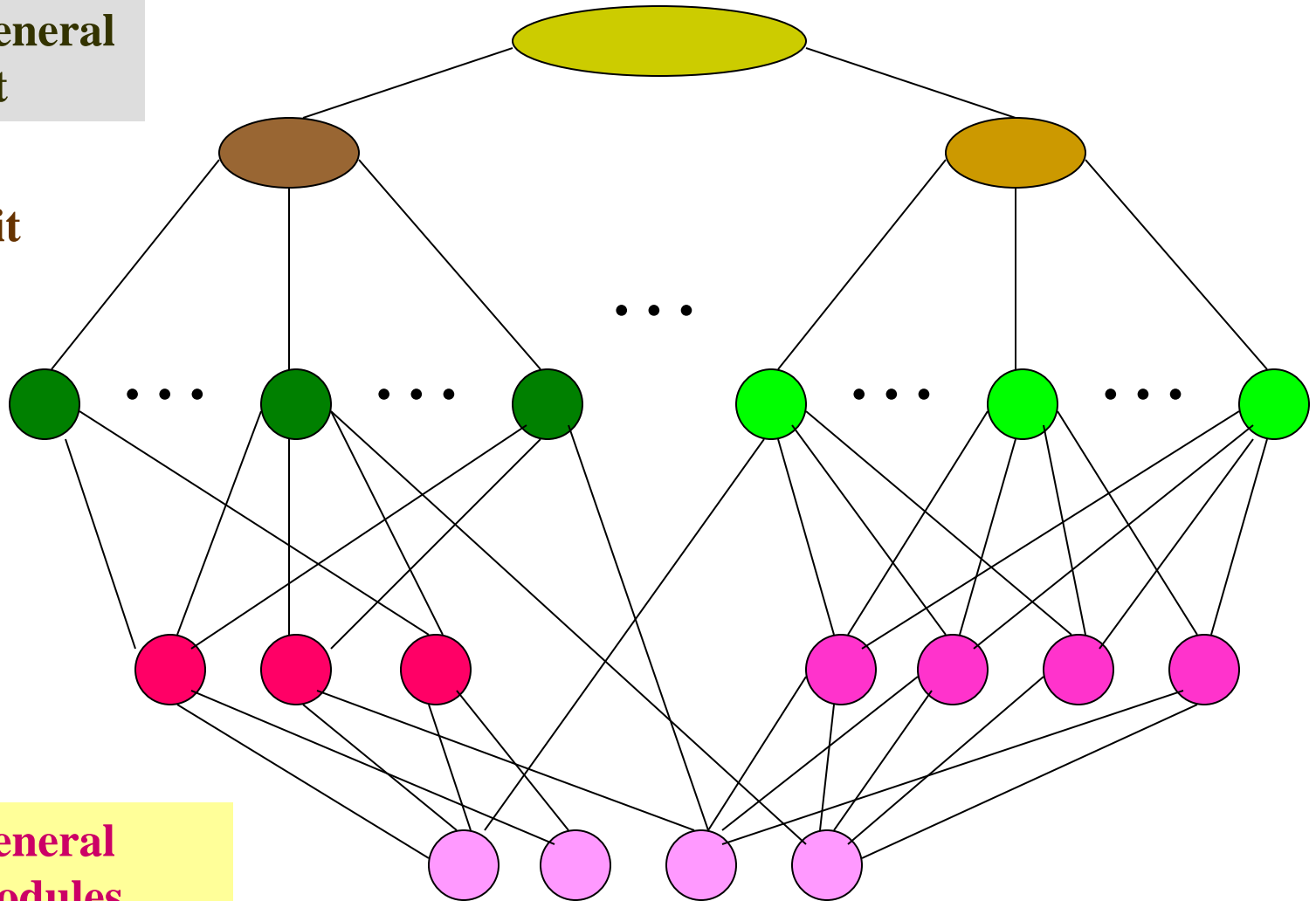
**Layer 0: General control unit**

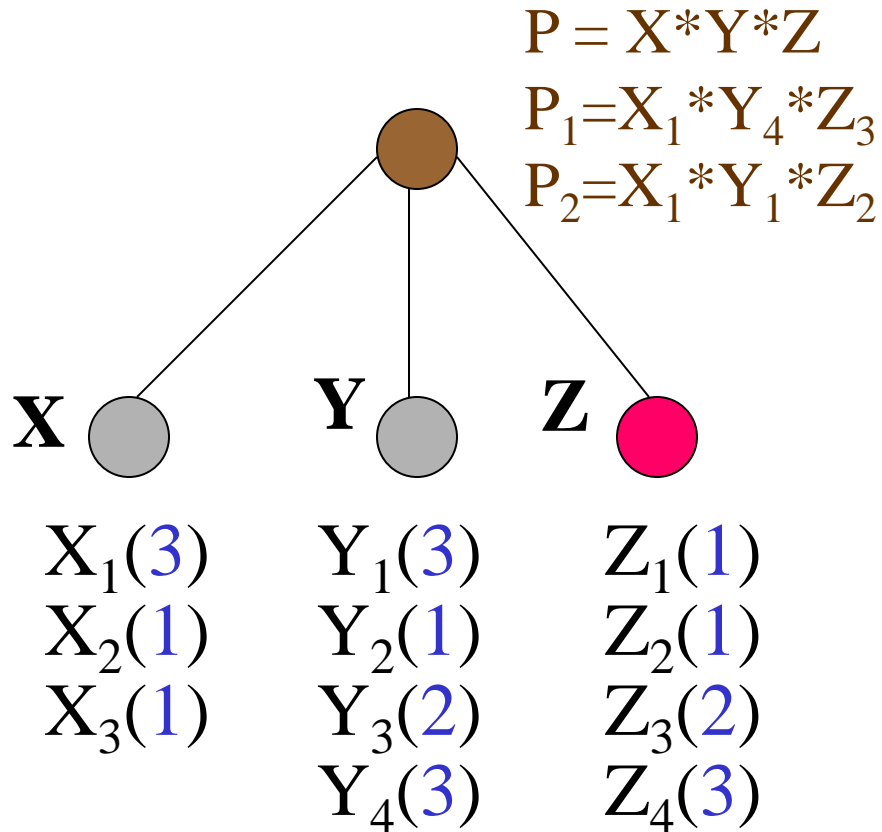
**Layer 1: Control unit**

**Layer 2: Function units**

**Layer 3: Common modules**

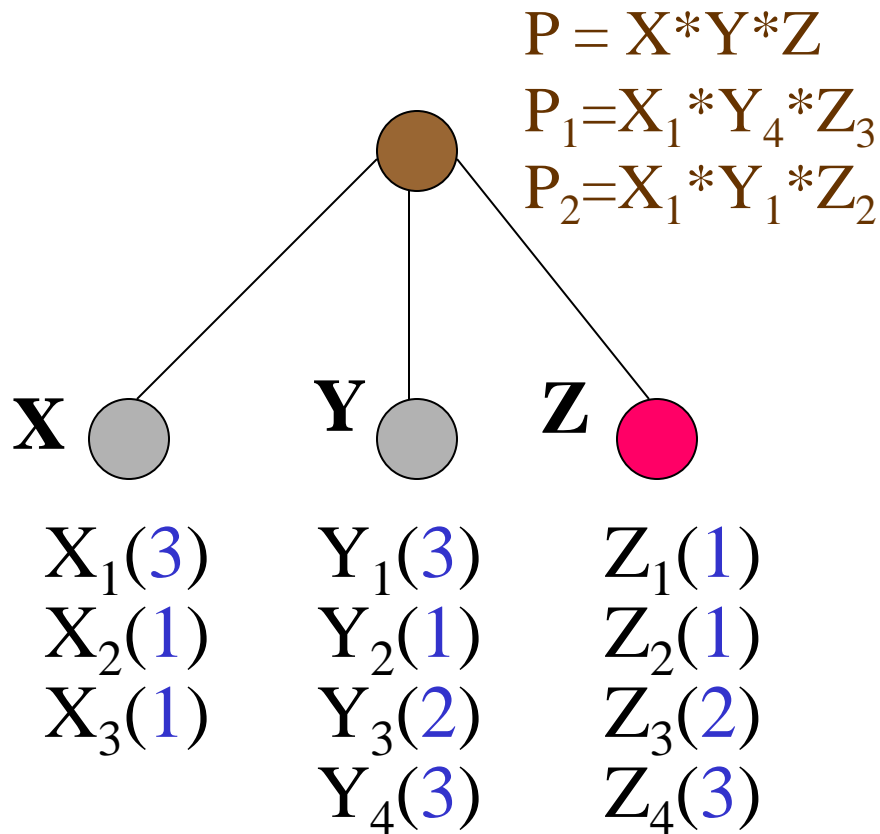
**Layer 4: General common modules**



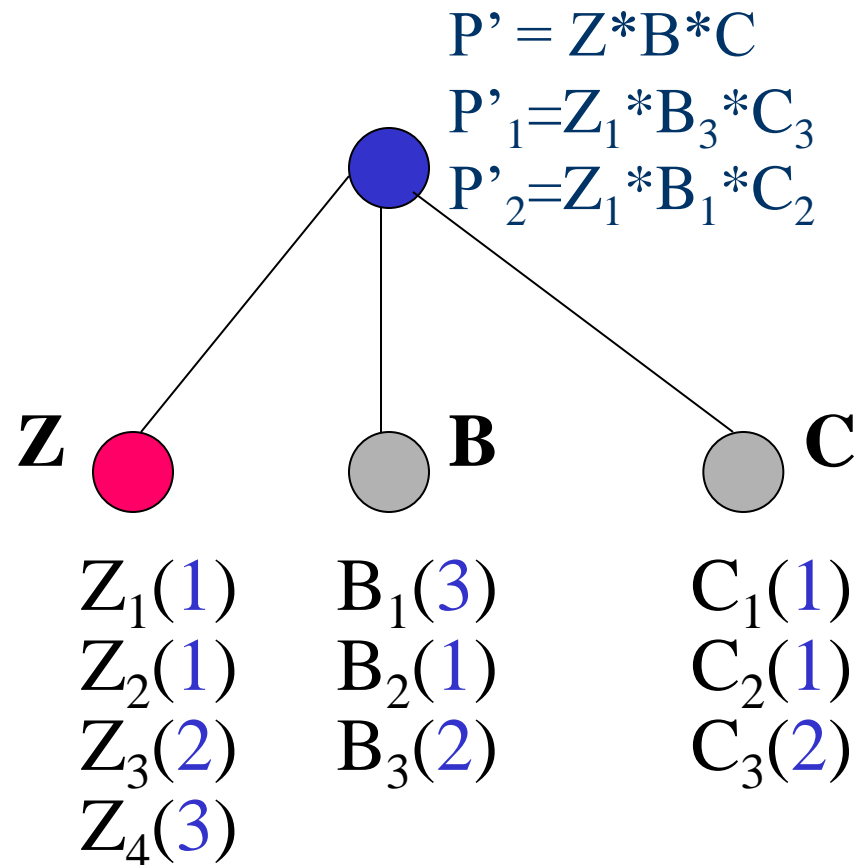
The 1<sup>st</sup> Product

2-product system

The 1<sup>st</sup> Product

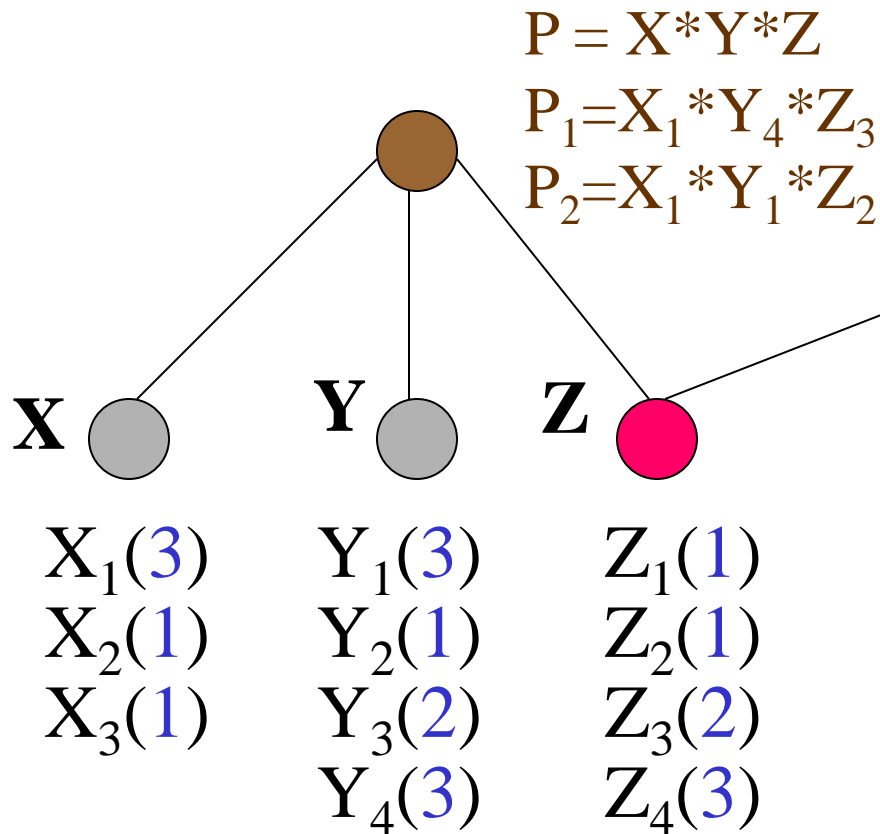


The 2<sup>nd</sup> Product

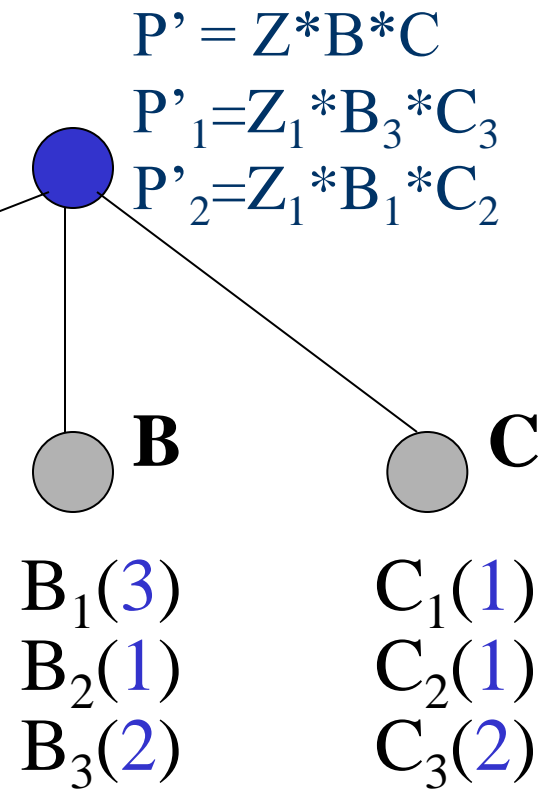


2-product system (one common module)

The 1<sup>st</sup> Product



The 2<sup>nd</sup> Product



2-product system (one common module)

$$S = P * P'$$

The 1<sup>st</sup> Product

The 2<sup>nd</sup> Product

$$P = X * Y * Z$$

$$P_1 = X_1 * Y_4 * Z_3$$

$$P_2 = X_1 * Y_1 * Z_2$$

$$P' = Z * B * C$$

$$P'_1 = Z_1 * B_3 * C_3$$

$$P'_2 = Z_1 * B_1 * C_2$$



- |          |          |          |
|----------|----------|----------|
| $X_1(3)$ | $Y_1(3)$ | $Z_1(1)$ |
| $X_2(1)$ | $Y_2(1)$ | $Z_2(1)$ |
| $X_3(1)$ | $Y_3(2)$ | $Z_3(2)$ |
|          | $Y_4(3)$ | $Z_4(3)$ |

- |          |          |
|----------|----------|
| $B_1(3)$ | $C_1(1)$ |
| $B_2(1)$ | $C_2(1)$ |
| $B_3(2)$ | $C_3(2)$ |



2-product system (two common modules)

$$S = P * P'$$

The 1<sup>st</sup> Product

The 2<sup>nd</sup> Product

$$P = X * Y * Z$$

$$P_1 = X_1 * Y_4 * Z_3$$

$$P_2 = X_1 * Y_1 * Z_2$$

$$P' = Y * Z * B * C$$

$$P'_1 = Y_1 * Z_1 * B_3 * C_3$$

$$P'_2 = Y_2 * Z_1 * B_1 * C_2$$



- |          |          |          |
|----------|----------|----------|
| $X_1(3)$ | $Y_1(3)$ | $Z_1(1)$ |
| $X_2(1)$ | $Y_2(1)$ | $Z_2(1)$ |
| $X_3(1)$ | $Y_3(2)$ | $Z_3(2)$ |
|          | $Y_4(3)$ | $Z_4(3)$ |

- |          |          |
|----------|----------|
| $B_1(3)$ | $C_1(1)$ |
| $B_2(1)$ | $C_2(1)$ |
| $B_3(2)$ | $C_3(2)$ |

## Recent English References

- 1. B. Agard, A. Kusiak, Data-mining-based methodology for the design of product family. Int. J. of Prod. Res., 42(15), 2955-2969, 2004.**
- 2. C.Y. Baldwin, K.B. Clark, Design Rules: The Power of Modularity. MIT Press, 2000.**
- 3. J. Dahmus, J.P. Gonzalez-Zugasti, K.N. Otto, Modular product architecture, Design Studies 22(5), 409-424, 2001.**
- 4. G. Dobrescu, Y. Reich, Progressive sharing of modules among product variants. Computer-Aided Design 35(9), 791-806, 2003.**
- 5. X. Du, J. Jiao, M.M. Tseng, Architecture of product family: Fundamentals and methodology. Concurrent Eng.: Res. and Appl. 9(4), 309-325, 2001.**
- 6. J.K. Gershenson, G.J. Prasad, S. Allamneni, Modular product design: A life-cycle view. Trans. of the SDPS 3(4), 13-26, 1999.**
- 7. J.P. Gonzalez-Zugasti, K.N. Otto, J.D. Baker, A method for architecting product platform. Res. in Eng. Des. 12(2), 61-72, 2000.**
- 8. T.K.P. Holmqvist, M.L. Person, Analysis and improvement of product modularization methods: Their ability to deal with complex products. Systems Engineering 6(3), 195-209, 2003.**
- 9. C.C. Huang, A. Kusiak, Modularity in design of products and systems. IEEE Trans. on Syst., Man and Cybern. - Part A, 28(1), 66-77, 1998.**
- 10. M.Sh. Levin, Modular system synthesis: Example for packaged composite software, IEEE Tr. on SMC-Part C, 35(4), 544-553, 2005.**
- 11. M.Sh. Levin, Combinatorial design of multiproduct system: common modules. Elsevier Server of Preprints in CS, 2003.**

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12. **M. Kokkolaras, R. Fellini, H.M. Kim, N. Michelena, and P. Papalambros, Extension of the target cascading formulation to the design of product family, Structural and Multidisciplinary Optimization, 24(4), 293-301, 2002.**
13. **A. Kusiak, Integrated product and process Design: a modularity perspective. J. of Eng. Des., 13(3), 223-231, 2002.**
14. **A. Messac, M.P. Martinez, T.W. Simpson, Effective product family design using physical programming. Engineering Optimization 34(3), 245-261, 2002.**
15. **M.H. Meyer, A.P. Lehnerd, The Power of Product Platforms, The Free Press, New York, 1997.**
16. **J.H. Mikkola, O. Gassmann, Managing modularity of product architectures: Toward an integrated theory. IEEE Trans. on Eng. Manag. 18(3), 204-218, 2003.**
17. **D. Robertson, K. Ulrich, Planning for product platforms, Sloan Manag. Review 39(4), 19-34, 1998.**
18. **M.S. Sawhney, Leverage high-variety strategies: From portfolio thinking to platform thinking. J. of the Academy of Marketing Science 26(1), 54-61, 1998.**
19. **D.M. Sharman, A.A. Yassine, Charactrizing complex product architecture. Systems Engineering 7(1), 35-60, 2004.**
20. **Z. Siddique, D.W. Rosen, On combinatorial design spaces for the configuration design of product family. AI EDAM 15(2) 91-108, 2001.**
21. **T.W. Simpson, J.R.A. Maier, F. Mistree, Product platform design: methods and application. Res. in Eng. Des. 13(1), 2-22,2001.**

## **LECTURE 26. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Institute of Physics and Technology (University)**

**Mark Sh. Levin**

**Inst. for Information Transmission Problems, RAS**

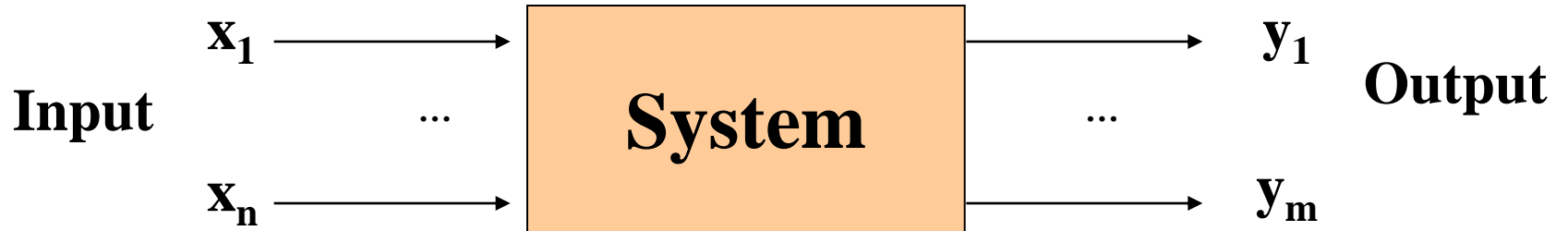
**Email: [mslevin@acm.org](mailto:mslevin@acm.org) / [mslevin@iitp.ru](mailto:mslevin@iitp.ru)**

### **L.26. System testing**

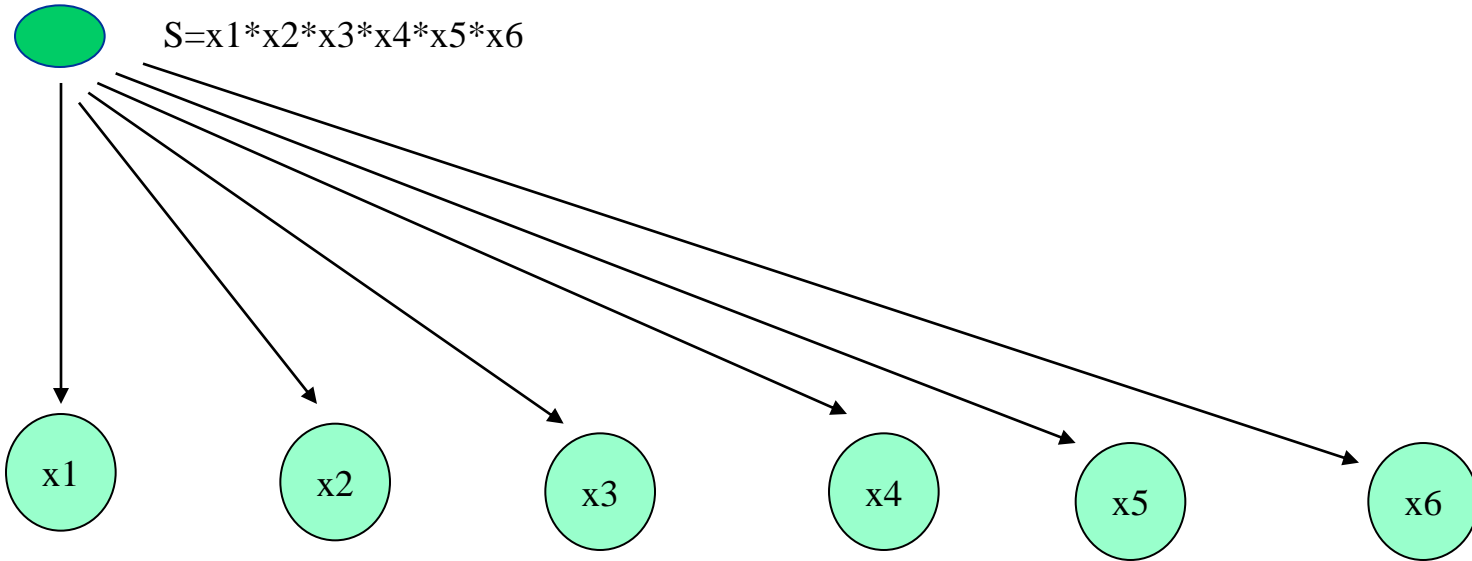
#### ***PLAN:***

1. System Testing: main approaches: \*white-box testing (system structure is well-known)
  - \*black-box testing as model checking
  - \*black-box testing as multi-function testing
2. Multi-function system testing: basic combinatorial problems:
  - \*preliminary analysis of system, \*composition of test cases
  - \*design of chain of test cases, \*covering of digraph of function clusters by chains
3. Illustrative example for multi-function system testing

Black-box system



Example of test case structure (cluster: functions 1, 3, and 4)



0:  $x_1 = \text{'Any'}$  (2)    0:  $x_2 = \text{'Any'}$  (2)    0:  $x_3 = \text{'Any'}$  (4)    0:  $x_4 = \text{'Any'}$  (4)    0:  $x_5 = \text{'Any'}$  (2)    0:  $x_6 = \text{'Any'}$  (4)

1:  $x_1 = 1$  (3)    ● 1:  $x_2 = 1$  (3)    1:  $x_3 = 1$  (1)    ● 1:  $x_4 = 1$  (1)    ● 1:  $x_5 = 1$  (3)    1:  $x_6 = 1$  (2)

● 2:  $x_1 = 2$  (3)    2:  $x_2 = 2$  (3)    2:  $x_3 = 2$  (1)    2:  $x_4 = 2$  (1)    2:  $x_5 = 2$  (3)    2:  $x_6 = 2$  (2)

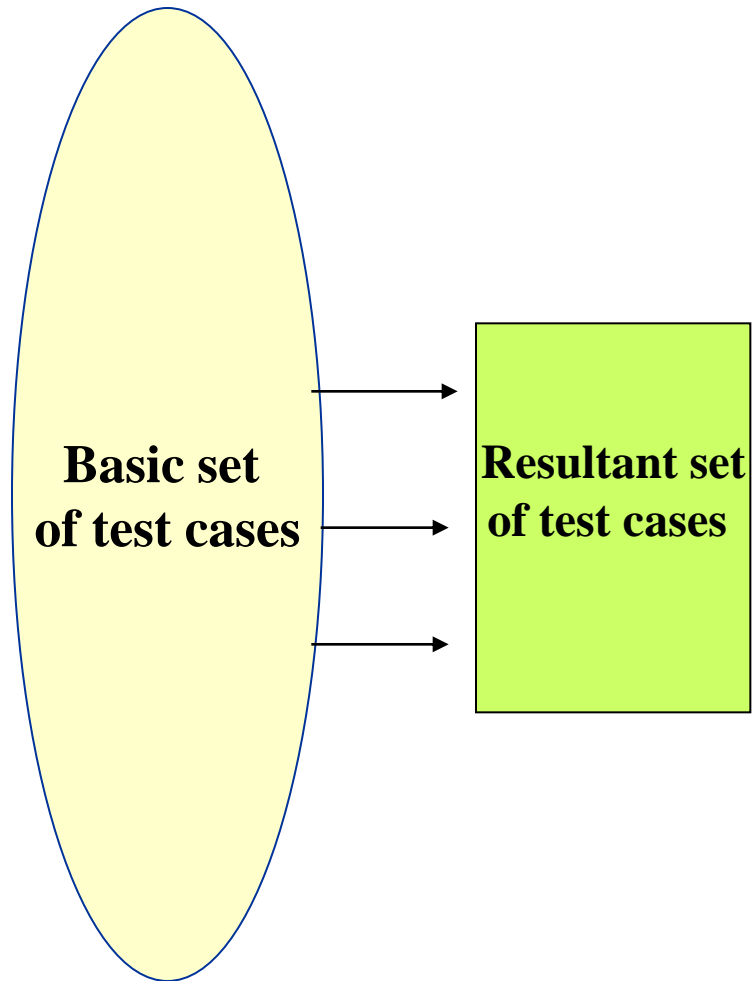
● 3:  $x_3 = 3$  (3)    3:  $x_6 = 3$  (2)

● 4:  $x_6 = 4$  (3)

5:  $x_6 = 5$  (1)

TEST CASE

## Selection of test cases (basic approaches)



### **Basic algorithms:**

- 1.Reducing of the initial space (by equivalence)**
- 2.Design of a test cases set which covers the reduced set**

Additional "dimension"

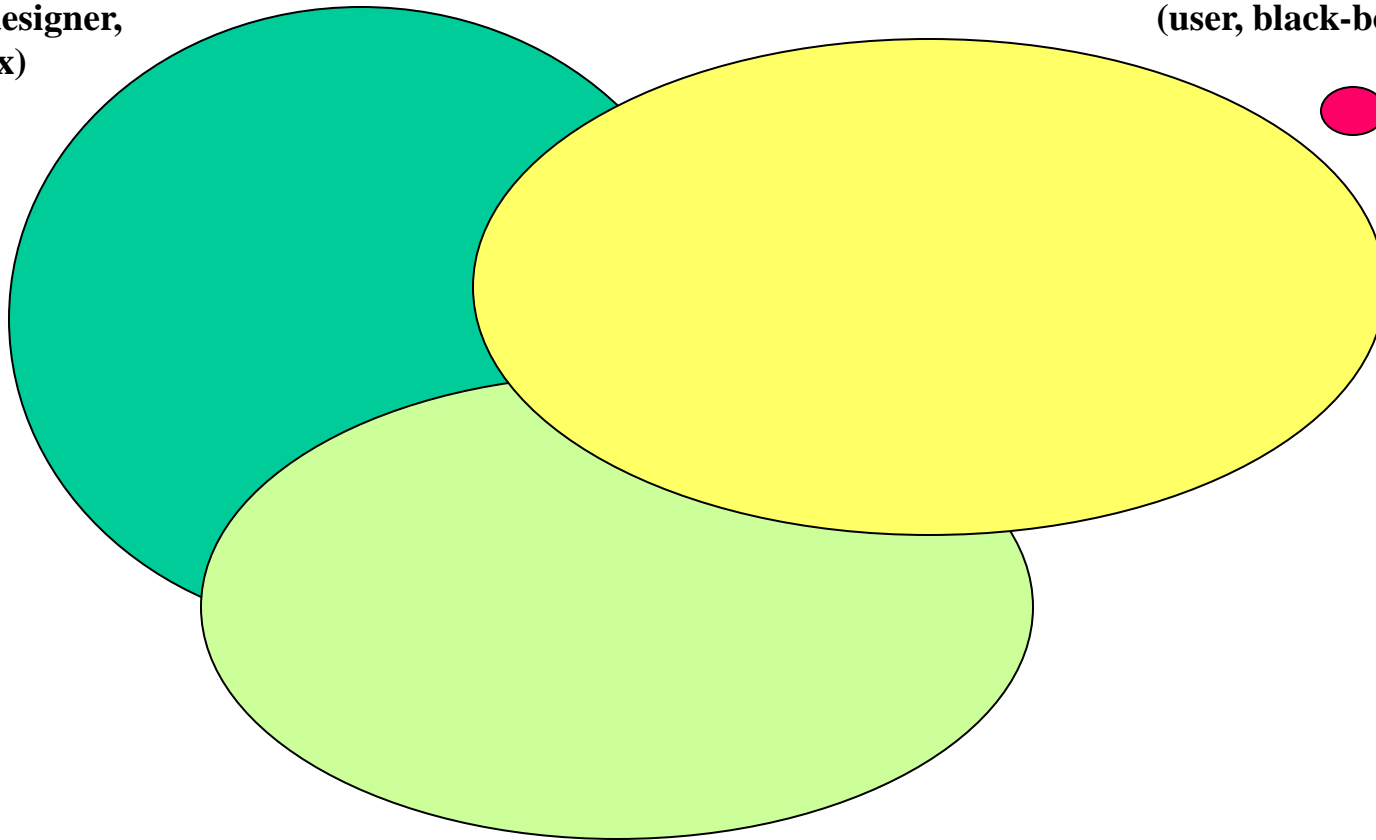
**Specification based  
inputs (designer,  
white box)**

**Real system behavior  
(user, black-box)**

**We are here**

**Designed test cases  
(tester, black-box,  
Model checking, etc.)**

**PLUS: Dynamics as system  
development / evolution**





## 1.Preliminaries: Main Roles and Responsibility (system testing)

### **SYSTEM EXPERT(SPECIALIST) :**

- \* system performance
- \* system safety
- \* system life cycle
- \* new requirements
- \* new generations
- \* new standards

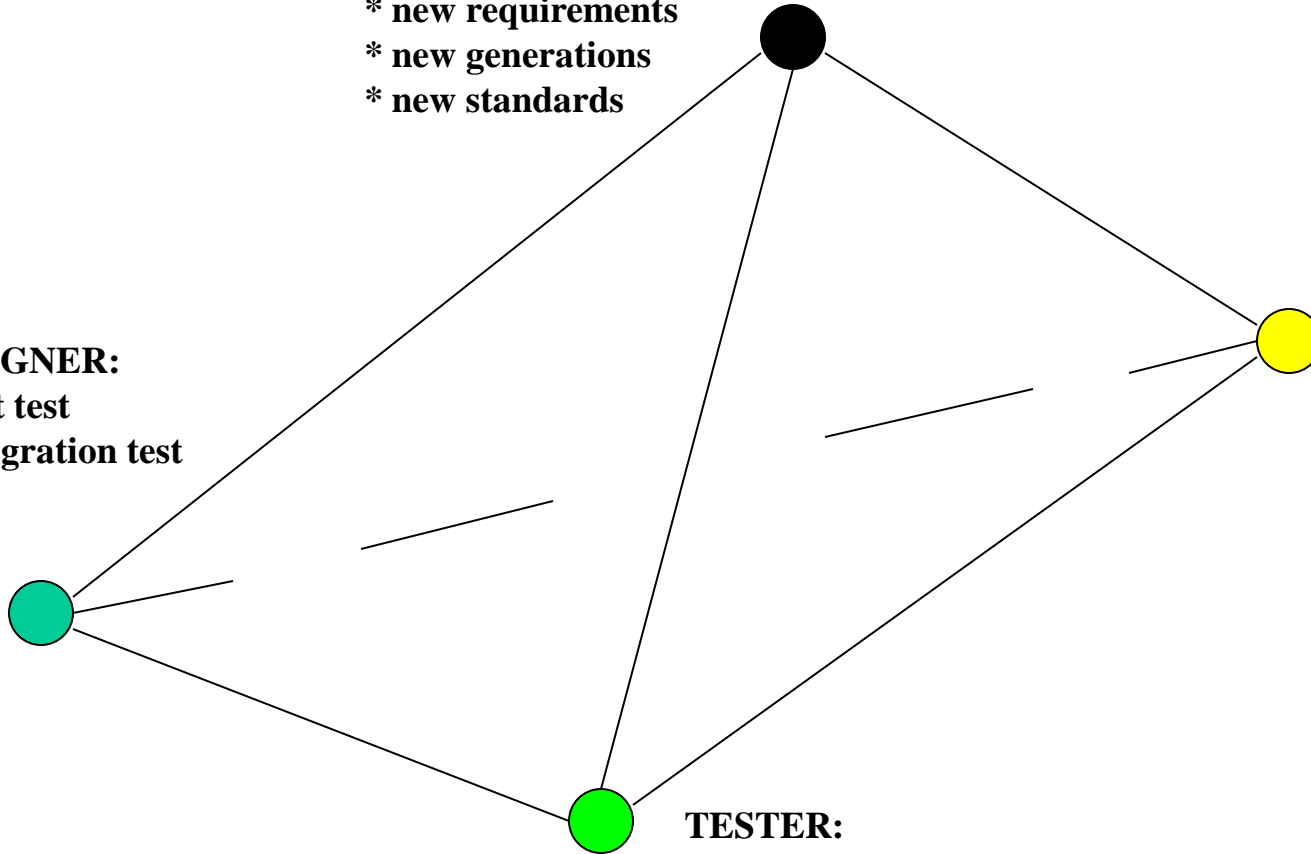
**USER(S):**  
\* functional test

### **DESIGNER:**

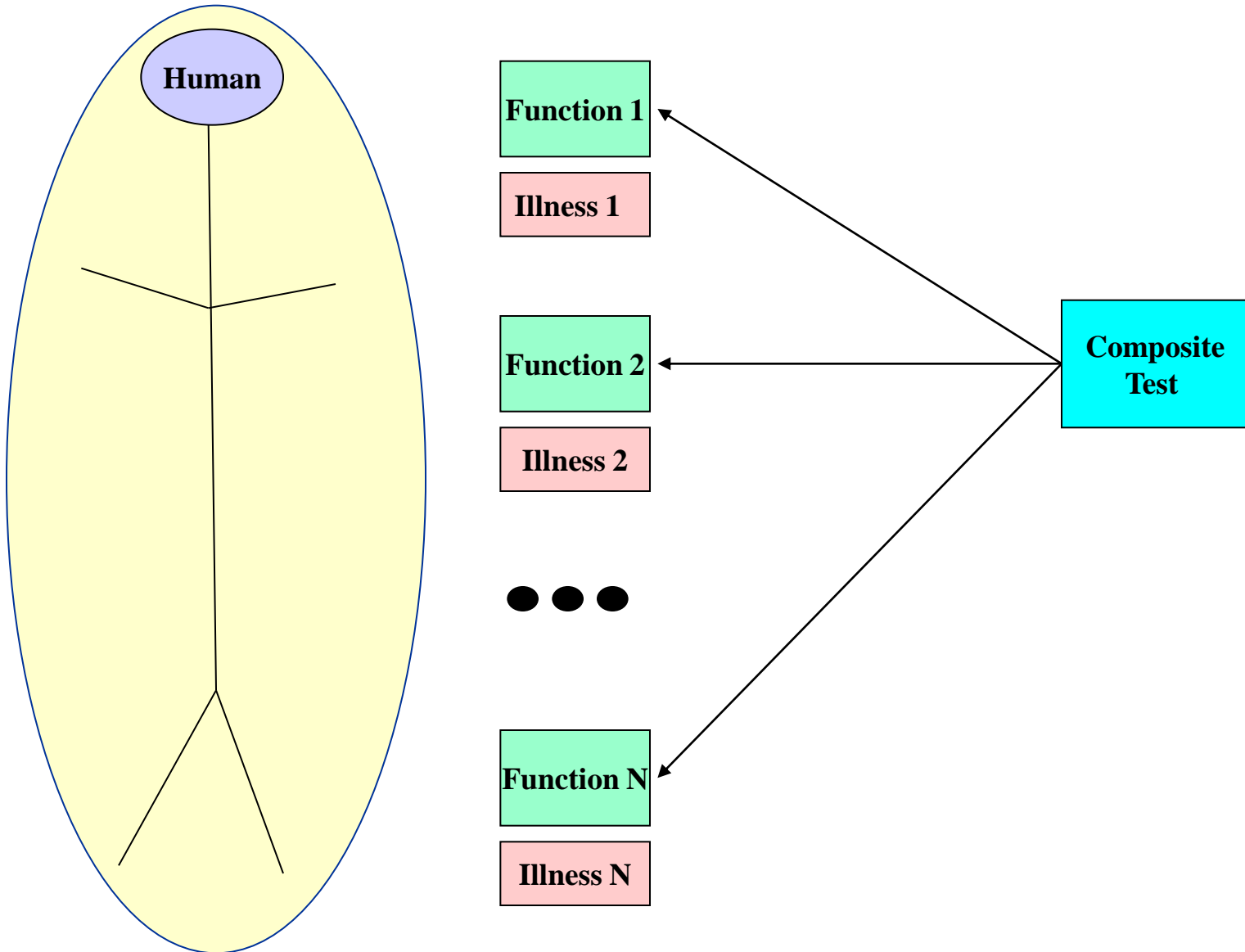
- \* unit test
- \* integration test

### **TESTER:**

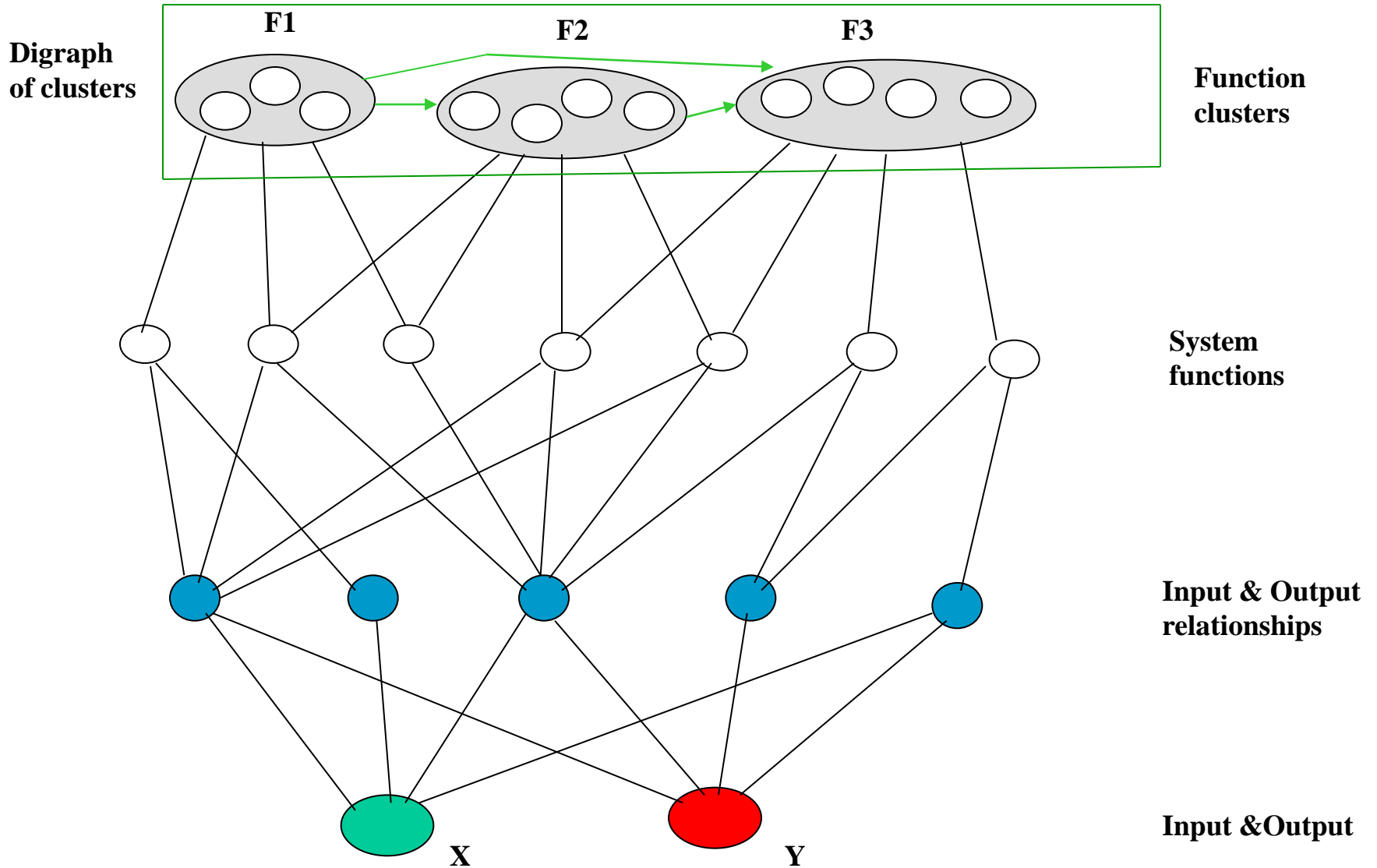
- \* model checking
- \* etc..



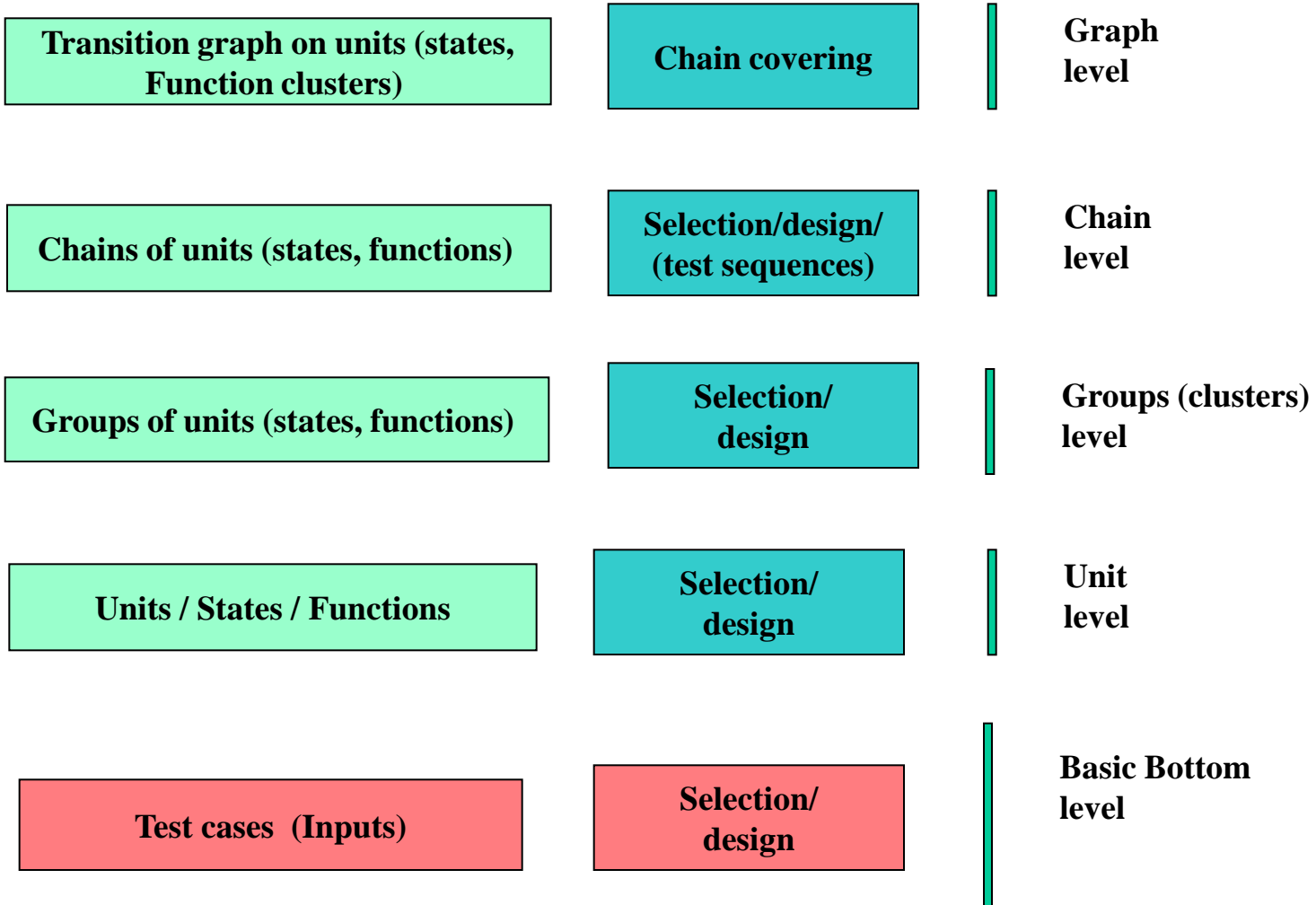
Human-based illustration for multi-function testing: N functions & composite test



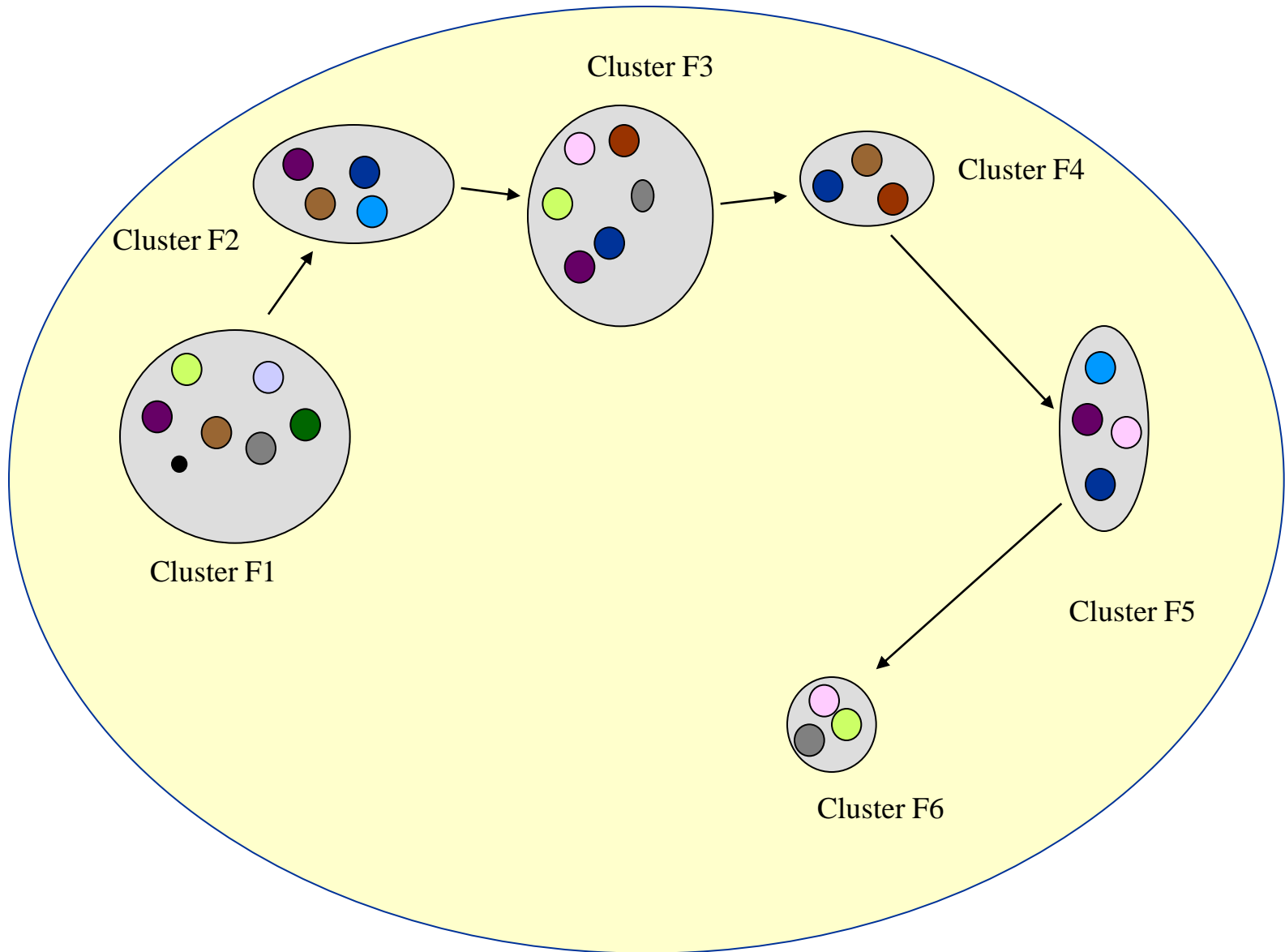
# Hierarchy of system characteristics in multi-function system testing 4



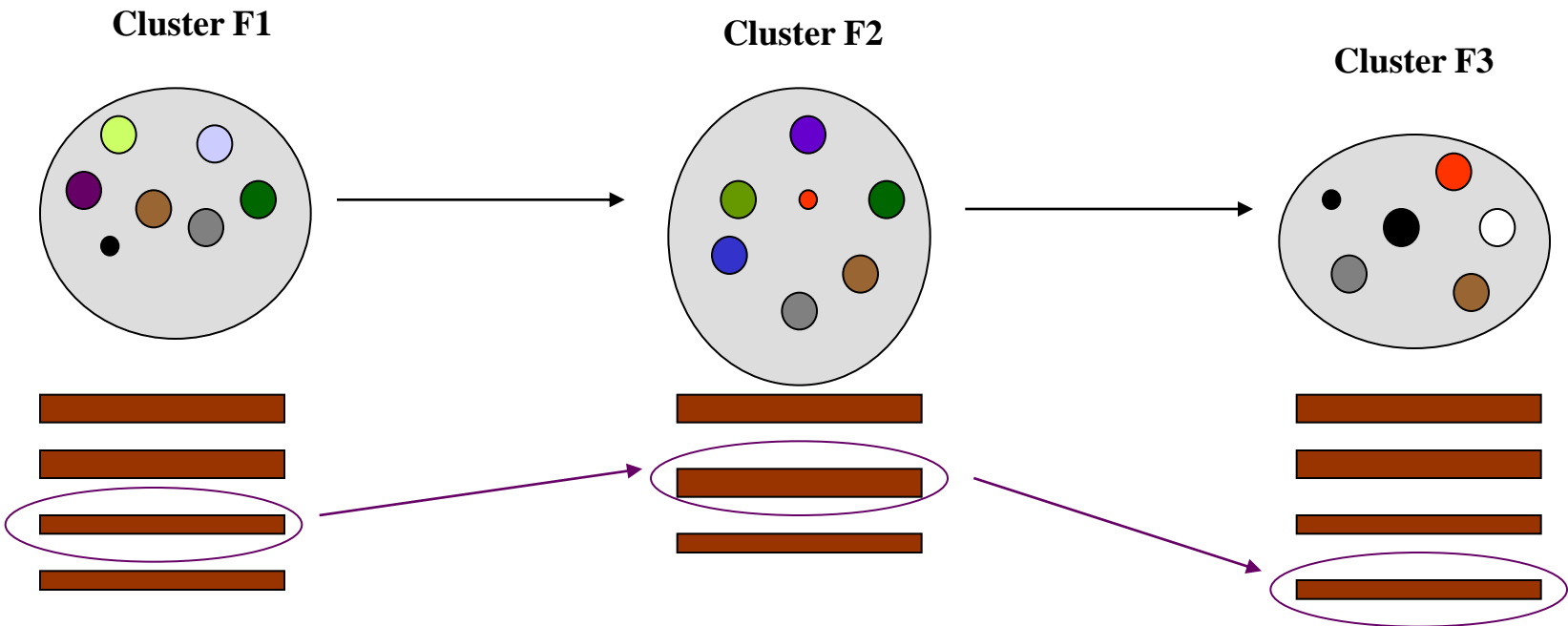
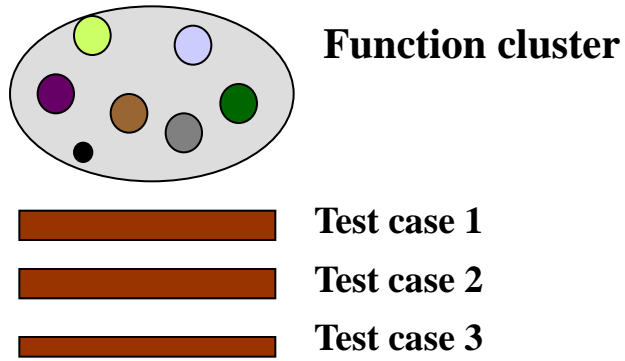
## Levels of testing process and problems



# Space of system functions and function clusters

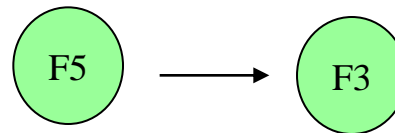
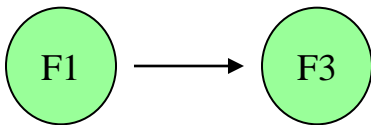
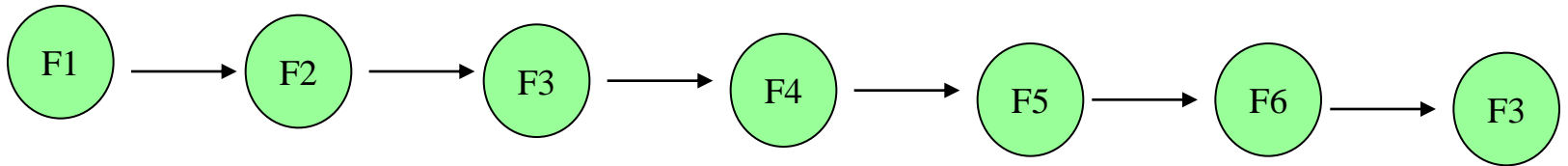
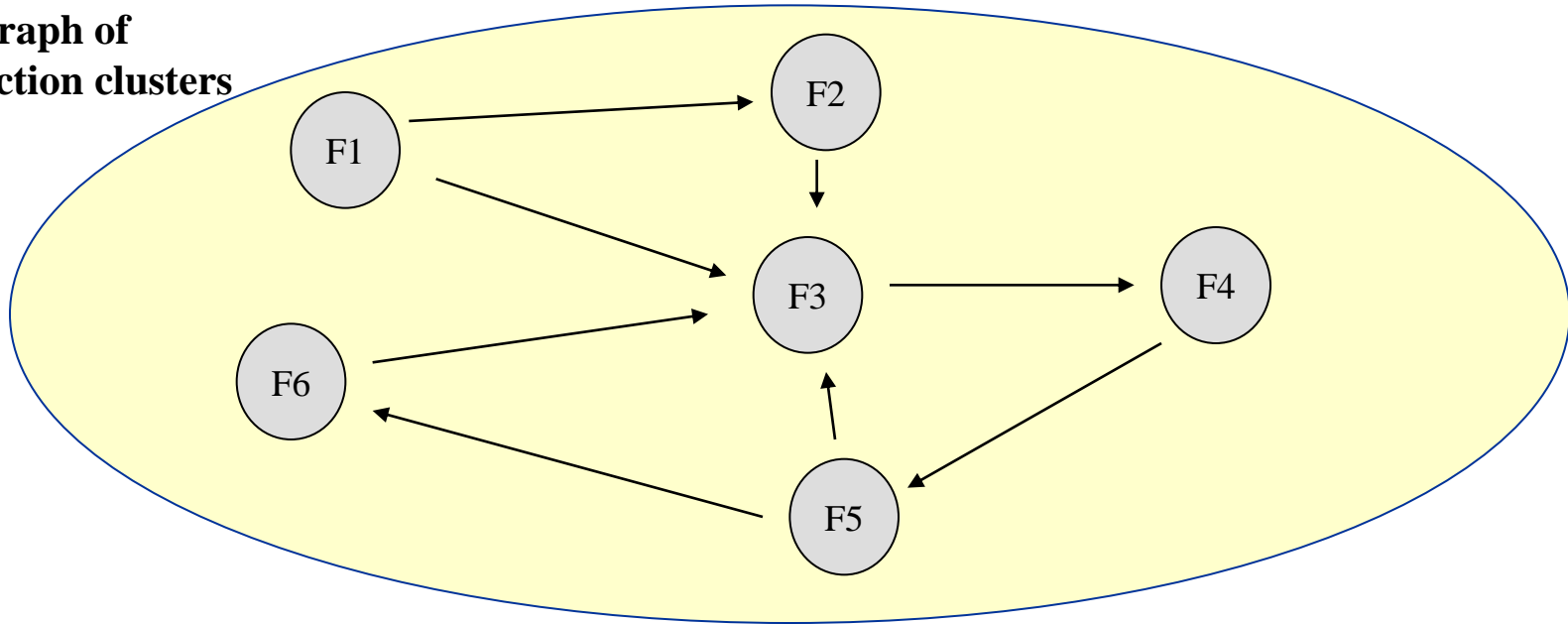


Test case for function cluster and a sequence of test cases for a chain of function clusters

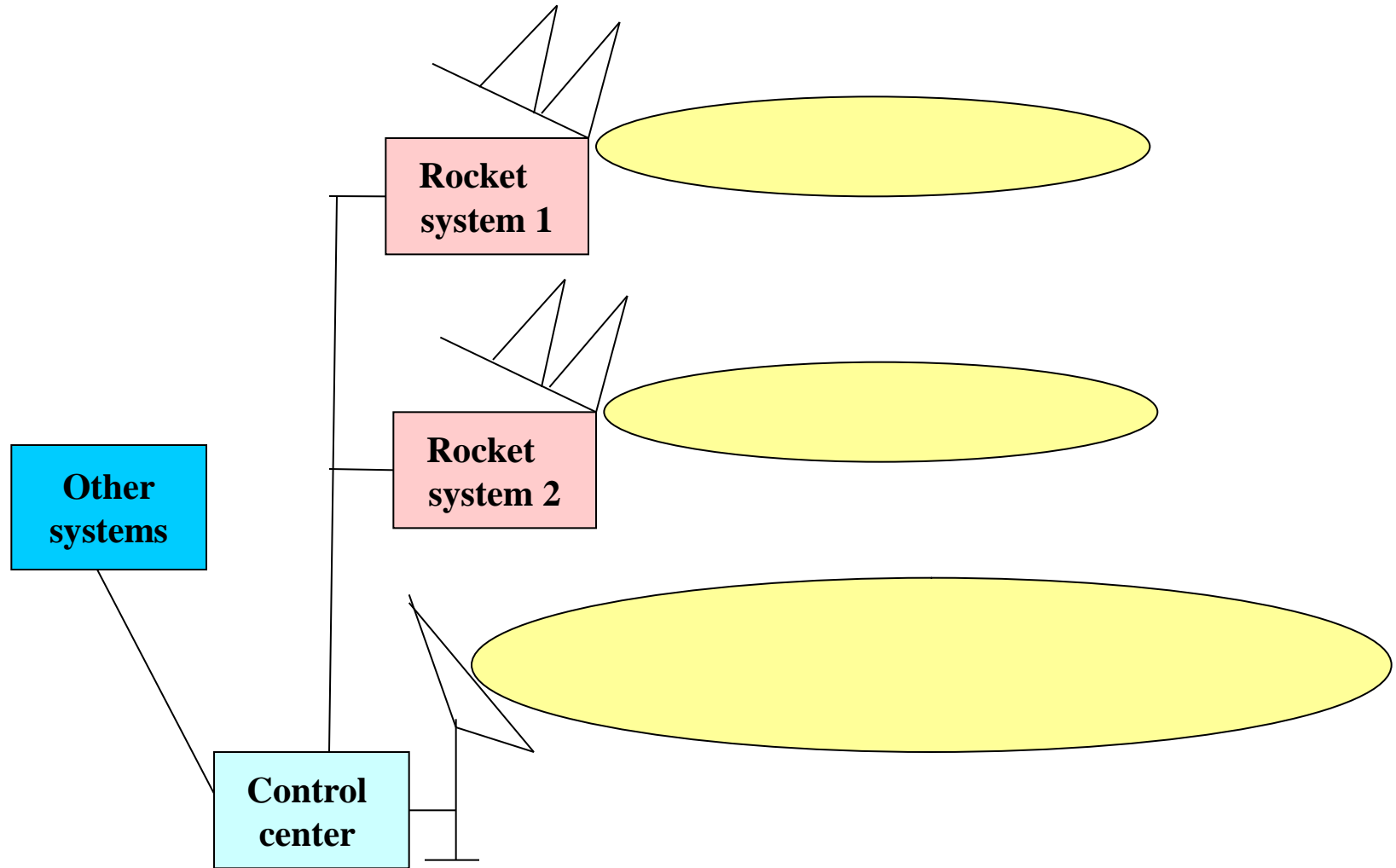


# Chains of function clusters and covering

**Digraph of  
function clusters**



Applied missile defense / anti-aircraft system





## Functions and function clusters

### Functions:

1. Scanning the examined area f1

2. Initialization of targets f2

3. Identification of targets f3

4. Tracking/maintenance of targets f4

5. Multi-target multi-track  
assignment f5

6. Fair control (assignment of  
rockets into targets) f6

7. Deletion of non-dangerous targets f7

8. Receiving data from other systems f8

9. Sending data to other systems f9

### Function clusters

F1: f1

F2: f1,f4

F3: f2,f3, f4

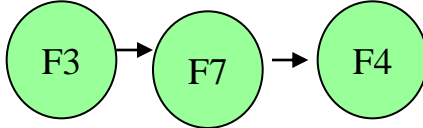
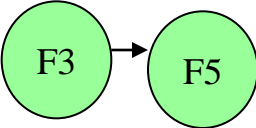
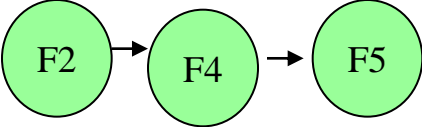
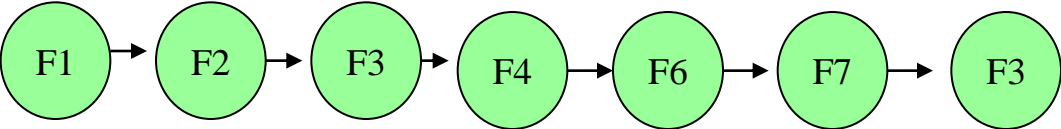
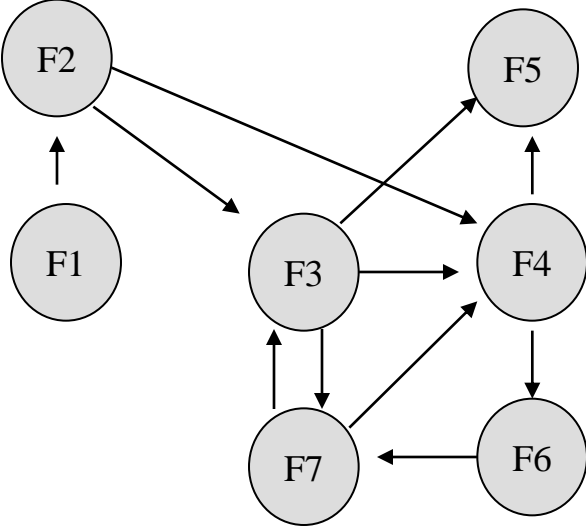
F4: f4,f5

F5: f4,f7

F6: f5,f6

F7: f5,f8, f9

Functions cluster digraph



1. M. Sh. Levin, M. Last, *Multi-Function System Testing: Composition of Test Sets*. 8<sup>th</sup> IEEE Int. Conf. HASE 2004, Tampa, FL, 99-108, 2004.
2. M. Sh. Levin, M. Last, *Test Case Sequences in System Testing: Selection of Test Cases for a Chain (Sequence) of Function Clusters*. 17<sup>th</sup> Int. Conf. IEA/AIE, Ottawa, LNCS 3029, Springer, 895-904, 2004.
3. M. Sh. Levin, M. Last, *Collection of Test Case Sequences Covering of Function Cluster Digraph*. IASTED Int. Conf. "AI and Applications", Innsbruck, 806-810, Febr. 2004.

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**Inst. for Information Transmission Problems, RAS**

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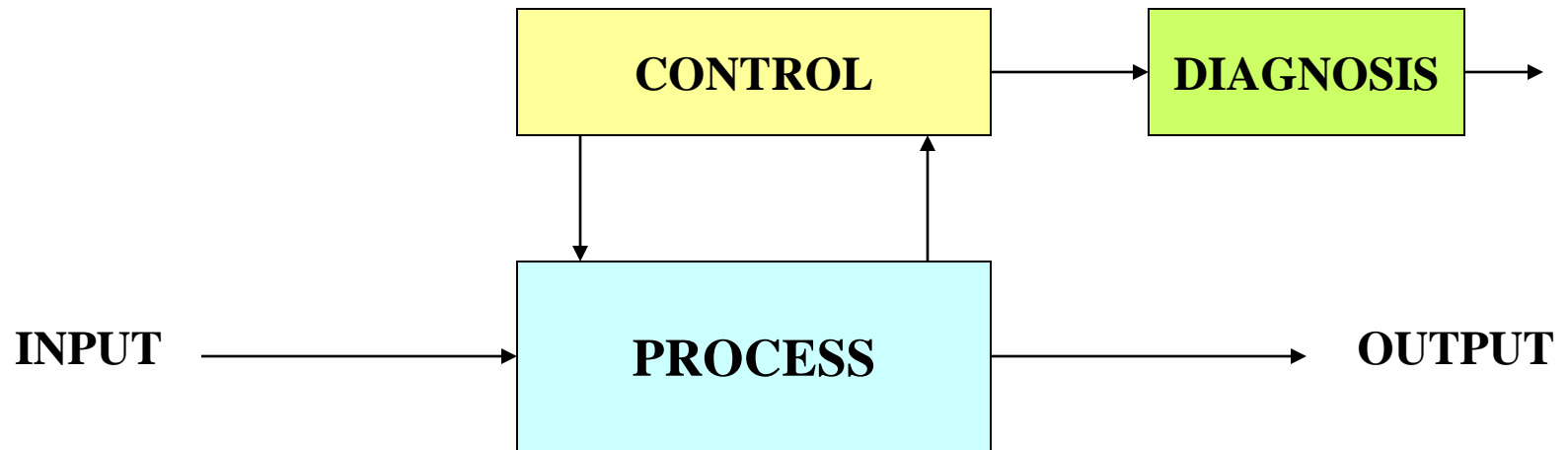
**L.27. System diagnosis, evaluation, improvement.**

***PLAN:***

1. Hierarchical approach to diagnosis of complex systems
2. Hierarchical evaluation of composable system: example for building:
  - \*models of building and corresponding evaluation scales for building parts
    - \*method of integration tables
    - \*usage of hierarchical combinatorial synthesis
    - \*change operations and planning an upgrade process

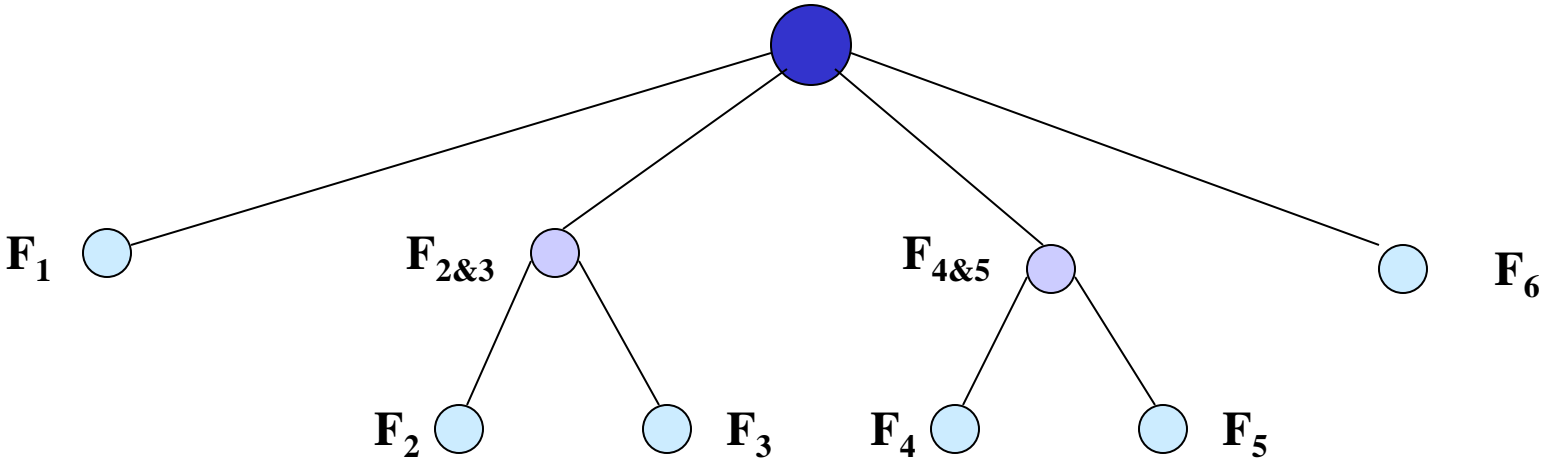
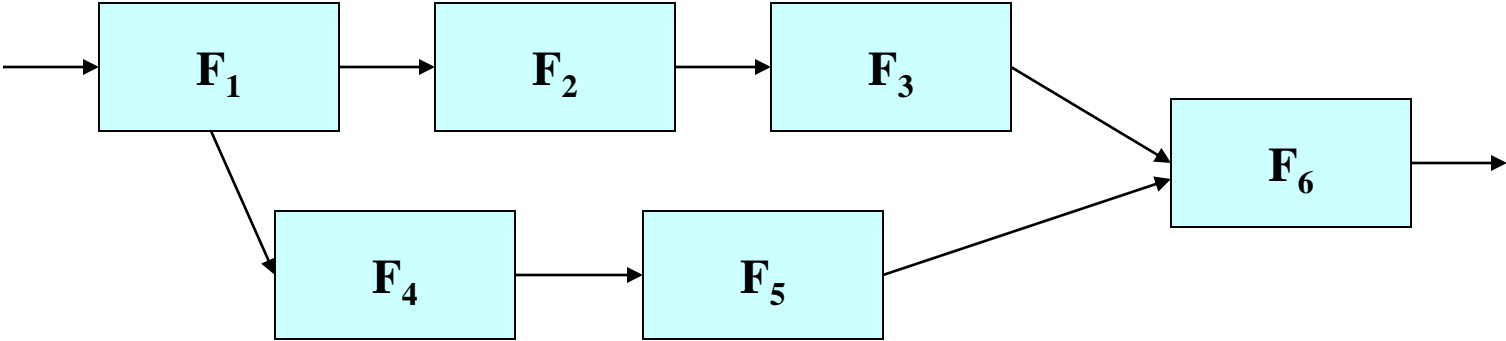
Nov. 12, 2004

## Multi-level diagnosis of complex systems



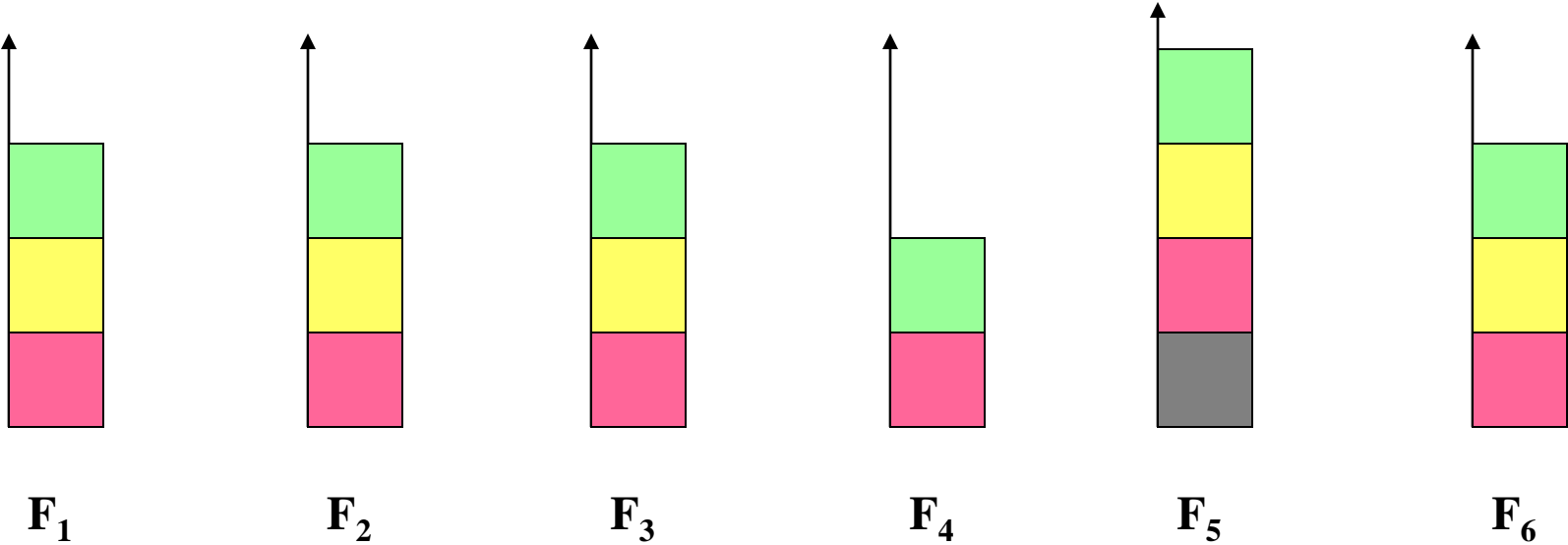
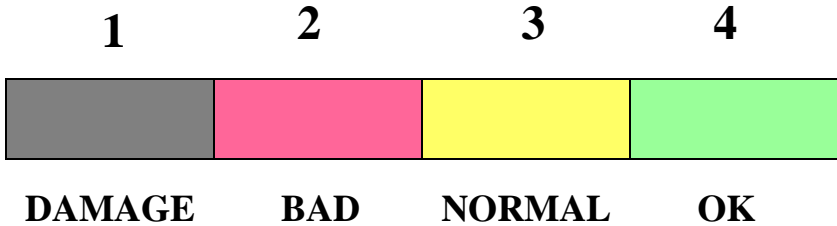
Multi-level diagnosis of complex systems

PROCESS

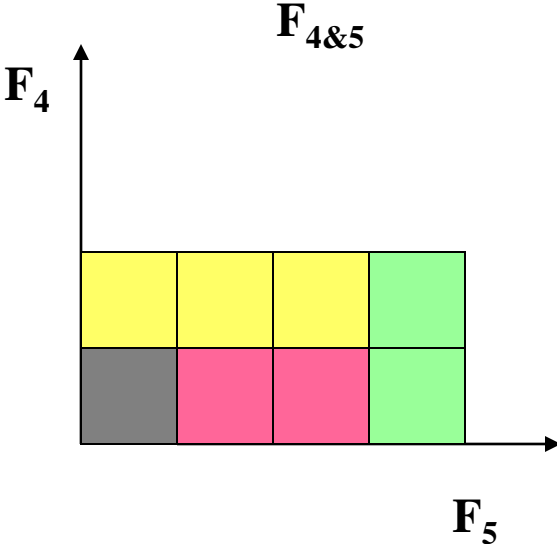
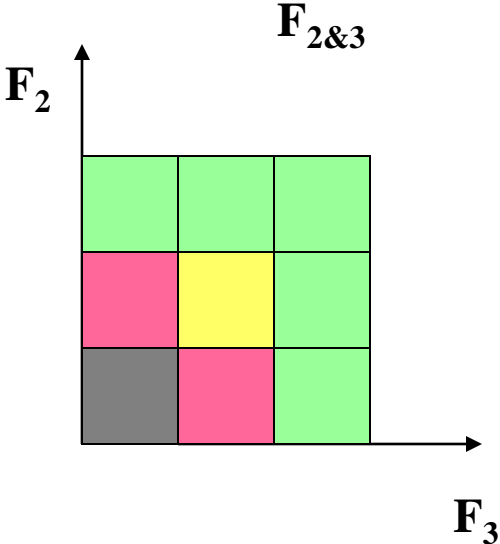


Multi-level diagnosis of complex systems

# SCALE



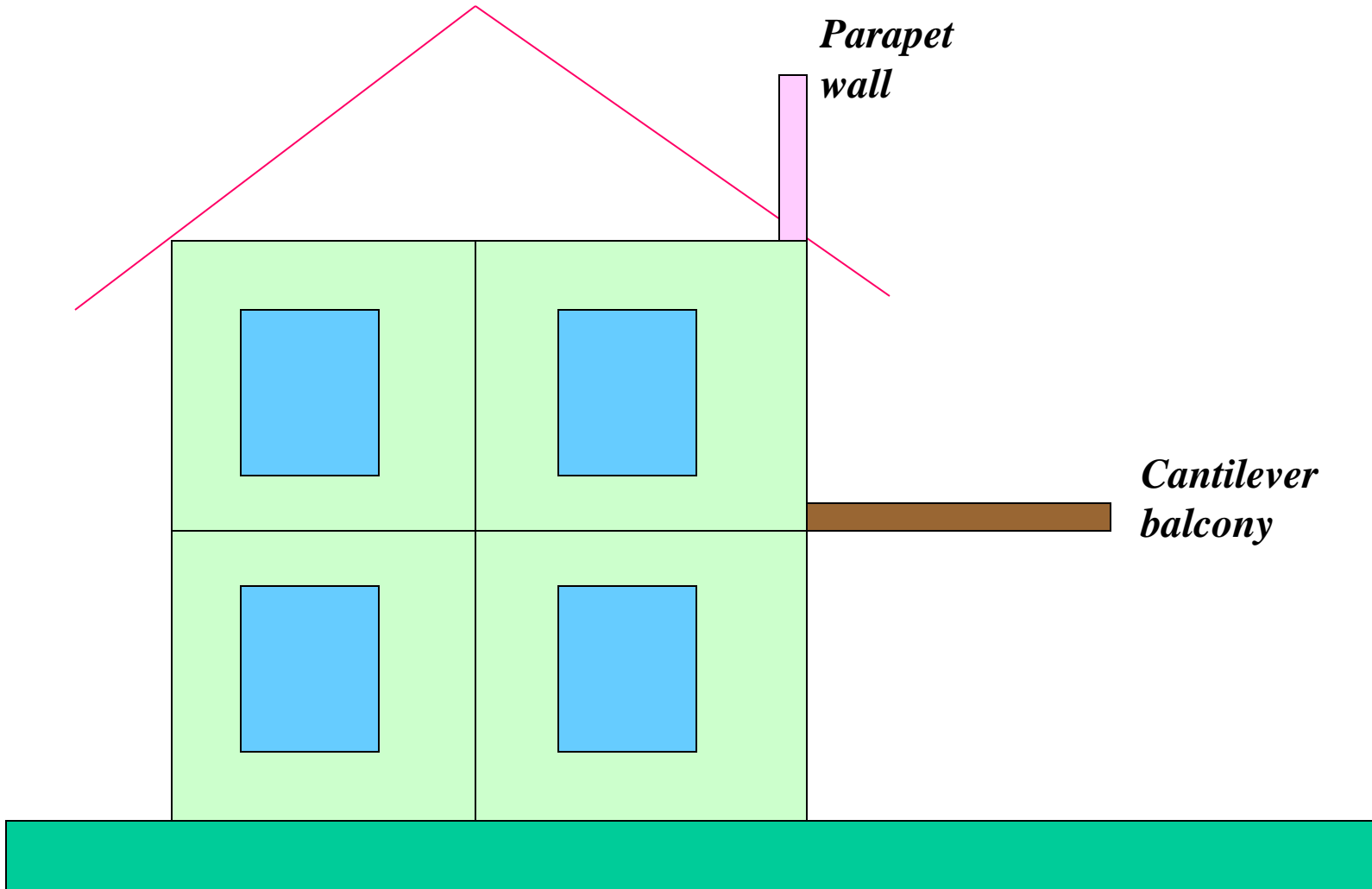
Multi-level diagnosis of complex systems



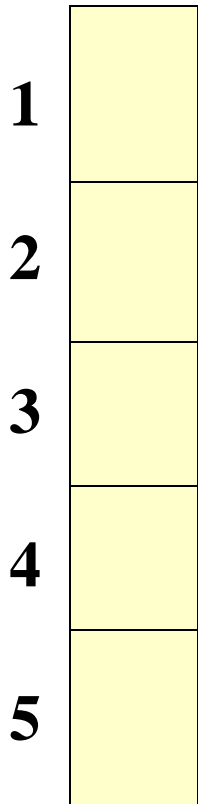
$F_1$  and  $F_{2\&3}$  and  $F_{4\&5}$  and  $F_6$   $\longrightarrow$  TOTAL ESTIMATE



Example of building (evaluation from the viewpoint of earthquake engineering)



## Generalized ordinal scale for damage



**1. Distriction (global)**

**2. Distriction (local)**

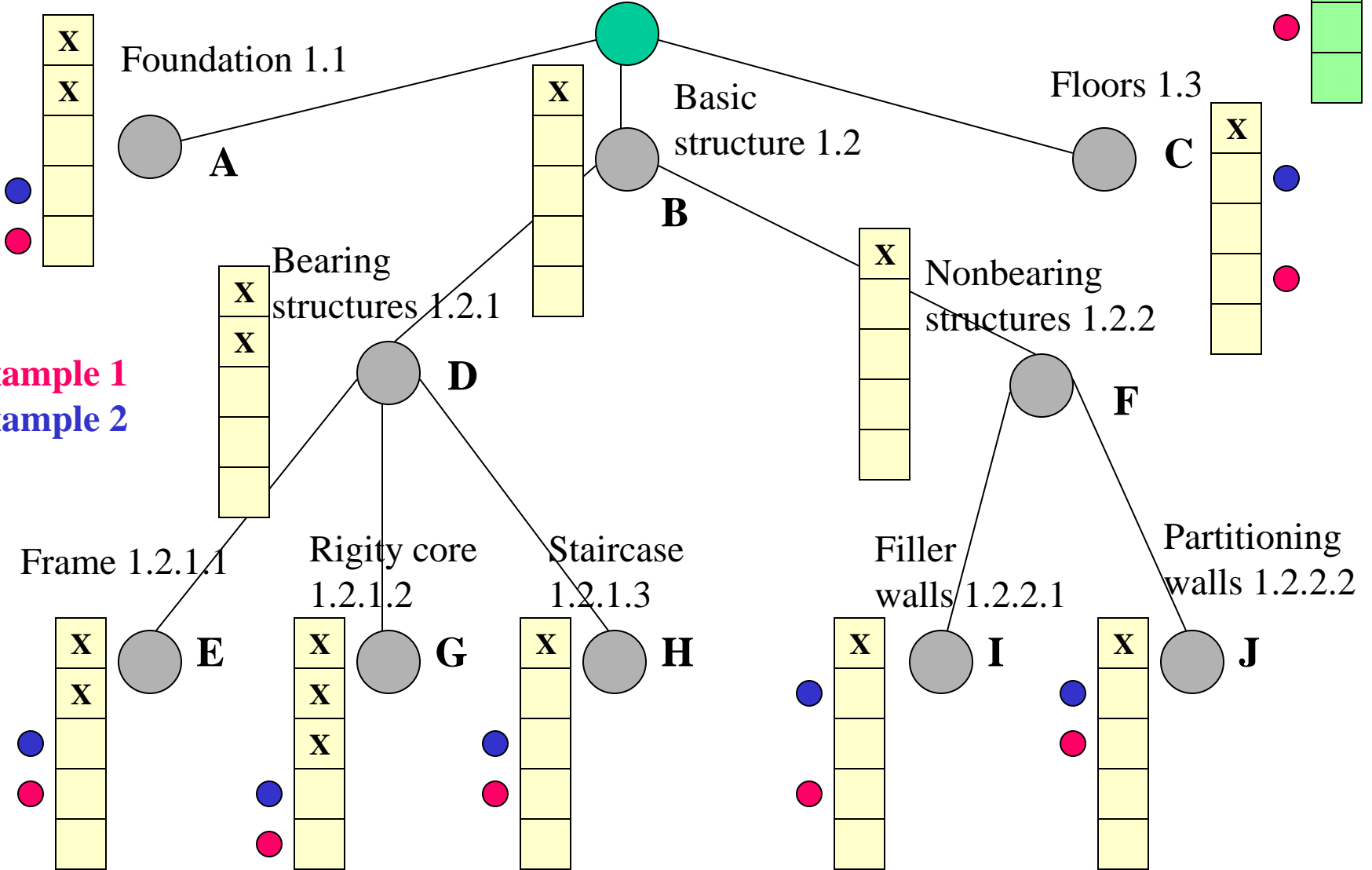
**3. Chinks**

**4. Small chinks (hair like)**

**5. Without damage**

Hierarchical model of building and corresponding scales

Building:  $S = A * B * C$



Example 1  
Example 2

Method 1: integration tables

Bearing structures D (1.2.1), scale [3,4,5]

<b>E</b>	<b>G</b>	<b>H</b>	<b>D</b>
<b>3</b>	<b>4</b>	<b>3</b>	<b>3</b>
<b>3</b>	<b>4</b>	<b>4</b>	<b>3</b>
<b>3</b>	<b>4</b>	<b>5</b>	<b>-</b>
<b>3</b>	<b>5</b>	<b>3</b>	<b>3</b>
<b>3</b>	<b>5</b>	<b>4</b>	<b>3</b>
<b>3</b>	<b>5</b>	<b>5</b>	<b>-</b>
<b>5</b>	<b>4</b>	<b>3</b>	<b>3</b>
<b>5</b>	<b>4</b>	<b>4</b>	<b>4</b>
<b>5</b>	<b>4</b>	<b>5</b>	<b>4</b>
<b>5</b>	<b>5</b>	<b>3</b>	<b>4</b>
<b>5</b>	<b>5</b>	<b>4</b>	<b>4</b>
<b>5</b>	<b>5</b>	<b>5</b>	<b>5</b>
<b>4</b>	<b>4</b>	<b>3</b>	<b>3</b>
<b>4</b>	<b>4</b>	<b>4</b>	<b>4</b>
<b>4</b>	<b>4</b>	<b>5</b>	<b>-</b>
<b>4</b>	<b>5</b>	<b>3</b>	<b>3</b>
<b>4</b>	<b>5</b>	<b>4</b>	<b>4</b>
<b>4</b>	<b>5</b>	<b>5</b>	<b>4</b>

## Method 1: integration tables

Nonbearing structures F (1.2.2), scale [2,3,4,5]

2	2	-	-	2	<b>I</b>
3	3	-	-	3	
3	3	4	-	4	
-	4	4	5	5	
<hr/>					
2	3	4	5		
<b>J</b>					

## Method 1: integration tables

**Basic structure B (1.2), scale [2,3,4,5]**

<b>2</b>	<b>3</b>	<b>-</b>	<b>-</b>	<b>3</b>	<b>D</b>
<b>3</b>	<b>4</b>	<b>4</b>	<b>-</b>	<b>4</b>	
<b>-</b>	<b>4</b>	<b>5</b>	<b>5</b>	<b>5</b>	
<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>		

**F**

Method 1: integration tables

Building S, scale [2,3,4,5]

A	B	C	S
3	2	2	2
3	2	3	-
3	2	4	-
3	2	5	-
3	3	2	2
3	3	3	3
3	3	4	3
3	3	5	-
3	4	2	-
3	4	3	-
3	4	4	-
3	4	5	-
3	5	2	-
3	5	3	-
3	5	4	-
3	5	5	-

A	B	C	S
5	2	2	2
5	2	3	-
5	2	4	-
5	2	5	-
5	3	2	-
5	3	3	-
5	3	4	3
5	3	5	3
5	4	2	-
5	4	3	-
5	4	4	4
5	4	5	4
5	5	2	-
5	5	3	-
5	5	4	-
5	5	5	5

A	B	C	S
4	2	2	2
4	2	3	-
4	2	4	-
4	2	5	-
4	3	2	-
4	3	3	3
4	3	4	3
4	3	5	-
4	4	2	-
4	4	3	-
4	4	4	4
4	4	5	4
4	5	2	-
4	5	3	-
4	5	4	-
4	5	5	-

Method 2: Hierarchical morphological design (combinatorial synthesis)

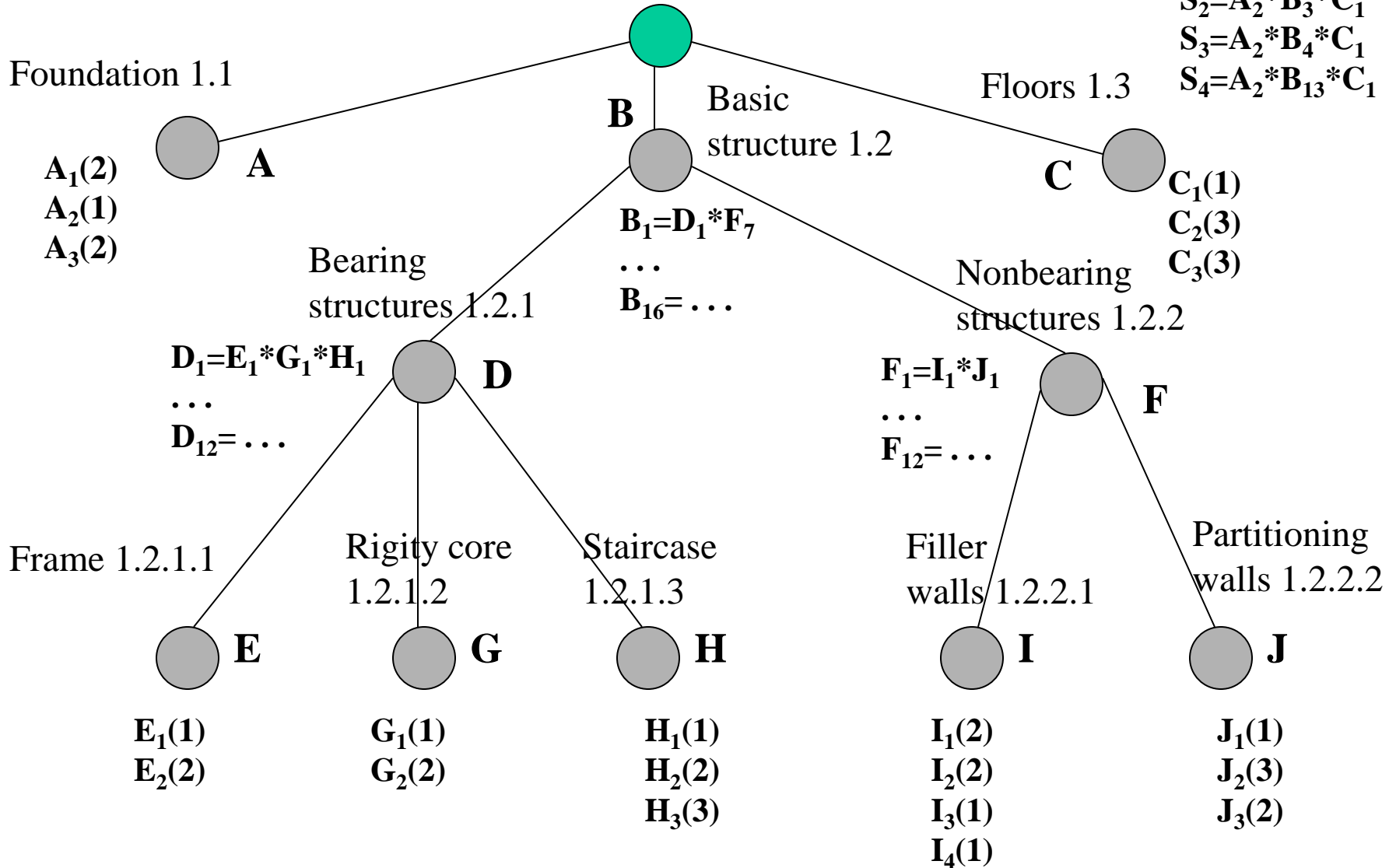
Building:  $S = A * B * C$

$$S_1 = A_2 * B_1 * C_1$$

$$S_2 = A_2 * B_3 * C_1$$

$$S_3 = A_2 * B_4 * C_1$$

$$S_4 = A_2 * B_{13} * C_1$$





## Design Alternatives for Building

**Foundation A :  $A_1$  (strip foundation),  $A_2$  (bedplate foundation),  $A_3$  (isolated parts)**

**Frame E :  $E_1$  (monolith frame),  $E_2$  (precast frame)**

**Rigidity core G :  $G_1$  (monolith rigid core),  $G_2$  (precast rigid core)**

**Stair case H :  $H_1$  (monolith staircase),  $H_2$  (precast staircase),  $H_3$  (composite staircase)**

**Filler walls I :  $I_1$  (small elements),  $I_2$  (curtain panel walls),  
 $I_3$  (precast enclose panel walls),  $I_4$  (frame walls)**

**Partitioning walls J :  $J_1$ (precast panel walls),  $J_2$  (small elements),  $J_3$  (frame walls)**

**Floors C :  $C_1$  (monolith slabs),  $C_2$  (composite slabs),  $C_3$  (precast slabs)**

## Method 2: Hierarchical morphological design (combinatorial synthesis)

### Compatibility

	<b>G<sub>1</sub></b>	<b>G<sub>2</sub></b>	<b>H<sub>1</sub></b>	<b>H<sub>2</sub></b>	<b>H<sub>3</sub></b>
<b>E<sub>1</sub></b>	<b>3</b>	<b>2</b>	<b>3</b>	<b>1</b>	<b>2</b>
<b>E<sub>2</sub></b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>2</b>
<b>G<sub>1</sub></b>			<b>3</b>	<b>2</b>	<b>1</b>
<b>G<sub>2</sub></b>			<b>2</b>	<b>1</b>	<b>1</b>

	<b>I<sub>1</sub></b>	<b>I<sub>2</sub></b>	<b>I<sub>3</sub></b>	<b>I<sub>4</sub></b>
<b>J<sub>1</sub></b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>3</b>
<b>J<sub>2</sub></b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
<b>J<sub>3</sub></b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>3</b>

**NOTE: 3 corresponds to the best level of compatibility**  
**0 corresponds to incompatibility**

## Method 2: Hierarchical morphological design (combinatorial synthesis)

### Compatibility

	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>
D <sub>1</sub>	3	3	3	2	2	2	2	2	2	2	2	3
D <sub>2</sub>	2	2	2	2	2	2	2	2	2	2	2	2
D <sub>3</sub>	2	2	2	2	2	2	2	2	2	2	2	2
D <sub>4</sub>	2	2	2	2	2	2	2	2	2	2	2	2
D <sub>5</sub>	2	2	2	2	2	2	2	2	2	2	2	2
D <sub>6</sub>	2	2	2	2	2	2	2	2	2	2	2	2
D <sub>7</sub>	2	2	2	2	2	2	2	2	2	2	2	2
D <sub>8</sub>	2	2	2	2	2	2	2	2	2	2	2	2
D <sub>9</sub>	2	2	2	2	2	2	2	2	2	2	2	2
D <sub>10</sub>	2	2	2	2	2	2	2	2	2	2	2	2
D <sub>11</sub>	1	1	1	3	2	2	3	2	2	2	2	2
D <sub>12</sub>	2	2	2	2	2	2	2	2	2	2	2	2

**NOTE: 3** corresponds to the best level of compatibility  
**0** corresponds to incompatibility

## Method 2: Hierarchical morphological design (combinatorial synthesis)

### Compatibility

	<b>C<sub>1</sub></b>	<b>C<sub>2</sub></b>	<b>C<sub>3</sub></b>	<b>B<sub>1</sub></b>	<b>B<sub>3</sub></b>	<b>B<sub>4</sub></b>	<b>B<sub>13</sub></b>
<b>A<sub>1</sub></b>	2	2	2	2	2	2	1
<b>A<sub>2</sub></b>	3	2	2	3	3	3	2
<b>A<sub>3</sub></b>	2	2	2	2	2	2	1
<b>C<sub>1</sub></b>				3	3	3	2
<b>C<sub>2</sub></b>				3	3	3	2
<b>C<sub>3</sub></b>				2	2	2	3

**NOTE: 3 corresponds to the best level of compatibility**  
**0 corresponds to incompatibility**

**Examples for building :**

$$S^i = A_1 * (E_1 * G_1 * H_1) * (I_3 * J_1) * C_1 \quad \text{estimate 2} \quad (\text{Pareto-layer})$$

$$S^{ii} = A_2 * (E_2 * G_2 * H_2) * (I_3 * J_1) * C_1 \quad \text{estimate 2} \quad (\text{Pareto-layer})$$

$$S^{iii} = A_1 * (E_2 * G_2 * H_2) * (I_3 * J_1) * C_3 \quad \text{estimate 3}$$

$$S^{iv} = A_2 * (E_2 * G_2 * H_2) * (I_3 * J_1) * C_3 \quad \text{estimate 3}$$

$$S^v = A_1 * (E_2 * G_1 * H_1) * (I_3 * J_3) * C_3 \quad \text{estimate 4}$$

**Operation group I (frames):**

- O<sub>1</sub> increasing a geometrical dimension and active reinforcement**
- O<sub>2</sub> increasing of active reinforcement**

**Operation group II (joints):**

- O<sub>3</sub> increasing a level for fixing a longitudinal active reinforcement in zone of joints**
- O<sub>4</sub> decreasing the step of reinforced cross rods in zone of joint**

**Operation group III (cantilever and cantilever balcony):**

- O<sub>5</sub> decreasing the projection cantilever**
- O<sub>6</sub> supplementary supporting the cantilever**

**Operation group IV (fronton and parapet wall):**

- O<sub>7</sub> fixing a bottom part**
- O<sub>8</sub> designing a 3D structure (special)**

**Operation group V (connection between frame and filler walls):**

- O<sub>9</sub> design of shear keys**
- O<sub>10</sub> design of mesh reinforcement**
- O<sub>11</sub> partition of filler walls by auxiliary frame**

## **BINARY RELATIONS OVER IMPROVEMENT OPERATIONS**

**Binary relation “equivalence” and “nonequivalence”**

**Binary relation “complementarity” and “noncomplementarity”**

**Binary relation “precedence”**

## **CRITERIA FOR IMPROVEMENT OPERATIONS**

**Group 1. Improvement of earthquake resistance**

**Group 2. Quality of architecture and plan decisions**

**Group 4. Utilization properties**

**Group 4. Expenditure**

**COMBINATORIAL MODELS FOR PLANNING OF IMPROVEMENT**

**Model 1: Knapsack**

**Model 2: Multiple choice problem**

**Model 3: Multiple criteria ranking**

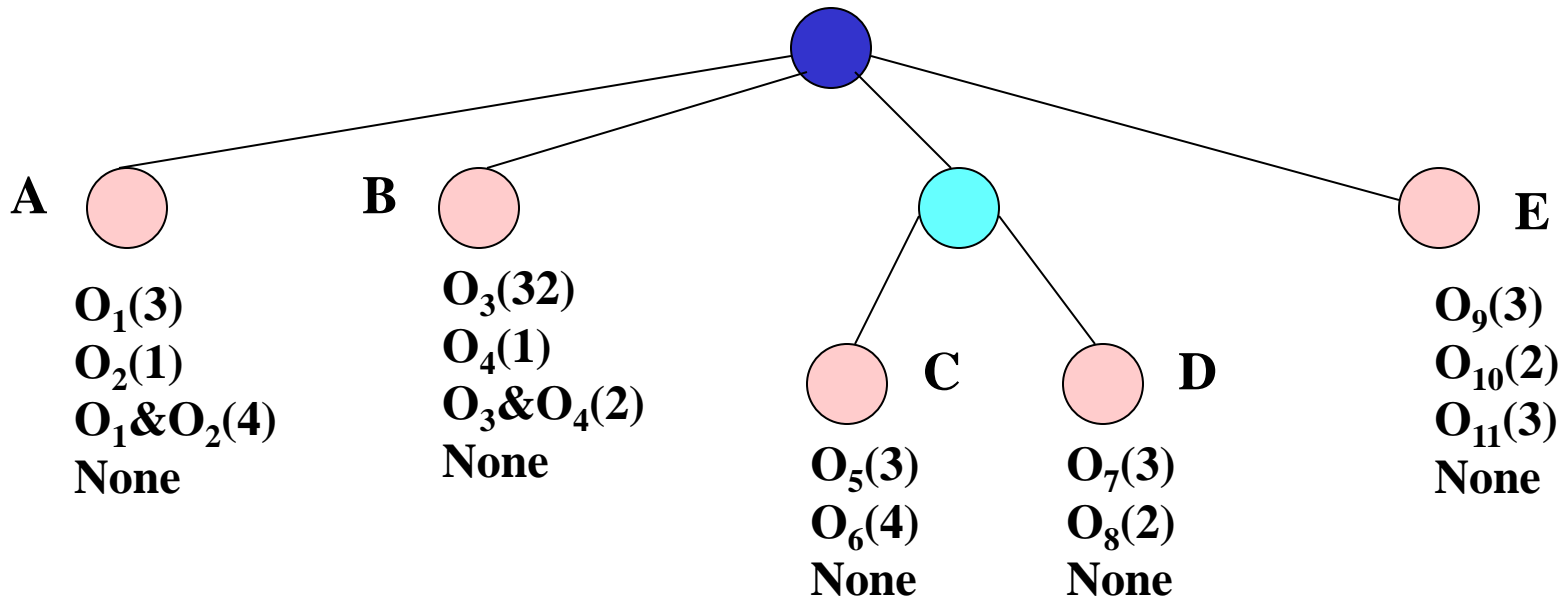
**Model 4: Morphological clique problem**

**Model 5: Scheduling**

**ETC.**



Improvement :  $S = A * B * (C * D) * E$



**Strategy:** O<sub>2</sub> => O<sub>4</sub> => O<sub>5</sub>&O<sub>7</sub>(4) => O<sub>10</sub>

## **LECTURE 28. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Institute of Physics and Technology (University)**

**Mark Sh. Levin**

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### **L.28. System maintenance**

#### ***PLAN:***

1. Preliminaries: life cycle, systems, utilization, personnel, maintenance, roles

2. Framework of maintenance process:

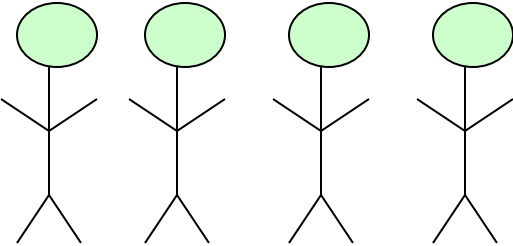
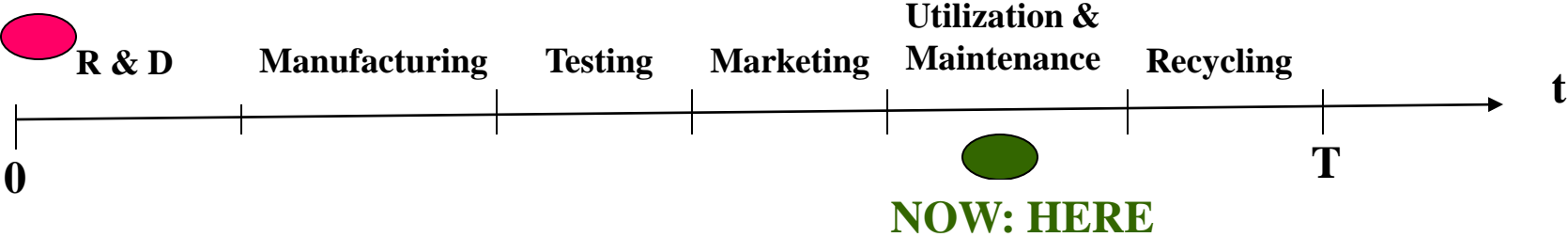
\*basic framework, \*systems under maintenance

\*maintenance operations (inspection as testing/analysis/diagnosis, repair, \*replacement) \*etc.

3. Illustrations: basic analogue (monitoring system), trajectory of fault.

Nov. 19, 2004

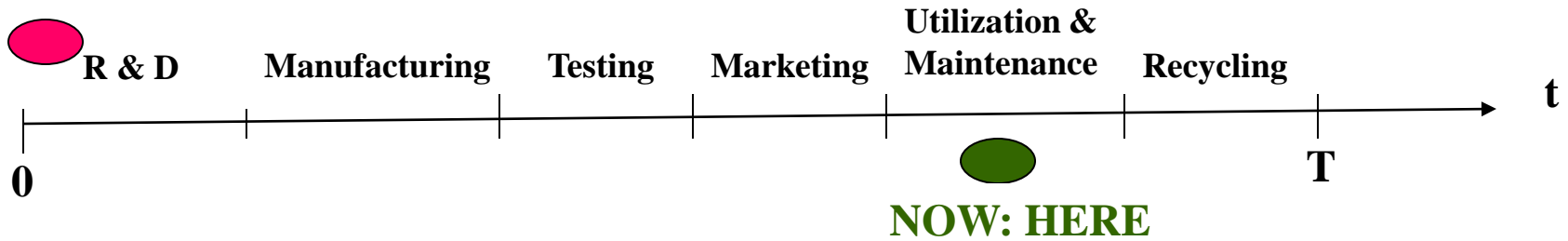
1.Preliminaries



PERSONNEL

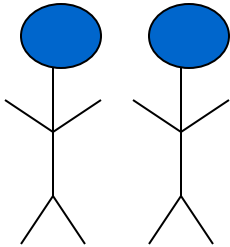


## 1.Preliminaries

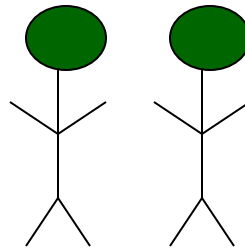


### UTILIZATION

(e.g., driver, pilot, operator)



**SYSTEM**



**MAINTENANCE**  
(support of system & personnel)

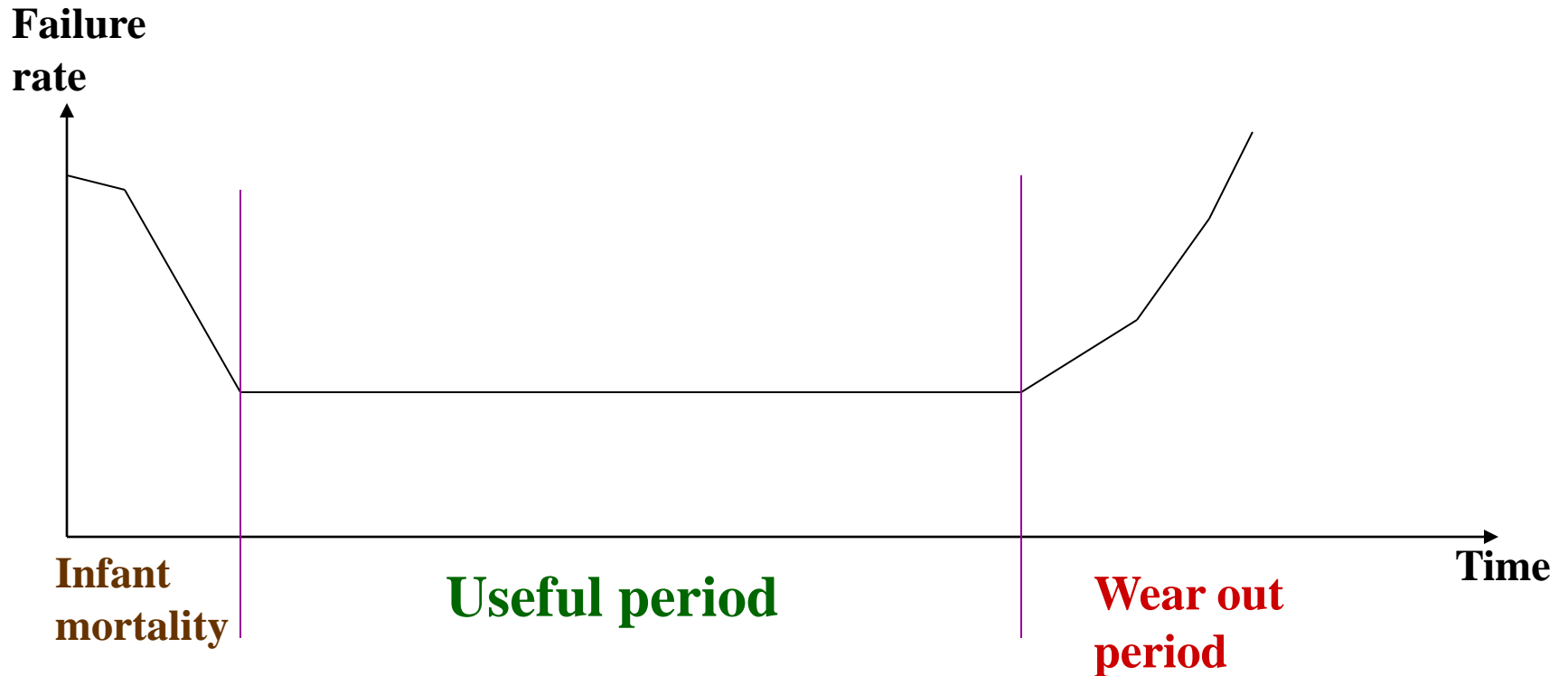
### MAINTENANCE ACTIONS:

- \*system installation
- \*training of personnel
- \*regular testing of personnel
- \*planning / scheduling of maintenance operations
- \*execution of operations (inspection, repair, replacement)
- \*analysis of the system, accumulation of data, prediction

**EXAMPLES:**

<b>SYSTEM</b>	<b>UTILIZATION</b>	<b>MAINTENANCE</b>
<b>1.Car</b>	<b>Driver, passengers</b>	<b>Maintenance personnel</b>
<b>2.Airplane</b>	<b>Pilot, passengers</b>	<b>Maintenance personnel</b>
<b>3.Human</b>	<b>Human</b>	<b>Doctor, human</b>
<b>4.House</b>	<b>Owner</b>	<b>Maintenance personnel, owner</b>
<b>5.Computer</b>	<b>User(s)</b>	<b>Maintenance personnel, special software, user(s)</b>

## TYPICAL FAILURE RATE CURVE:



## **AN ORDINAL SCALE FOR FAULTS:**

**1.OK**

**2.Small fault**

**3.Significant fault**

**4.Damage (destruction)**

## **OBJECT UNDER MAINTENANCE:**

**1.System and / or system part (component, unit)**

**2.System state**

**3.System function or function cluster  
(as a group of interrelated functions)**



## **MAINTENANCE FRAMEWORK (problems):**

- 1.System analysis / evaluation (systems, its part, etc.)**
- 2.System prediction**
- 3.Operational management / preventive maintenance:**
  - \*testing**
  - \*evaluation**
  - \*additional information**
  - \*repair / replacement**
- 4.Design of information model for system and each part (object)**
- 5.System strategy:**
  - \*selection of object under maintenance**
  - \*selection of operation (i.e., inspection, repair, replacement)**
  - \*assignment of time for operation(s)**
  - \*execution of the operation(s)**

## 2.Framework

**Planning**

**Models  
for  
system,  
faults,  
prediction**

**SYSTEM**

**Maintenance operations  
(scanning, repair, replacement)**

**Analysis,  
accumulation,  
processing,  
integration  
(fusion),  
distribution  
of  
information**

### **MODELS:**

- \*selection, \*knapsack, \*routing, \*assignment / allocation, etc.**
- \*probabilistic models, Markov processes**
  - \*reliability evaluation / analysis**
    - \*safety analysis**
      - \*simulation**
        - \*etc.**

## **SYSTEMS UNDER MAINTENANCE:**

**1. Whole system**

**2. Multi-component (modular) system:**

**\*one-layer modular system**

**\*hierarchical (multi-layer) system**

**\*multi-layer modular system**

**with complex module interrelation**

**(including interrelation between**

**different layers and branches)**

**3. Developing systems (e.g., upgrade of components, structure, interconnection)**

**4. Change of external environment**

## **KINDS OF MAINTENANCE:**

### **1.BASIC MAINTENANCE:**

**faults => operation**

### **2.PREVENTIVE MAINTENANCE:**

**prediction of faults =>  
preliminary maintenance operation**

### **3.SELF-MAINTENANCE**

## **SYSTEM CHARACTERISTICS:**

**1.Reliability (stability, etc.)**

**2.Safety**

**3.Viability, survivability**

**4.Robustness**

**4.Performance**

## **PROBLEMS & MODELS OF PREVENTIVE MAINTENANCE:**

### **1.REVELATION (ALLOCATION) OF TEST POINTS**

**Models: multicriteria selection, knapsack-like problems, allocation problems, etc.**

### **2.PLANNING TEST OPERATIONS**

**Models: multicriteria selection, knapsack-like problems, scheduling, etc.**

## **MODELING & INFORMATION PROCESSING:**

**1.Modeling of faults**

**2.Diagnosis of faults**

**3.Monitoring of faults**

**4.Tracking of faults**

**5.Integration (fusion) of information on local faults**

**6.Distribution of information on faults for various information systems and specialists**



**SYSTEM**

**BASIC ACTIONS**

- 1.Scanning**
- 2.Small repair / replacement**
- 3.Essencial repair / replacement**

**SYSTEM LAYERS**

- 1.System**
- 2.System parts**  
(group of states,  
group of functions)
- 3.Components**  
(state, function)



### 3.Basic analogue: Monitoring systems

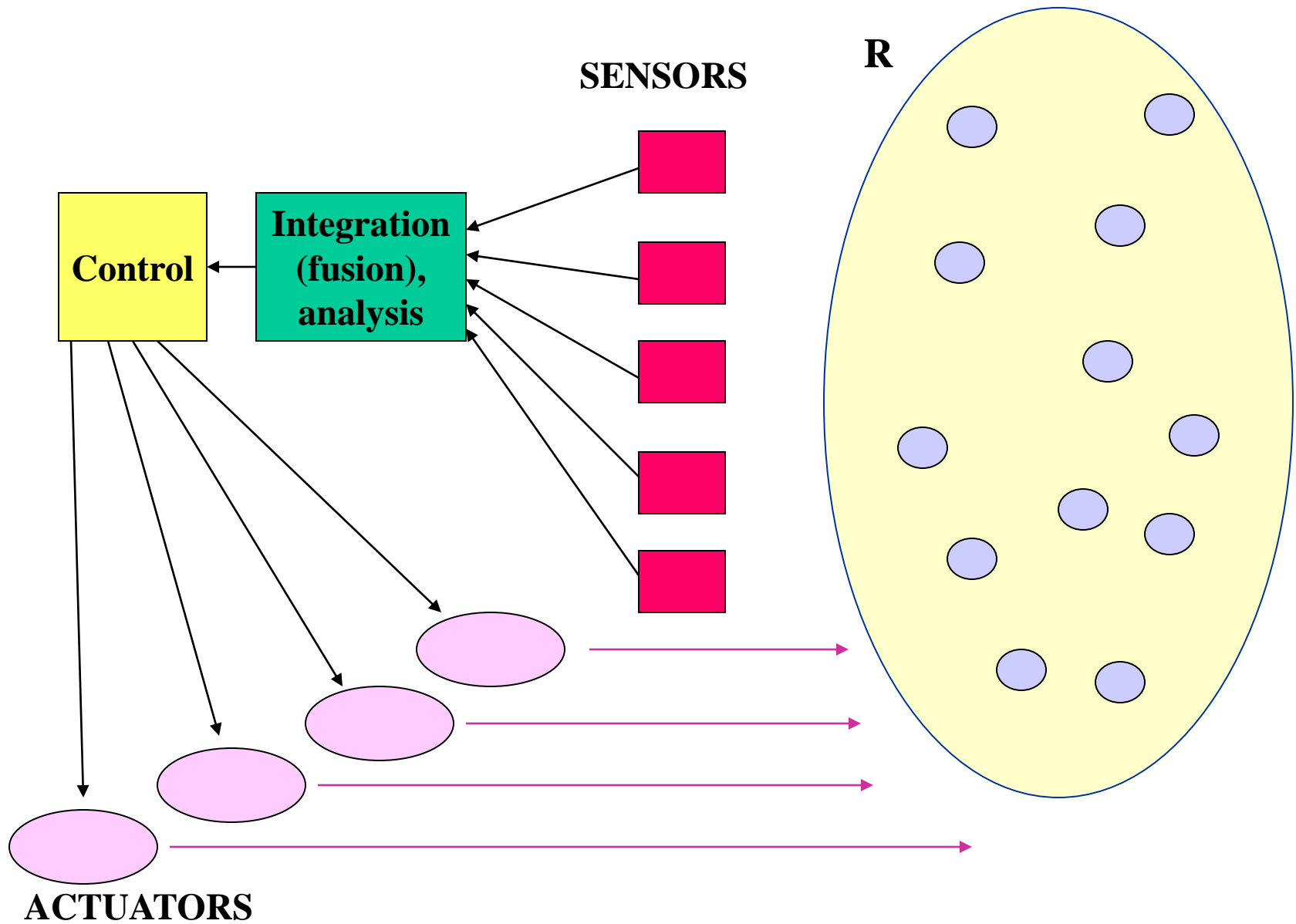
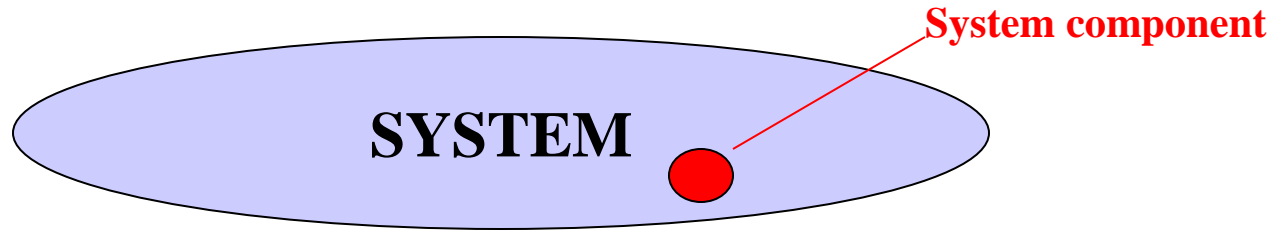
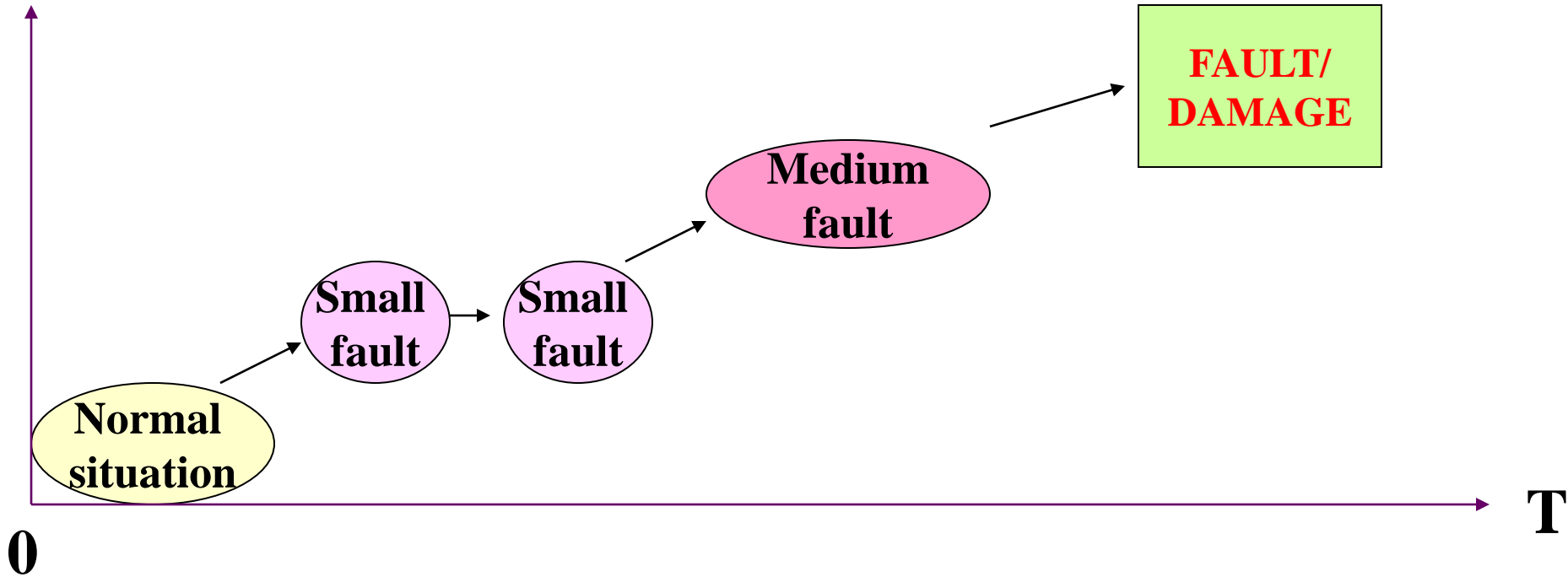


Illustration: Accumulation of information



Trajectory of faults (for components)



## **LECTURE 29. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Inst. of Physics and Technology (University)**

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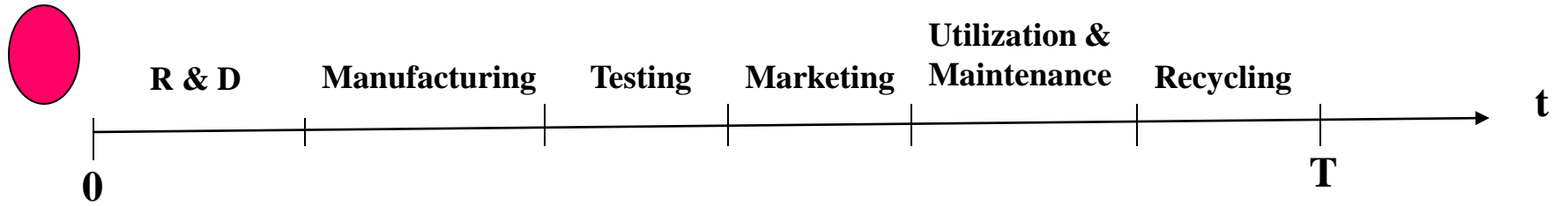
### **L.29. Requirements engineering**

#### ***PLAN:***

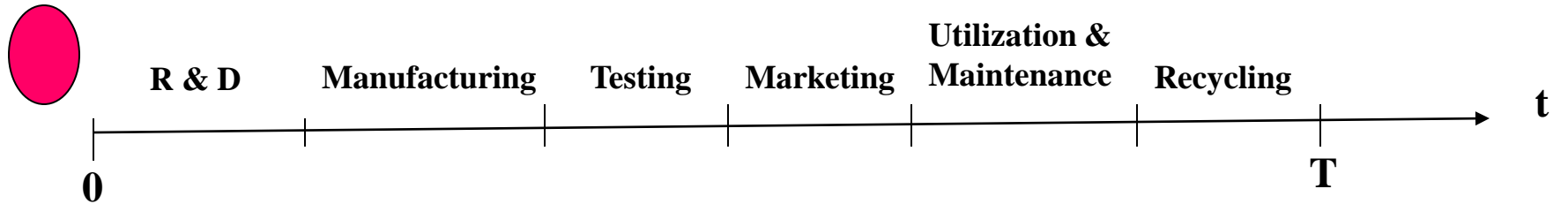
1. Requirements engineering: preliminaries
2. Types of requirements
  2. Additions
3. Systems under analysis
  4. Models

Nov. 26, 2004

# 1.Preliminaries

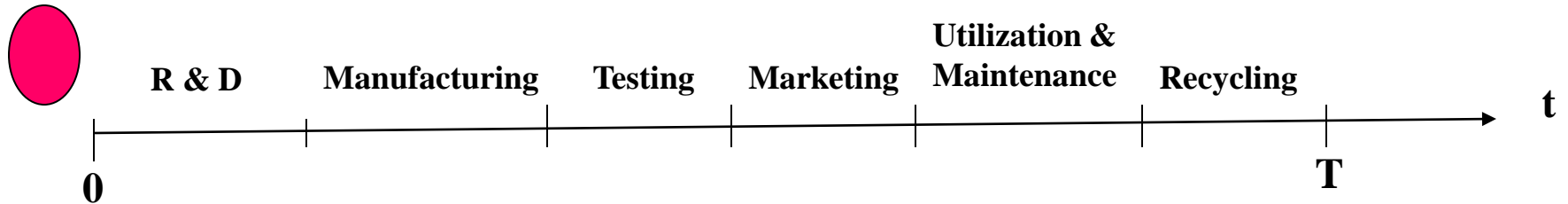


# 1.Preliminaries



**OLD RUSSIAN ENGINEERING EXPERIENCE: NB!!!**

## 1.Preliminaries

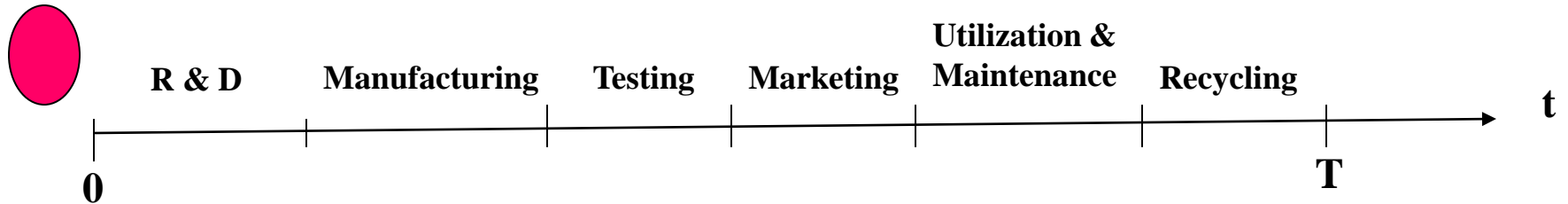


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### **WEST EXPERIENCE:**

- 1.Ralph R. Young, The Requirements Engineering Handbook, Artech House, 2004 (Carnegie Mellon Univ.)**
- 2.S. Robertson, J. Robertson, Mastering the Requirements Process. Addison-Wesley, 1999.**
- 3.K.E. Wiegers, Software Requirements. 2<sup>nd</sup> ed., Microsoft Press 2003.**

## 1.Preliminaries



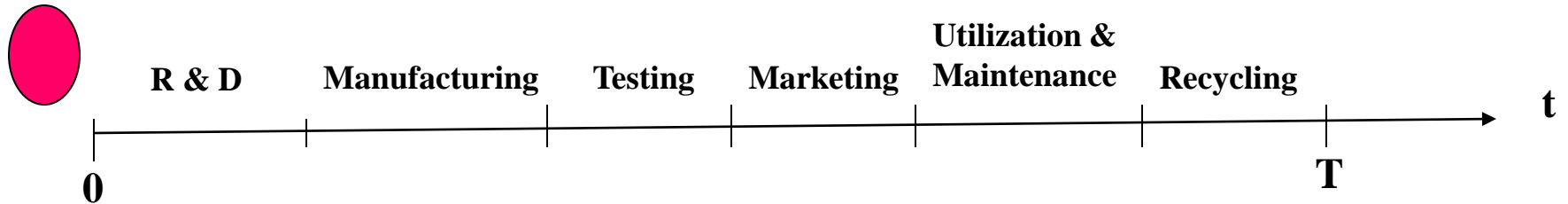
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**JOURNALS: “Requirement Engineering” (Springer), etc.**

## 1.Preliminaries



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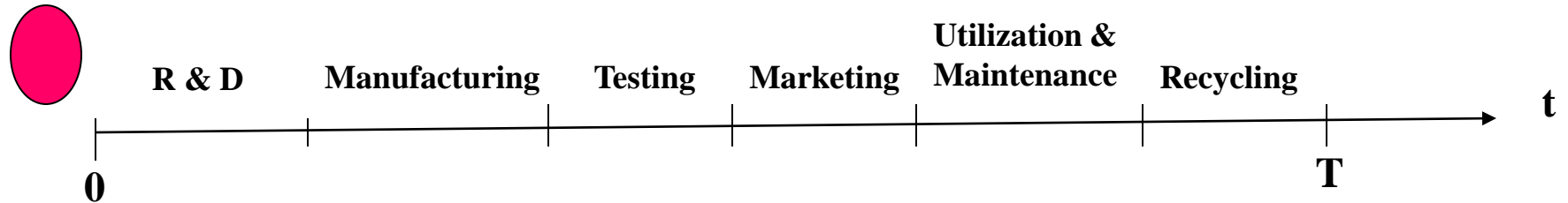
**JOURNALS: “Requirement Engineering” (Springer), etc.**

### **CONFERENCES:**

**IEEE Requirement Engineering Conference, etc.**



## 1.Preliminaries: Layers (product, requirements, standards)

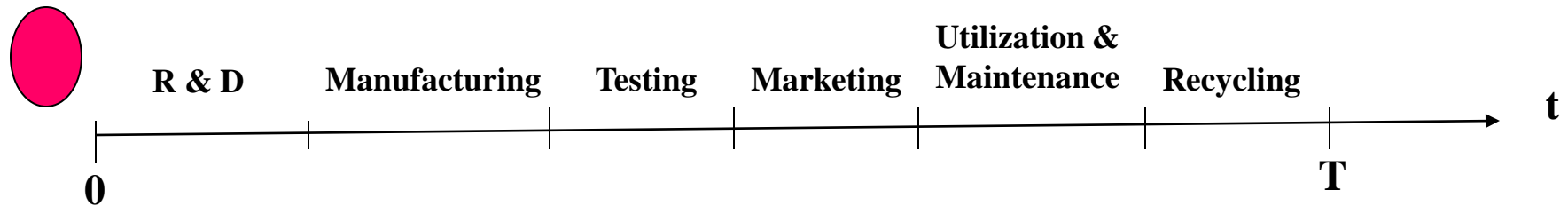


**1.Standards**

**2.Requirements**

**3.System (product, product family, platform)**

## 1.Preliminaries



### **Neighbor disciplines:**

**1.Systems engineering (& life cycle engineering)**

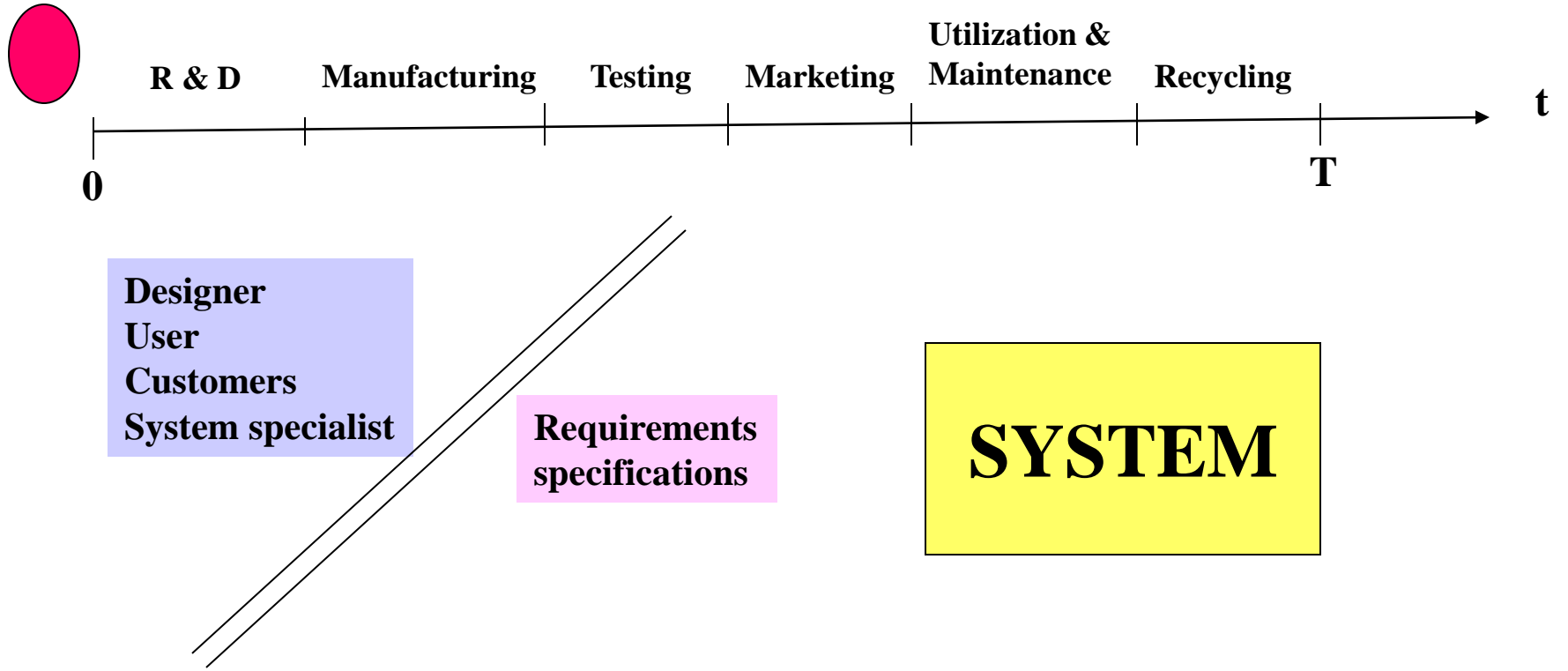
**2.Strategic management**

**3.Marketing**

**4.Forecasting**

**5.Knowledge engineering (acquisition of experience)**

# 1.Preliminaries

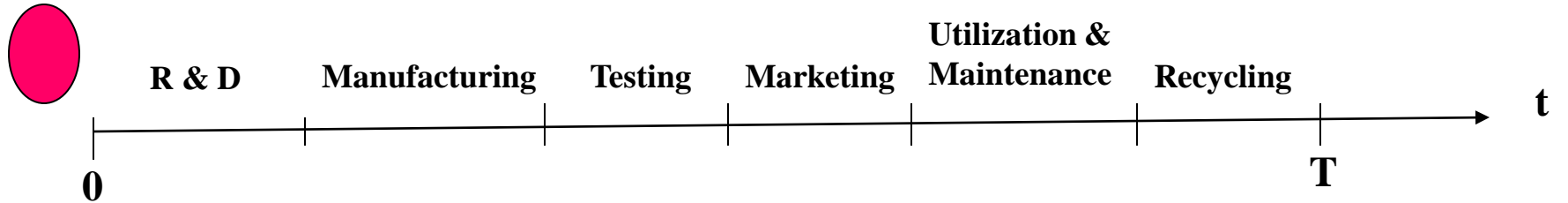


Designer  
User  
Customers  
System specialist

Requirements  
specifications

**SYSTEM**

# 1.Preliminaries



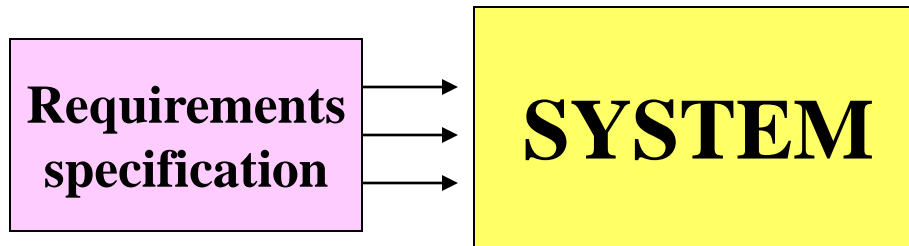
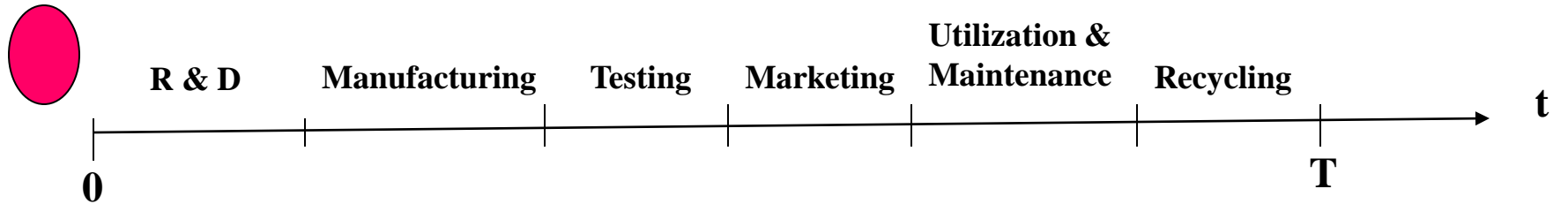
Designer  
User  
Customers  
System specialist

Requirements specifications

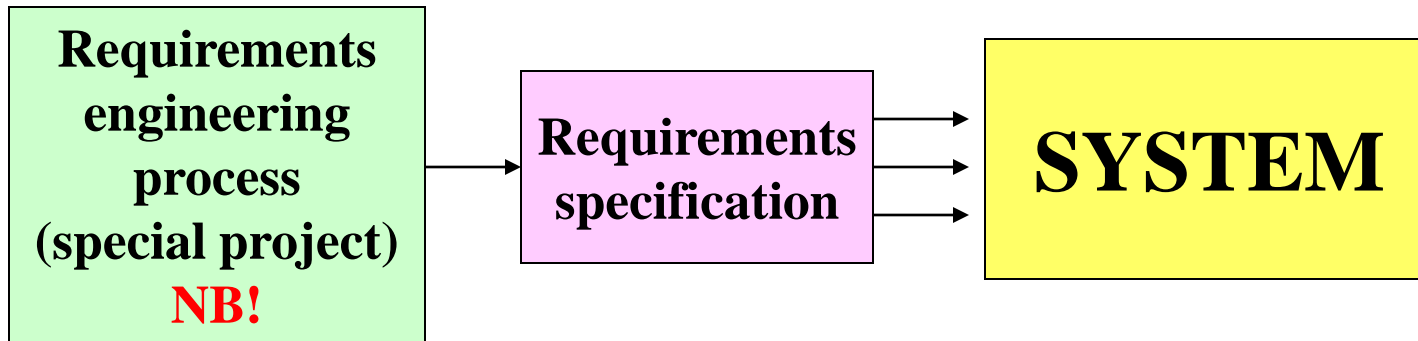
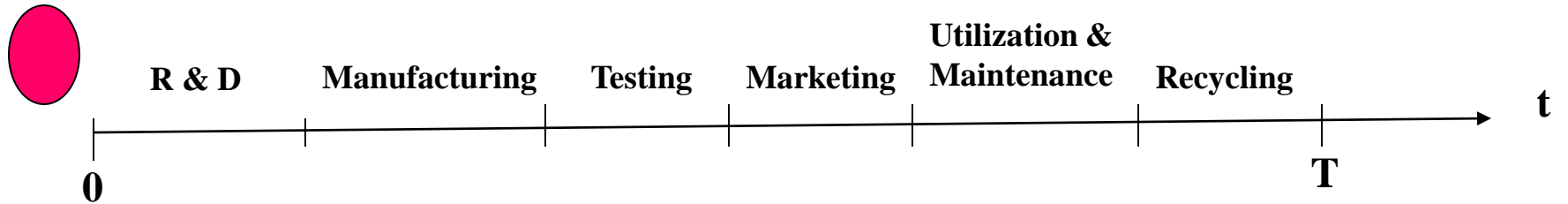
Personnel with skills (LITERACY)

**SYSTEM**

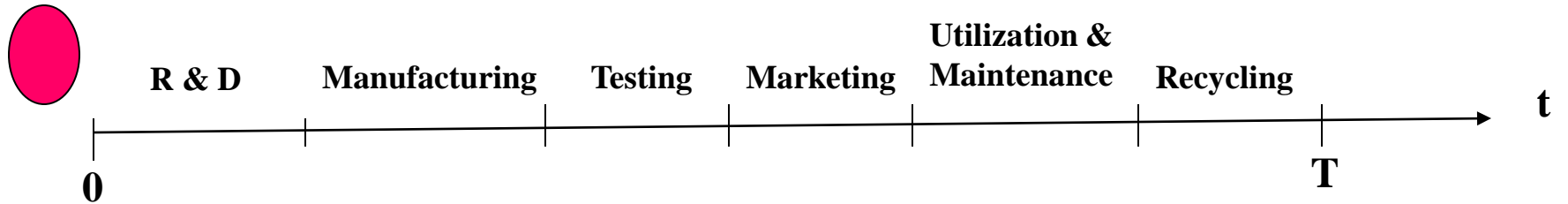
# 1.Preliminaries



# 1.Preliminaries

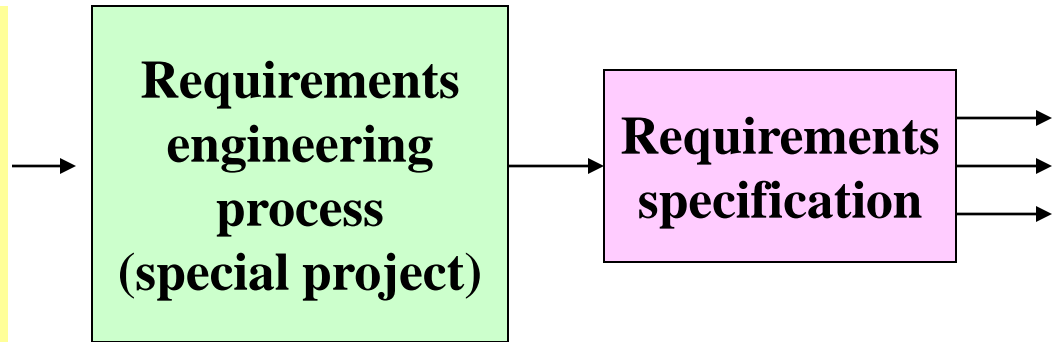


# 1.Preliminaries

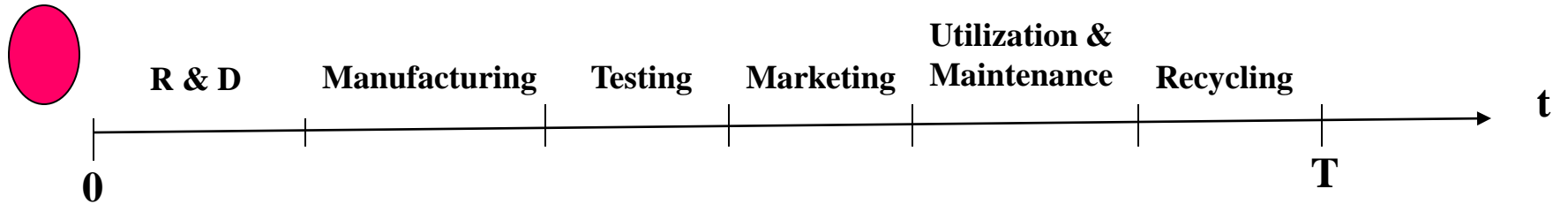


## SOURCES:

- 1.Information from users, etc.
- 2.Previous experience (e.g., design, manufacturing)
- 3.Analogue-Systems
- 4.Use Cases



# 1.Preliminaries



## SOURCES:

- 1.Information from users
- 2.Previous experience (e.g., design, manufacturing)
- 3.Analogue-Systems
- 4.Use cases

Requirements engineering process (special project)

Requirements specification

## PROBLEMS & TOOLS (TECHNIQUES):

- 1.System analysis
- 2.Discovering
- 3.Acquisition of knowledge, skills, experience
- 4.Structuring & Integration
- 5.Modeling & Representation
- 6.Analysis of dynamics
- 6.Testing
- 7.Forecasting



## 2.Types of requirements (a hierarchical system of requirements)

### **TYPES:**

- 1.Business requirements**
  - 2.User's requirements**
  - 3.High-level or system requirements**
  - 4.Functional requirements (things the system must do)**
  - 5.Non-functional requirements (properties the system must have)**
  - 6.Design requirements / design constraints**
  - 7.Manufacturing constraints**
  - 8.Performance requirements**
  - 9.Interface requirements (with other systems)**
  - 10.Qualification requirements**
  - 11.Logistics requirements**
  - 12.Environmental requirements**
  - 13.System, subsystem and component requirements**
  - 14.Reusing requirements**
- ETC.**

**ADDITIONS:**

**1.Criteria for evaluation of requirements**

**2.Prototyping**

**3.Scenarios**

**4.Reusing requirements**

## **OBJECT & HIERARCHY**

**1.System and / or system part (component, unit)**

**2.System state, group of states, state chart**

**3.System function, function cluster, digraph of function clusters**

## **OBJECT & HIERARCHY**

**1.System and / or system part (component, unit)**

**2.System state, group of states, state chart**

**3.System function, function cluster, digraph of function clusters**

**1.SYSTEM / PRODUCT**

**2.PRODUCT FAMILY**

**3.PLATFORM**

## **SCENARIOS:**

**1.STRUCTURE (e.g., chain, tree) of system states, functions**

**2.Qualitative scenarios**

**3.Integration of use cases & forecasting**

## **SCENARIOS:**

- 1.STRUCTURE (e.g., chain, tree) of system states, functions**
- 2.Qualitative scenarios**
- 3.Integration of use cases & forecasting**

## **MODELS:**

- 1.Entity relationship**
- 2.State transition model**
- 3.Entity relationship & state transition diagrams**

**MODELS:**

**I.HIERARCHY OF REQUIREMENTS**

- 1.Hierarchy of information**
- 2.Integration of information (fusion), etc.**

**II.SCENARIOS**

- 1.Coneptual maps**
- 2.Graph models, etc.**

**III.DYNAMICAL MODELING**

- 1.Simulation**
- 2.Testing, etc.**

**LECTURE 30 (compressed version). Course: “Design of Systems: Structural Approach”**  
**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**  
**Moscow Inst. of Physics and Technology (University)**

**Mark Sh. Levin**  
**Inst. for Information Transmission Problems, RAS**

Email: [mslevin@acm.org](mailto:mslevin@acm.org) / [mslevin@iitp.ru](mailto:mslevin@iitp.ru)

### **L.30. Allocation problems.**

#### ***PLAN:***

1. Allocation problem (problem formulations as assignment, matching, location):
  - \*assignment problem (marriage problem), \*quadratic assignment problem (QAP),
  - \*generalized assignment problem (GAP), \*string matching (illustration), \*multiple matching (illustration)
2. List of basic algorithm approaches
3. Evolution chart of allocation-like problems
4. List of application domains
5. Basic references (books, sites)

Dec. 3, 2004

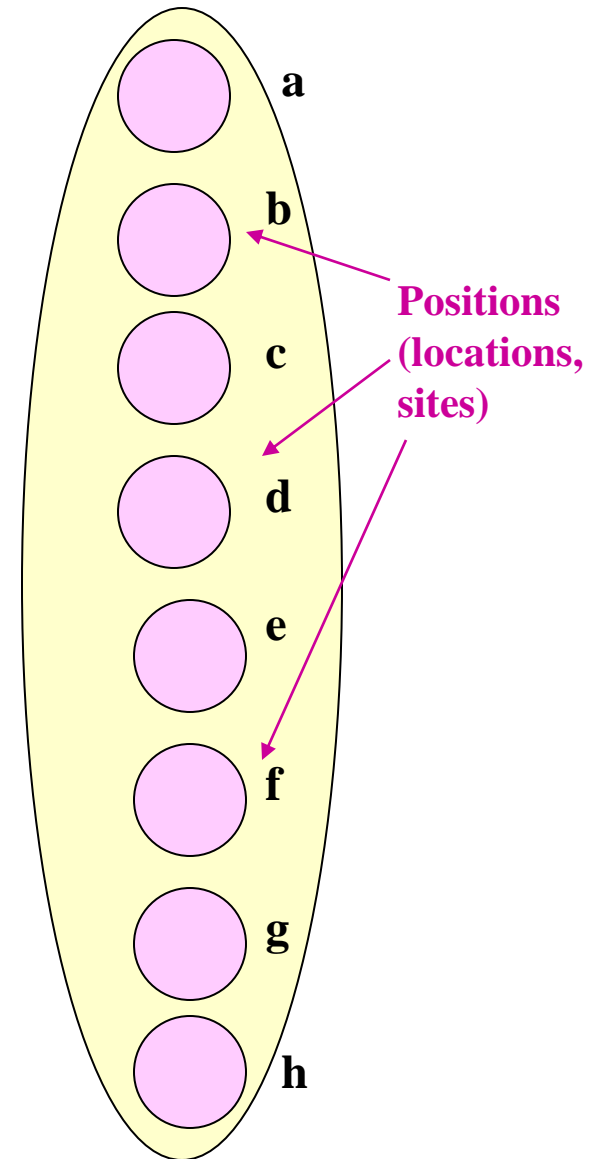
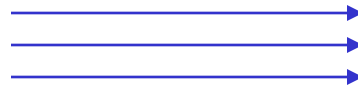
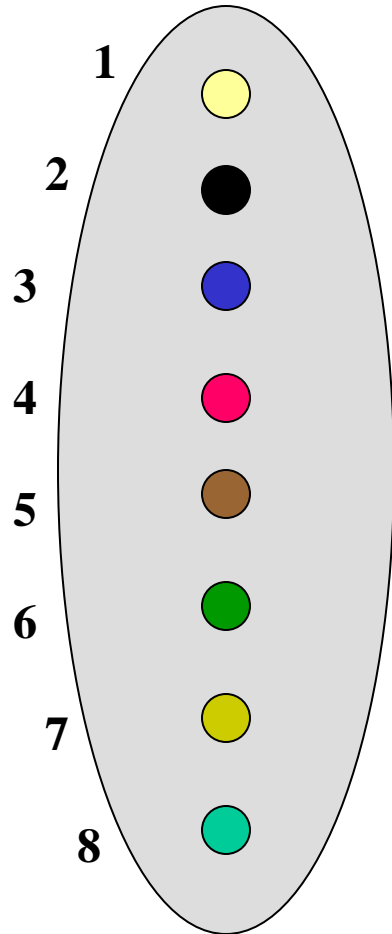


Allocation problem

Allocation (assignment, matching, location):

Set of elements  
(e.g., personnel, facilities)

MAPPING

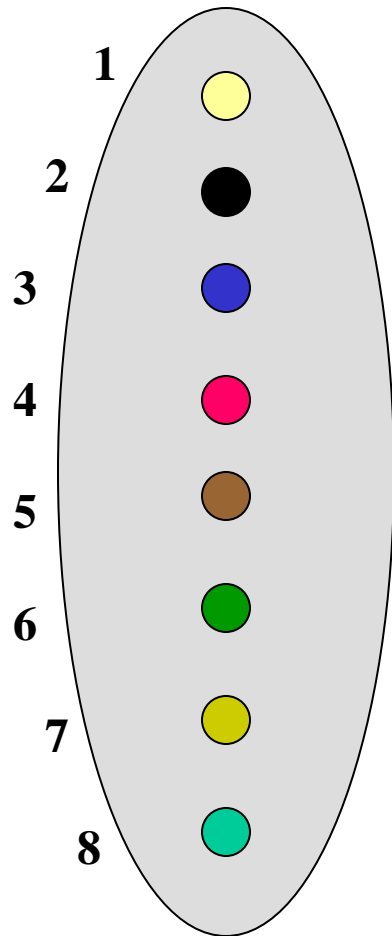


BIPARTITE GRAPH

# Allocation problem

## Allocation (assignment, matching, location):

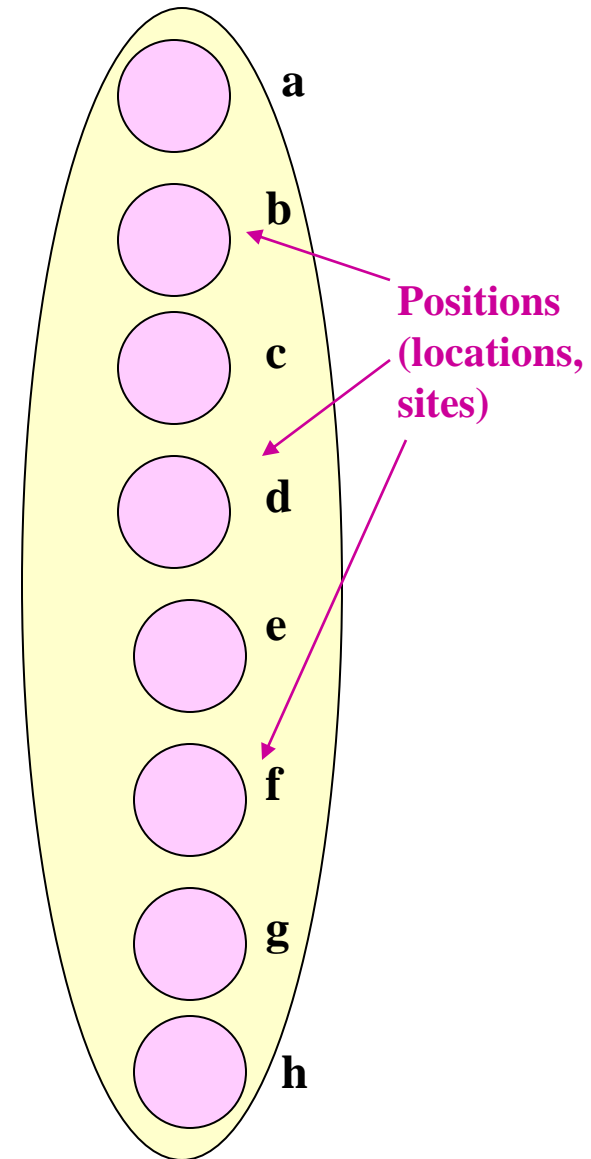
Set of elements  
(e.g., personnel, facilities)



matrix of weights  $c_{ij}$

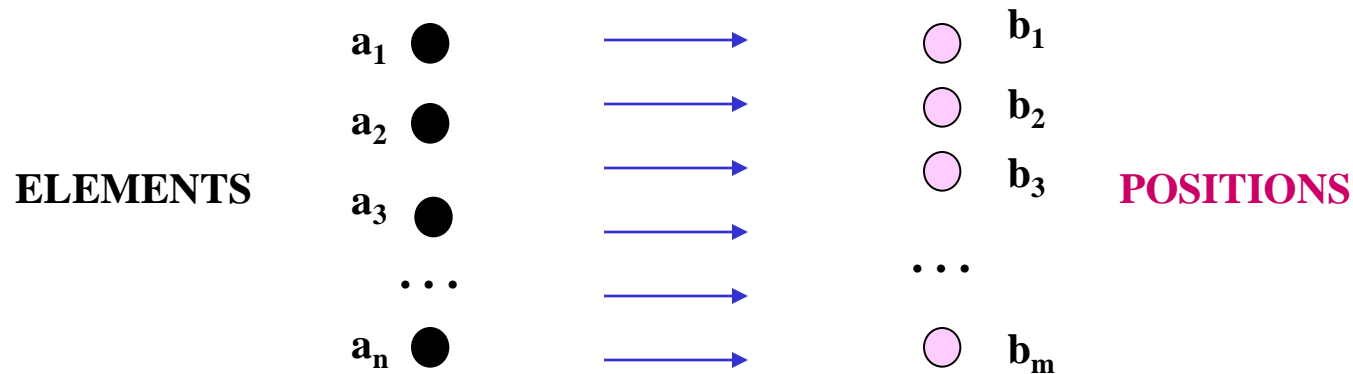
	a	b	c	d	e	f	g	h
1	.	.	.	.	.	.	.	.
2	.	.	.	.	.	.	.	.
3	.	.	.	.	.	.	.	.
4	.	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.
6	.	.	.	.	.	.	.	.
7	.	.	.	.	.	.	.	.
8	.	.	.	.	.	.	.	.

**BIPARTITE GRAPH**



- 1.Boys -- *Girls* (*marriage problem*)
  - 2.Workers -- *Work positions*
  - 3.Facilities -- *Positions in manufacturing system* (*facility layout*)
  - 4.Tasks -- *Processors in multiprocessor system*
  - 5.Anti-rockets -- *Target in defense systems*
  - 6.Files -- *Databases in distributed information systems*
- Etc.

## Assignment problem AP



### FORMULATION (combinatorial):

Set of elements (e.g., personnel, facilities, tasks):  $A = \{ a_1, \dots, a_i, \dots, a_n \}$

Set of positions (e.g., locations, processors)  $B = \{ b_1, \dots, b_j, \dots, b_m \}$

(now let  $n = m$ )

Effectiveness of pair  $a_i$  and  $b_j$  is:  $c(a_i, b_j)$

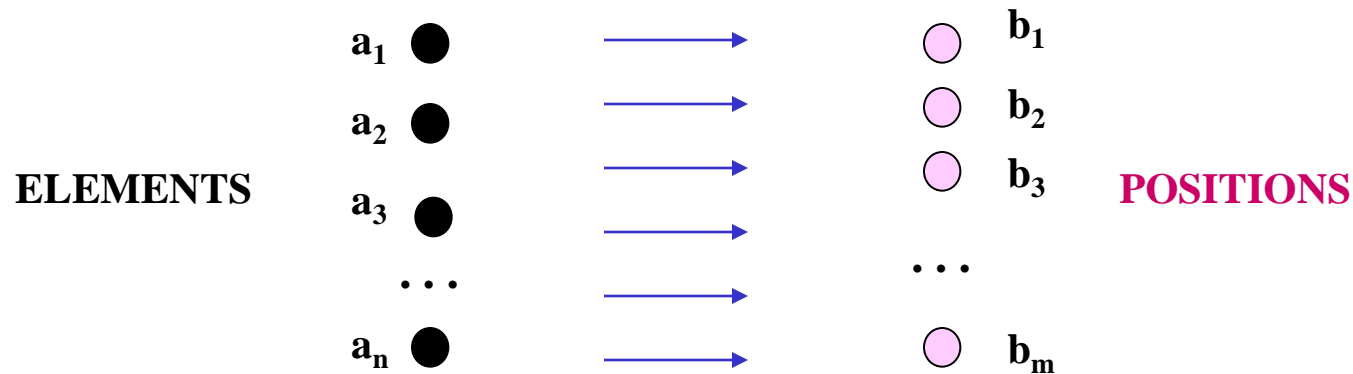
$\Pi = \{s\}$  is set of permutations (assignment) of elements of  $A$  into position set  $B$ :

$s^* = \langle (s^*[1]), \dots, (s^*[i]), \dots, (s^*[n]) \rangle$ , i.e., element  $a_i$  into position  $s[i]$

The goal is:  $\max \sum_{i=1}^n c(i, s[i])$

**Maximum (minimum) weight matching in a weighted bipartite graph**

## Assignment problem AP



### ANOTHER FORMULATION (algebraic):

Set of elements (e.g., personnel, facilities, tasks):  $A = \{ a_1, \dots, a_i, \dots, a_n \}$

Set of positions (e.g., locations, processors)  $B = \{ b_1, \dots, b_j, \dots, b_m \}$

(now let  $n = m$ )

Effectiveness of pair  $a_i$  and  $b_j$  is:  $c(a_i, b_j)$

$x_{ij} = 1$  if  $a_i$  is located into position  $b_j$  and  $0$  otherwise ( $x_{ij} \in \{0,1\}$ )

The problem is:  $\max \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$

s.t.  $\sum_{i=1}^n x_{ij} = 1 \quad \forall j$   
 $\sum_{j=1}^n x_{ij} = 1 \quad \forall i$

## Assignment problem AP

### **ALGORITHMS:**

**1. Polynomial exact algorithm (  $O(n^3)$  )**

**2. Hungarian method**

**Etc.**

### **OTHER VERSIONS:**

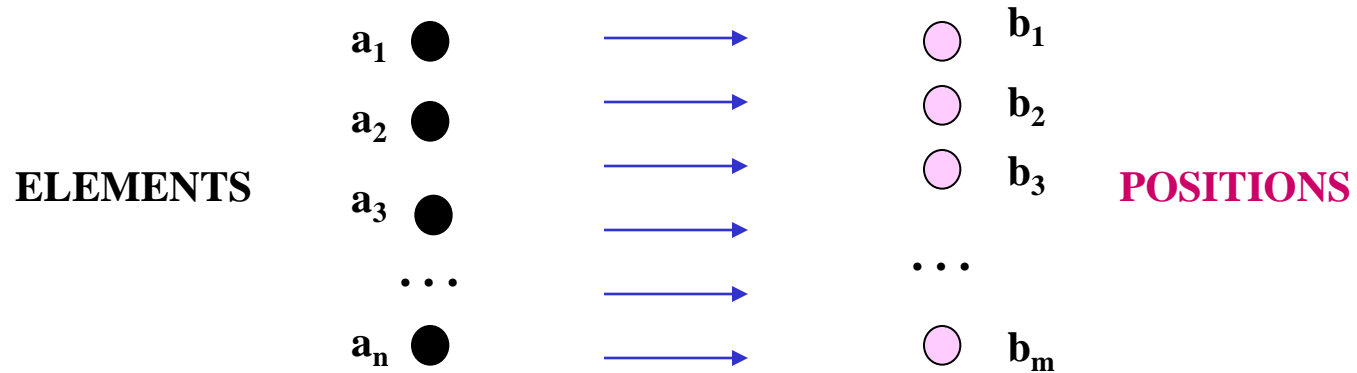
**1. "*Minimum*" problem**

**2. "*Min max*" problem**

**3. Multicriteria problem**

**Etc.**

## Quadratic Assignment Problem QAP



matrix of weights (“flow”)  $c_{ij}$

	$b_1$	$\dots$	$b_j$	$\dots$	$b_n$
$a_1$	.		.		.
$\dots$	.		.		.
$a_i$	.		$c_{ij}$		.
$\dots$	.		.		.
$a_n$	.		.		.

matrix of distance  $d_{ij}$

	$b_1$	$\dots$	$b_j$	$\dots$	$b_n$
$b_1$	.		.		.
$\dots$	.		.		.
$b_i$	.		$d_{ij}$		.
$\dots$	.		.		.
$b_n$	.		.		.

## A Basic (“flow”) Mathematical Formulation of the Quadratic Assignment Problem

An assignment of elements  $\{ i \mid i \in \{1, \dots, n\} \}$  to positions (locations)  $\{ p(i) \}$ , where  $p$  is permutation of numbers  $\{1, \dots, n\}$ , a set of all possible permutations is  $\Pi = \{ p \}$ .

Let us consider two  $n$  by  $n$  matrices:

- (i) a “flow” (or “utility”) matrix  $C$  whose  $(i, j)$  element represents the flow between elements (e.g., facilities)  $i$  and  $j$ , and
- (ii) a distance matrix  $D$  whose  $(i, j)$  ( $p(i), p(j)$ ) element represents the distance between locations  $p(i)$  and  $p(j)$ .

With these definitions, the QAP can be written as

$$\max_{p \in \Pi} \sum_{i=1}^n \sum_{j=1}^n c_{ij} d_{p(i)p(j)}$$



**ALGORITHMS:**

**1.Branch & Bound method**

**2.Relaxation approach**

**3.Greedy algorithms**

**4.Genetic algorithms (evolutionary computing)**

**5.Metaheuristics (i.e., local optimization, hybrid schemes)**

**BASIC BOOKS:**

**1.P.M. Pardalos, H. Wolkowicz, (Ed.), Quadratic Assignment and Related Problems. American Mathematical Society, 1994.**

**2.E. Cela, The Quadratic Assignment Problem. Kluwer, 1998.**

**SITES:**

**1.Quadratic assignment Problem Library: <http://www.opt.math.tu-graz.ac.at/qaplib/>**

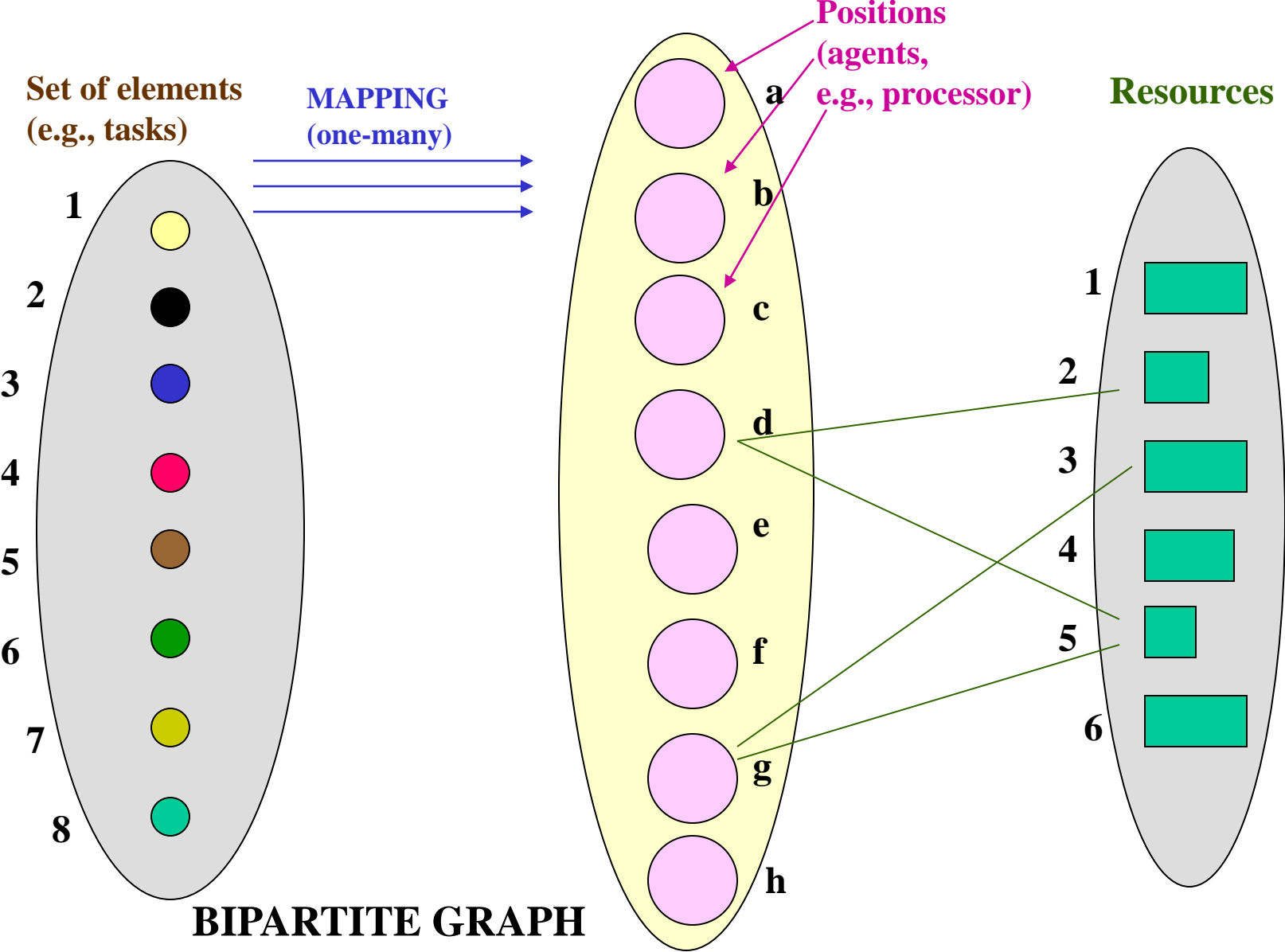
**Multi-objective (multicriteria) Assignment Problem  
(Quadratic Assignment Problem):**

**Elements in matrix of weights (“flows”) are vectors.**

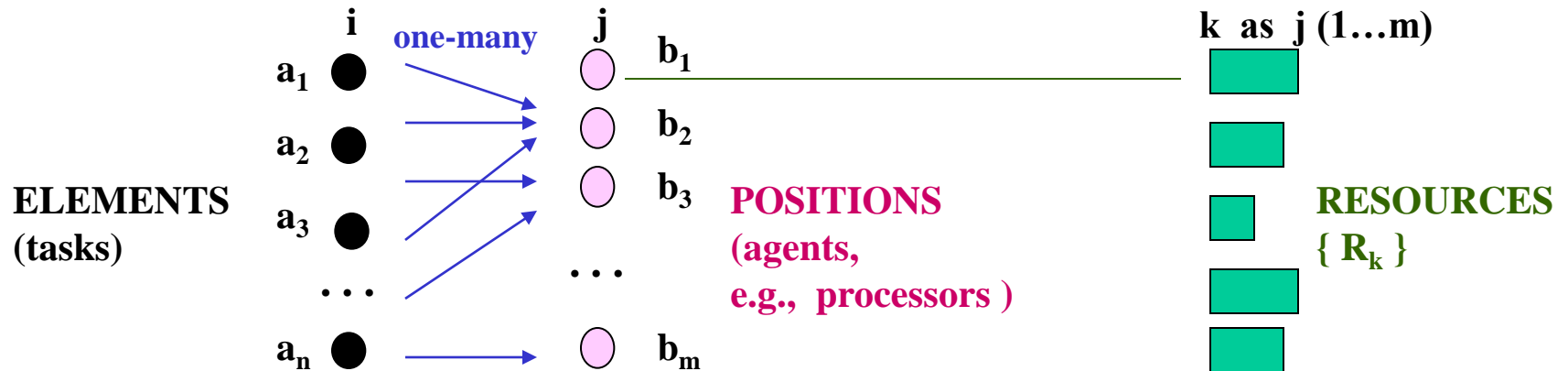
**ALGORITHMS**

- 1.Branch & Bound method**
- 2.Relaxation approach**
- 3.Greedy algorithms**
- 4.Metaheuristics (i.e., local optimization, hybrid schemes)**
- 5.Multi-objective evolutionary optimization**

Generalized Assignment Problem GAP



## Generalized Assignment Problem GAP



### FORMULATION (algebraic):

Set of elements (e.g., personnel, facilities, tasks):  $A = \{ a_1, \dots, a_i, \dots, a_n \}$

Set of positions (e.g., locations, processors)  $B = \{ b_1, \dots, b_j, \dots, b_m \}$

(now let  $n = m$ )

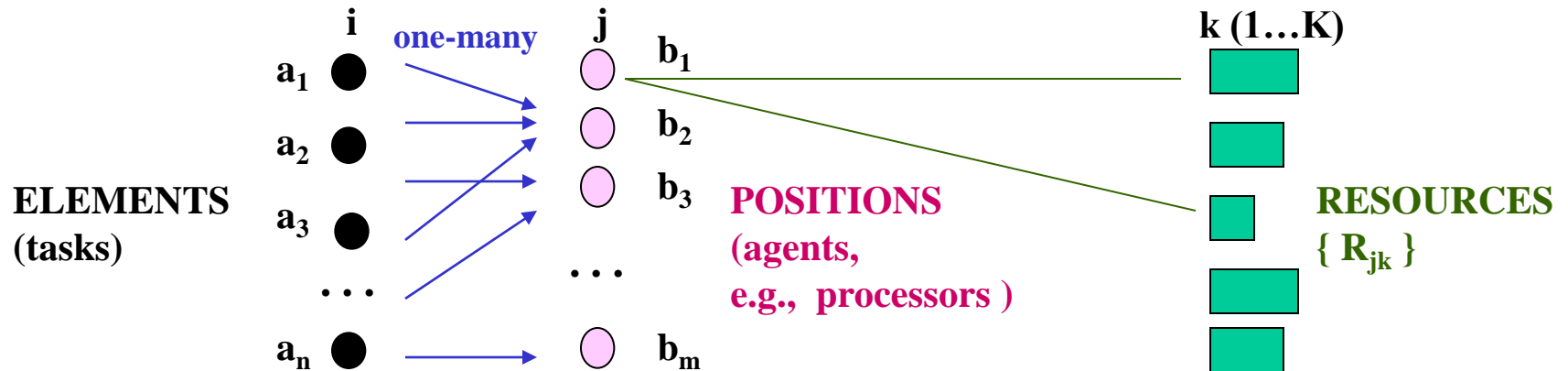
Effectiveness of pair  $a_i$  and  $b_j$  is:  $c(a_i, b_j)$

$x_{ij} = 1$  if  $a_i$  is located into position  $b_j$  and 0 otherwise ( $x_{ij} \in \{0,1\}$ )

The problem 1 is:  $\max \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$

s.t.  $\sum_{i=1}^n r_{ik} x_{ij} \leq R_j \quad \forall j$  ( $R_k$  is resource of agent  $k$ )  
 $\sum_{j=1}^n x_{ij} = 1 \quad \forall i$

## Generalized Assignment Problem GAP (multiple common resources)



### FORMULATION (algebraic):

Set of elements (e.g., personnel, facilities, tasks):  $A = \{ a_1, \dots, a_i, \dots, a_n \}$

Set of positions (e.g., locations, processors)  $B = \{ b_1, \dots, b_j, \dots, b_m \}$

(now let  $n = m$ )

Effectiveness of pair  $a_i$  and  $b_j$  is:  $c(a_i, b_j)$

$x_{ij} = 1$  if  $a_i$  is located into position  $b_j$  and 0 otherwise ( $x_{ij} \in \{0, 1\}$ )

The problem 2 is:  $\max \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$

s.t. 
$$\sum_{j=1}^m \sum_{i=1}^n r_{ik} x_{ij} \leq R_k \quad \forall k \text{ (common } K \text{ resources)}$$

$$\sum_{j=1}^m x_{ij} = 1 \quad \forall i$$

**ALGORITHMS:**

**1.Branch & Bound method**

**2.Relaxation approach**

**3.Heuristics (greedy algorithms, etc.)**

**4.Genetic algorithms (evolutionary computing)**

**5.Metaheuristics (i.e., local optimization, hybrid schemes)**

**EXTENSION:**

**1.Multicriteria cases**

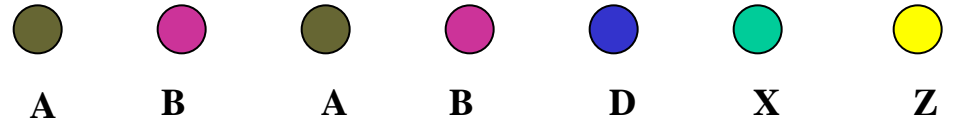
**2.Uncertainty**

**3.Etc. (e.g., dependence of problem parts,  
dependence on time)**

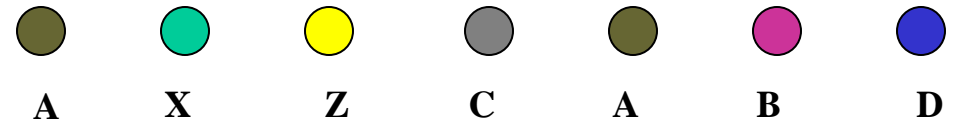
# String matching

## CASE OF 2 WORDS:

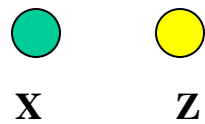
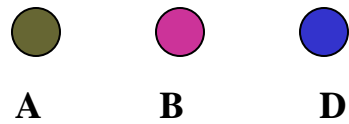
String (word) 1



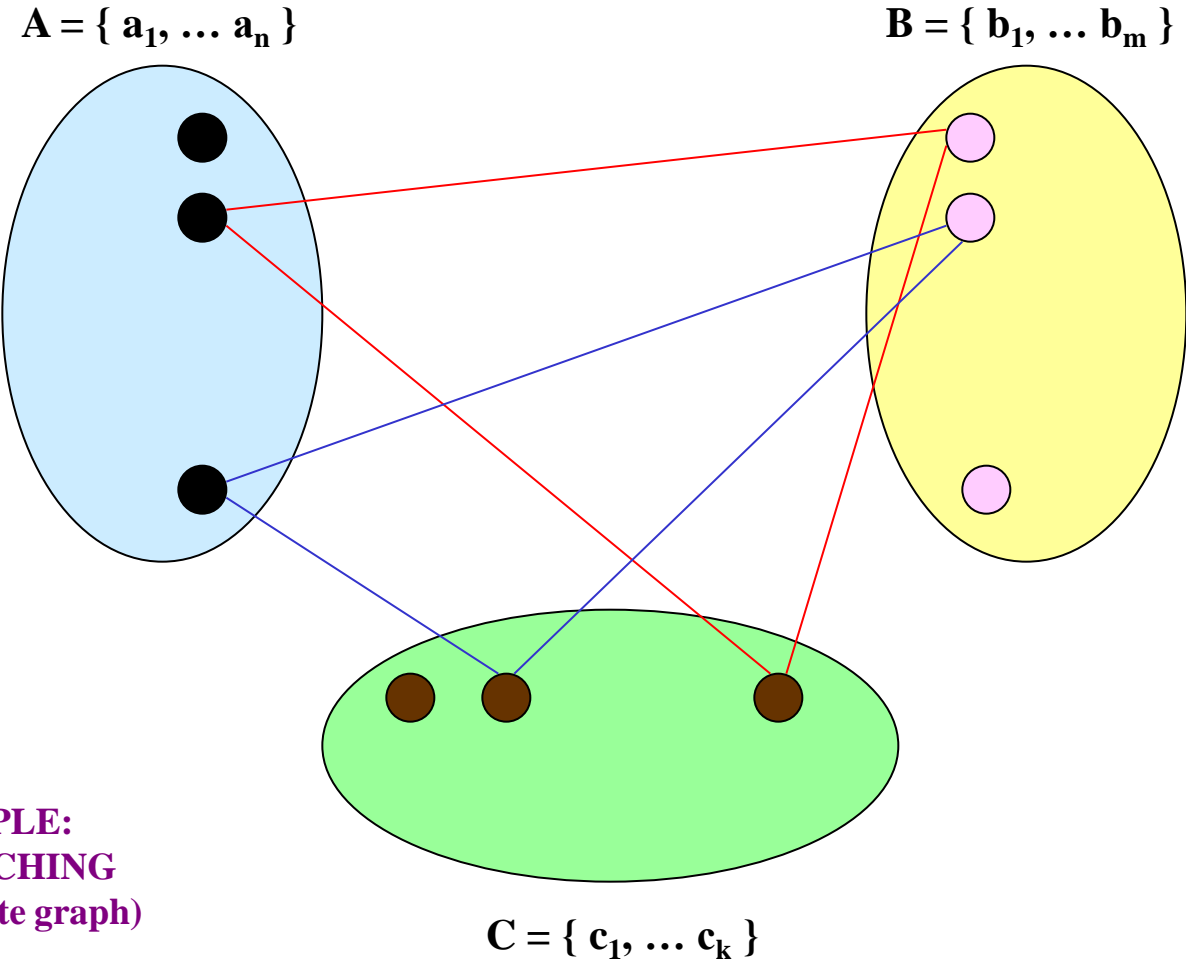
String (word) 2



## PATTERNS:



Multiple matching problem (Lecture 17-18)



**EXAMPLE:**  
**3-MATCHING**  
**(3-partite graph)**



**ALGORITHMS:**

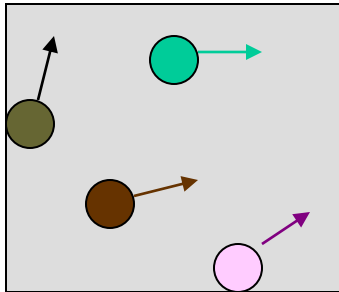
- 1.Enumerative algorithms (e.g., Branch-And-Bound)**
- 2.Heuristics (e.g., greedy algorithms, various local optimization techniques)**
- 3.Metaheuristics including hybrid methods**
- 4.Morphological approach**

**VERSIONS:**

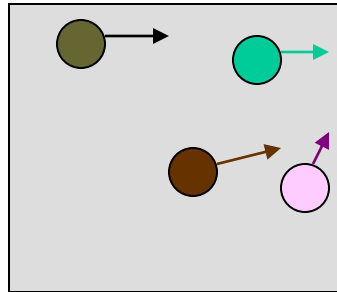
- 1.Dynamical (multi-stage) problem (multiple track assignment)**
  - 2.Problem with errors**
  - 3.Problem with uncertainty (probabilistic estimates, fuzzy sets)**
- Etc.**

Usage of assignment problem to define velocity of particles, etc. (Lecture 17-18)

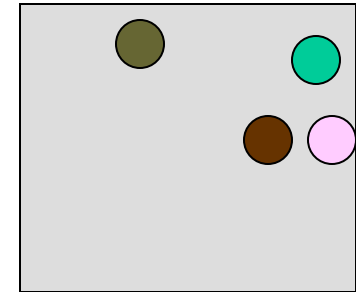
FRAME 1



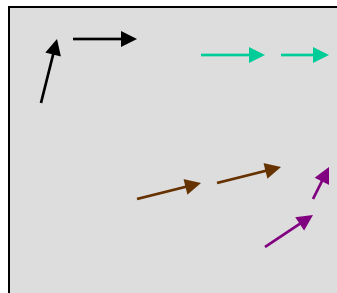
FRAME 2



FRAME 3



VELOCITY SPACE



**APPLICATIONS (air/ water environments):**

1. Physical experiments
2. Climat science
3. Chemical processes
4. Biotechnological processes

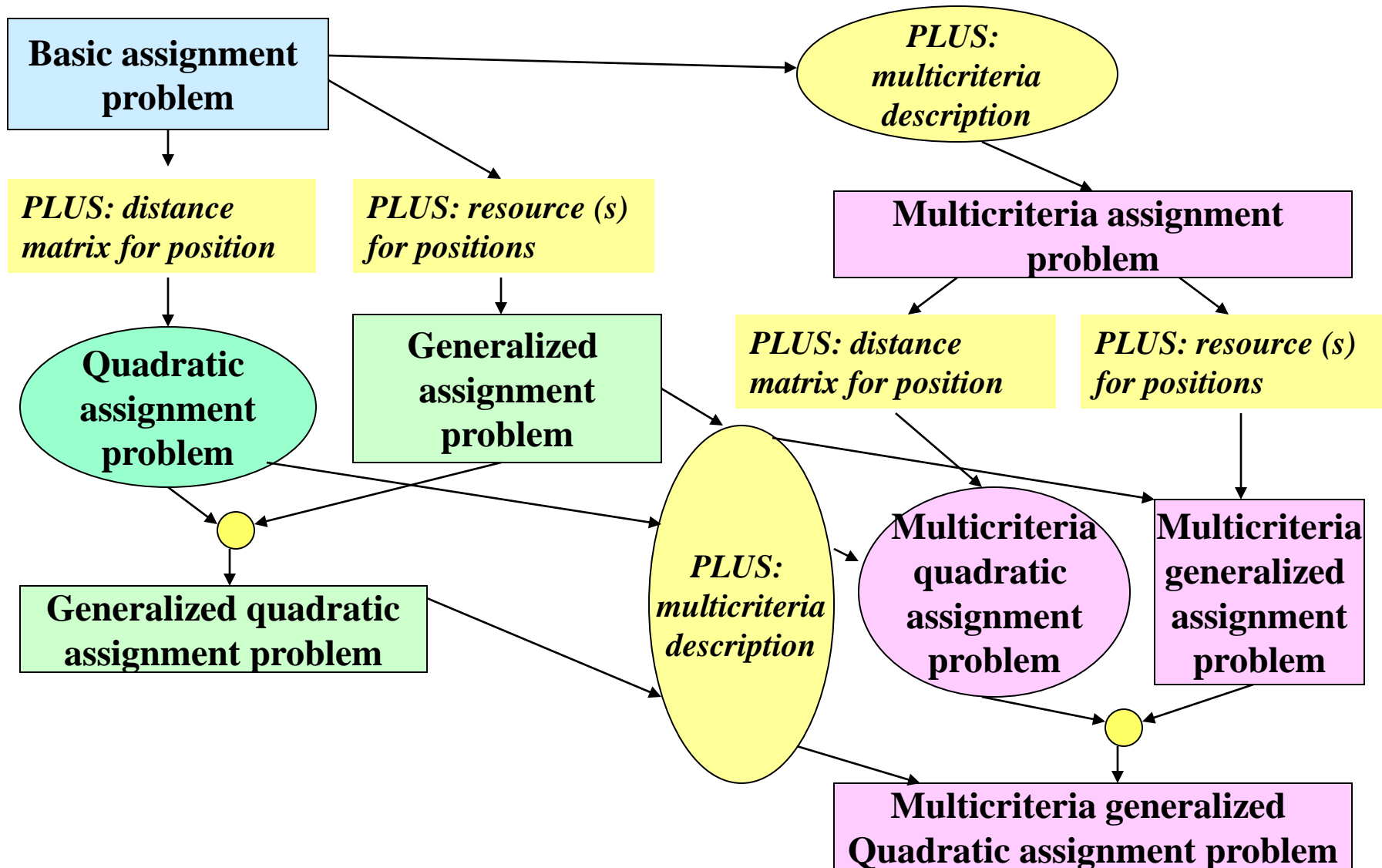
**Contemporary sources:**

1. PIV systems (laser/optical systems)
2. Sattelite photos
3. Electronic microscope, etc.

## List of algorithms for allocation-like problems

- 1. polynomial algorithms (e.g., Hungarian method, techniques on the basis of flow-like ideas, etc.)**
- 2. LP-based algorithms**
- 3. epsilon-approximation polynomial algorithms**
- 4. data-correction algorithms**
- 5. Branch-And-Bound method**
- 6. dynamic programming approach**
- 7. evolutionary and genetic algorithms**
- 8. Bender's decomposition scheme**
- 9. Local optimization techniques as various heuristics  
(Tabu-search algorithms, simulated annealing, hybrid schemes etc.)**
- 10. techniques of multiple criteria analysis (e.g., multi-attribute utility analysis, Analytic Hierarchy Process, ELECTRE outranking method)**
- 11. simulation based approaches**
- 12. polyhedral methods**
- 13. hierarchical approaches including morphological (decomposition) approach**
- 14. "constraint satisfaction problem" approach**
- 15. fuzzy methods**
- 16. knowledge-based algorithmic rules**
- 20. neural networks**

Evolution chart of allocation-like problems



## Application domains of allocation-like problems

- 1.facility allocation (layout) in manufacturing systems**
- 2.allocation of resources in investment actions**
- 3.allocation of facilities in supply chain management**
- 4.assignment routing problems**
- 5.allocation / layout in VLSI design**
- 6.allocation / layout in the space design of buildings including topological / layout in multi-floor building design**
- 7.allocation in architectural planning, e.g., in urban systems (allocation of buildings and / or components of infrastructure as railway stations, hospitals, shop-centers, schools, parks, etc.) including site search (selection) problem, i.e., allocation of sites on a map**
- 8.allocation of emergency service facilities**
- 9.allocation of buffer capacities in production lines (including distribution of maintenance operations)**
- 10.allocation of total maintenance investment among products**
- 11.allocation of inspection efforts in various systems (e.g., production systems), allocation of test positions and test time and optimal allocation of test resources**
- 12.allocation of operations for balance control in assembly lines**
- 13.allocation of human-machine functions**
- 14.portfolio selection (optimization) on the basis of assignment-like models (e.g., quadratic assignment problem)**
- 15.project tasks assignment**

## Application domains of allocation-like problems

- 16.allocation of resources in large team
- 17.allocation of traffic police resources
- 18.blood assignment in a donation-transfusion system
- 19.assignment of papers to reviewers
- 20.assignment in sport
- 21.dynamic storage allocation , note this problem is a complex one and often corresponds to a set of combinatorial problems including bin-packing, segmentation / partitioning, scheduling, etc.
- 22.task allocation / assignment in distributed (e.g., computer and / or information) systems
- 23.allocation of production tasks in a virtual organization
- 24.information (e.g., databases files, documents) allocation in distributed computer / information environments and networks, for example, allocation / reallocation of arrived data items in a distributed information system (a set of servers), placement and replacement for large-scale distributed caches in digital libraries or in Internet
- 25.capacity assignment / allocation in communication networks
- 26.allocation of Internet resources 27.allocation of resources in e-business infrastructure
- 28.design of standards
- 29.multisensor multitarget tracking on the basis of non-linear and multidimensional assignment problems (e.g., in radar systems)
- 30.allocation of reliability among components that are to be assembled into a system or component redundancy allocation
- 31.assignment problem in experimental high energy physics
- 32.locomotive assignment to train-segments

## Application domains of allocation-like problems

- 33.assignment / allocation problems in universities (e.g., assignment of students to exams, assignment of faculty members to courses, assignment of auditoriums for lectures, allocation of the most desirous projects to the most qualified students)**
- 34.assignment / allocation of particles / points (particle matching) in particle image velocimetry (PIV) and particle tracking velocimetry (PTV) for measurement in fluid mechanics**
- 35.personnel management systems (e.g., allocation of personnel, allocation of tasks, role assignment) including assignment of technicians to handle computer system faults (e.g., hardware, software, communication)**
- 36.allocation of rooms among people**
- 37.allocation of rights (e.g., in social networks for social choice and welfare, for participants of electronic financial markets)**
- 38.allocation of discrete resources**
- 39.channel and frequency assignment in mobile radio systems (including dynamical assignment)**
- 40.assigning cells to switches in cellular mobile networks**
- 41.allocation of tolerances in manufacturing systems**
- 42.wavelength allocation on trees of rings for optical communication networks**

## Application domains of allocation-like problems

**43.allocation in medical organizations, for example:**

- (a) allocation of surgeries to operating rooms, (b) allocation of inpatient resources,**
- (c) staff assignment problem, (d) the workshift and rest assignment of nursing personnel,**
- (e) hospital facility layout, (f) bed allocation, (g) organ allocation,**
- (h) patient assignment, and (i) medical service allocation and reallocation**

**44.bottleneck allocation methodology for scheduling in manufacturing systems**

**45.traffic assignment in transportation networks and  
in communication / computer networks**

**46.hierarchical location-allocation problems on networks**

**47.dynamic allocation of resources in multi-project research and development systems**

**48.allocation of ecological facilities**

**49.hierarchical location-allocation of banking facilities**



**BASIC BOOKS:**

- 1.M.S. Daskin, **Networks and Discrete Location. Models, Algorithms, and Applications.** Wiley, 1995.
- 2.G.Y. Handler, P.B. Mirchandrani, **Location on Networks: Theory and Algorithms.** MIT Press, 1979.
- 3.E. Minieka, **Optimization Algorithms for Networks and Graphs.** Marcel Dekker, 1978.
- 4.P.B. Mirchandrani, R.L. Francis, (Eds.), **Discrete Location Theory,** Wiley, 1990.
- 5.P.M. Pardalos, H. Wolkowicz, (Ed.), **Quadratic Assignment and Related Problems.** American Mathematical Society, 1994.
- 6.E. Cela, **The Quadratic Assignment Problem.** Kluwer, 1998.
- 7.M.I. Rubinshtein, **Optimal Grouping of Interconnected Objects,** Nauka, (in Russian), 1989.
- 8.D.Gusfield, R.W. Irwing, **The Stable Marriage Problem: Structure and Algorithms,** The MIT Press, 1989.
- 9.J.Aoe, (Ed.), **Computer Algorithms: String Pattern Matching Strategies.** IEEE CS Press, 1994.
- 10.A.I. Barros, **Discrete and Fractional Programming Techniques for Location Models.** Kluwer, 1998.

**SITES:**

- 1.Dictionary of Algorithms and Data Structure (NIST): <http://www.nist.gov/dads/>
- 2.OR-Library by J.E. Beasley: <http://www.brunel.ac.uk/depts/research/jeb/info.html>
- 3.Quadratic Assignment Problem Library: <http://www.opt.math.tu-graz.ac.at/qaplib/>

**LECTURE 31. Course: “Design of Systems: Structural Approach”**

**Dept. “Communication Networks & Systems”, Faculty of Radioengineering & Cybernetics**

**Moscow Institute of Physics and Technology (University)**

**Mark Sh. Levin**

**Inst. for Information Transmission Problems, RAS**

**Email: [mslevin@acm.org](mailto:mslevin@acm.org) / [mslevin@iitp.ru](mailto:mslevin@iitp.ru)**

**L.31. Satisfiability problem. Six basic combinatorial problems. Timetabling.**

***PLAN:***

**1. Satisfiability problem:**

**\*formulation and illustration , \*modification, \*applications**

**2. Basic combinatorial problems**

**(satisfiability, 3-satisfiability, vertex covering, 3-matching, clique, Hamiltonian cycle,  
partitioning,)**

**3. Timetabling problems**

**\*formulation and illustration, \*applications, \*algorithms and solving schemes, \*basic references (papers, sites)**

**Dec. 10, 2004**

# Satisfiability problem

## BOOLEAN VARIABLES:

$U = x_1, x_2, x_3, \dots$  ( 0 or 1 )

## LOGICAL OPERATIONS (AND, OR, NOT):

\*logical negation NOT \*conjunction AND,  
\*parentheses for grouping

## SET OF BOOLEAN EXPRESSIONS C:

Examples:  $C_1 = (\text{NOT } x_1) \text{ OR } x_2$   
 $C_2 = x_1 \text{ OR } (\text{NOT } x_3)$   
 $C_3 = (\text{NOT } x_5) \text{ OR } (\text{NOT } x_7) \text{ OR } x_9$   
 $C_4 = x_1 \text{ OR } x_{10} \text{ OR } (\text{NOT } x_{11}) \text{ OR } x_{22}$

ALL TRUE (1)

**Satisfiability is *the* original NP-complete problem.**

**Despite its applications to**

**\*constraint satisfaction,**

**\*logic, and**

**\*automatic theorem proving,**

**it is perhaps most important theoretically as  
the root problem**

**from which all other NP-completeness proofs originate.**

## Satisfiability

**Satisfiability, or SAT for short, is the following problem:**

**Given an expression in propositional logic is there a satisfying assignment?**

**For example, can we assign a value of true or false to each of the variables x, y, z such that the following expression is true?**

**$(x \text{ or } y) \ \& \ (x \text{ or not } y) \ \& \ (\text{not } x \text{ or } z) \ \& \ (\text{not } z \text{ or not } x \text{ or not } y)$**

**In this case the answer is yes: the satisfying assignment is x=true, y=false, z=true**

**There is no algorithm known that is guaranteed to solve any problem of this kind in a time polynomial in the number of variables.**

**The amazing discovery of Cook and others in the early 70s was that many natural problems such as scheduling, and many questions in networks, graphs, etc. can be transformed into the above form (see Garey & Johnson, 1979). Recently a popular area of interest is SAT-based planning (Kautz and Selman, 92).**

**In this approach the task of finding a plan in a given domain is converted to the task of finding a model for a certain satisfiability problem.**

## **MAXIMUM SATISFIABILITY**

**INSTANCE:** Set  $U$  of variables,  
collection  $C$  of disjunctive clauses of literals,  
where a literal is a variable or a negated variable in  $U$ .

**SOLUTION:** A truth assignment for  $U$ .

**MEASURE:**

**Number of clauses satisfied by the truth assignment.**

**APPLICATIONS:**

**1. Software Verification**

**2. Electronic Design Automation and Verification**

**3. Model Analysis**

**4. Model Checking**

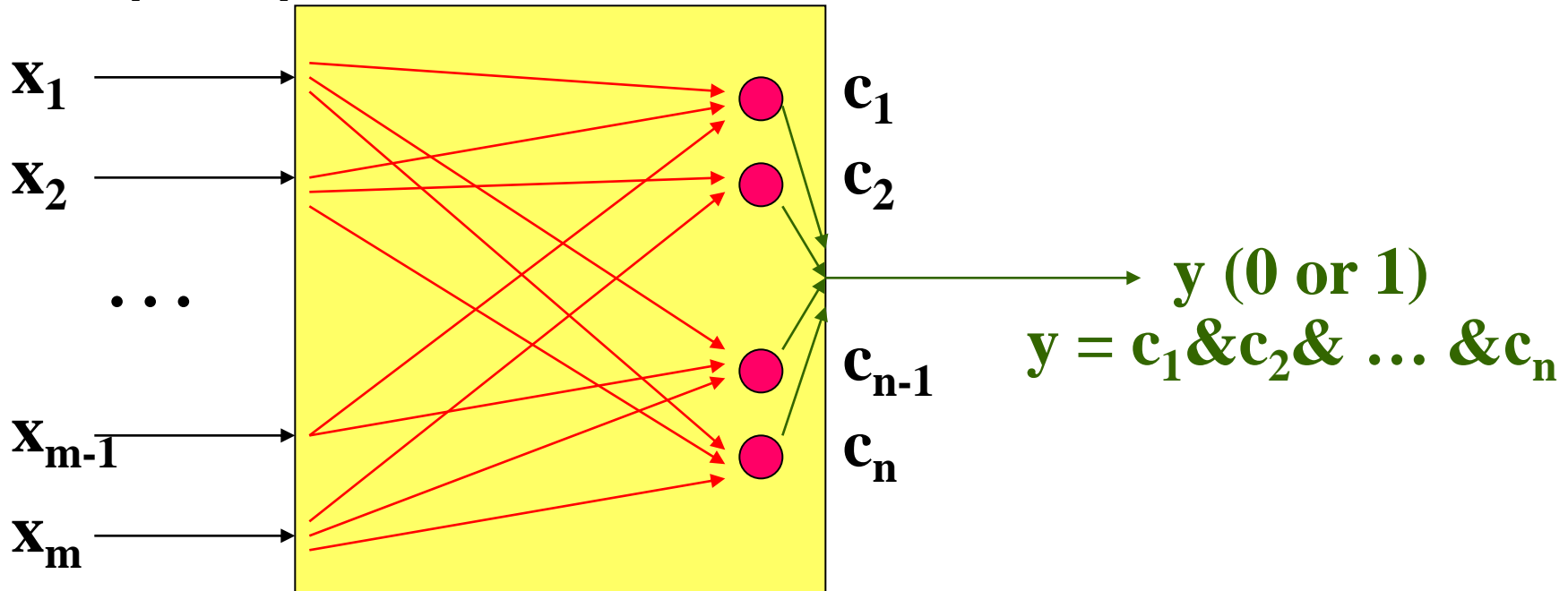
**5. Theorem Prover**

**6. AI Planning**

**PROBLEM: Exist  $x_0=(x_1,\dots,x_n)$  that  $y(x_0) = 1$  OR not**

Literal:  $x_i$  / not  $x_i$

**SYSTEM**



**Example:**

$$c_1 = \text{not } x_1 \text{ OR } x_2 \text{ OR } x_4 \text{ OR not } x_5 \text{ OR } x_7$$

$$c_2 = x_1 \text{ OR not } x_2 \text{ OR not } x_3 \text{ OR } x_5 \text{ OR } x_7$$

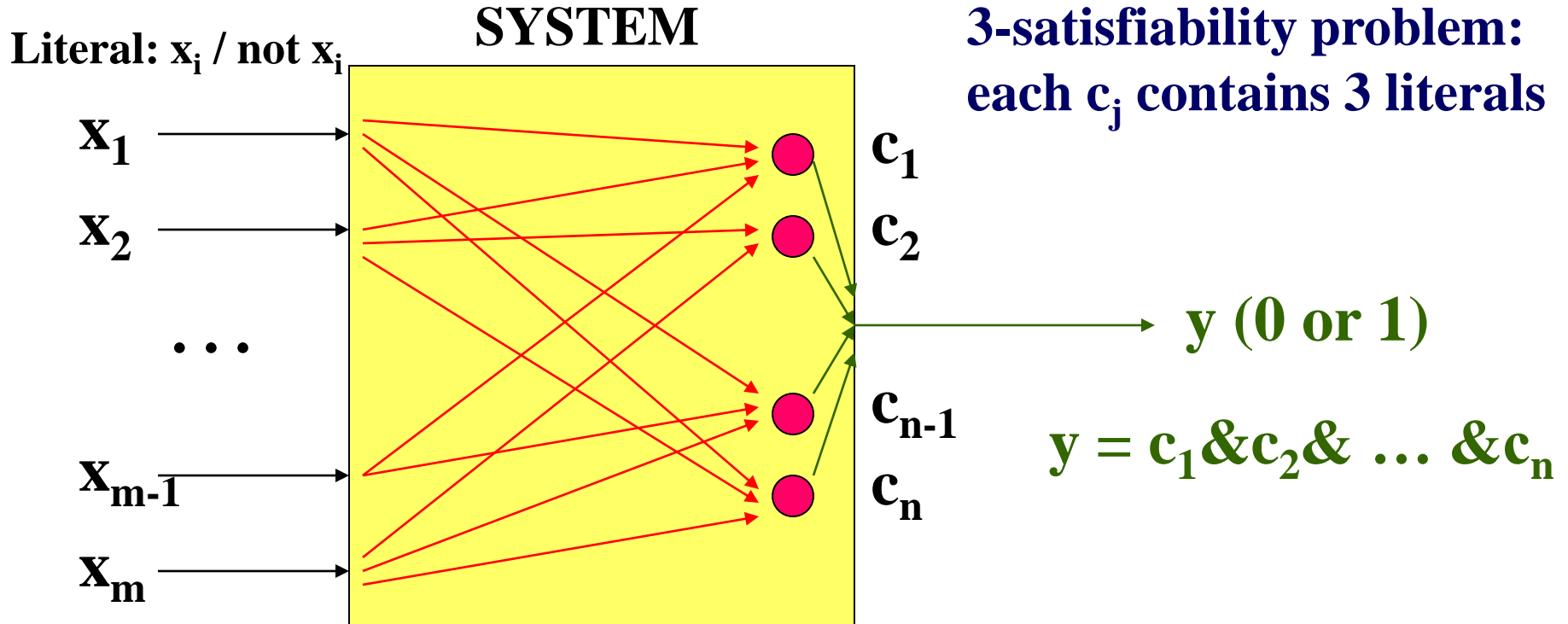
$$c_3 = \text{not } x_1 \text{ OR not } x_2 \text{ OR } x_3 \text{ OR not } x_5 \text{ OR not } x_n$$

$$c_4 = \text{not } x_2 \text{ OR } x_3 \text{ OR } x_7 \text{ OR } x_{n-2} \text{ OR } x_{n-1}$$

...



# 3-Satisfiability problem



**Example:**

$$c_1 = \text{not } x_1 \text{ OR } x_2 \text{ OR } x_4$$

$$c_2 = x_2 \text{ OR not } x_3 \text{ OR } x_7$$

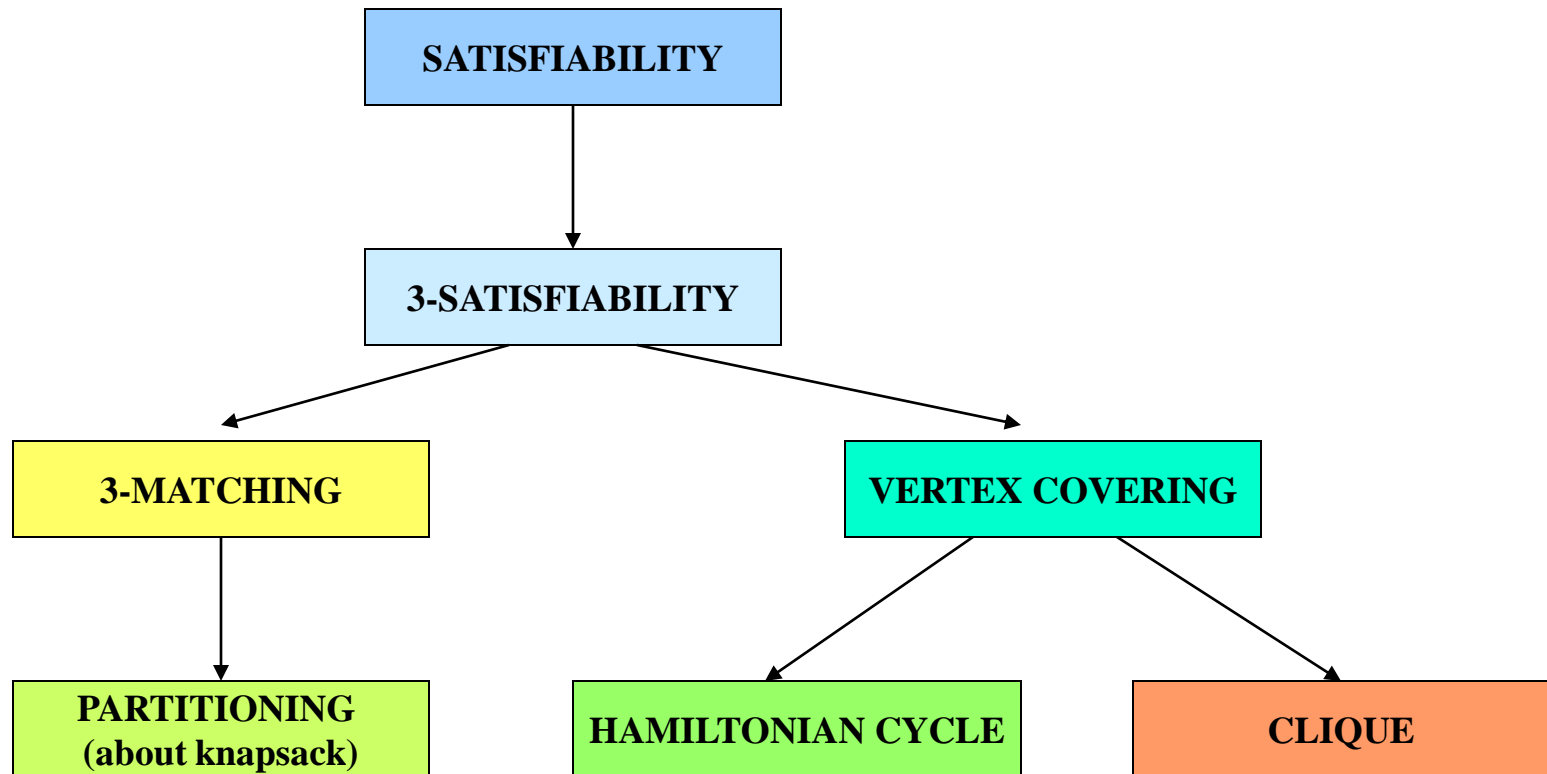
$$c_3 = \text{not } x_1 \text{ OR not } x_5 \text{ OR not } x_n$$

$$c_4 = \text{not } x_2 \text{ OR } x_{n-2} \text{ OR } x_{n-1}$$

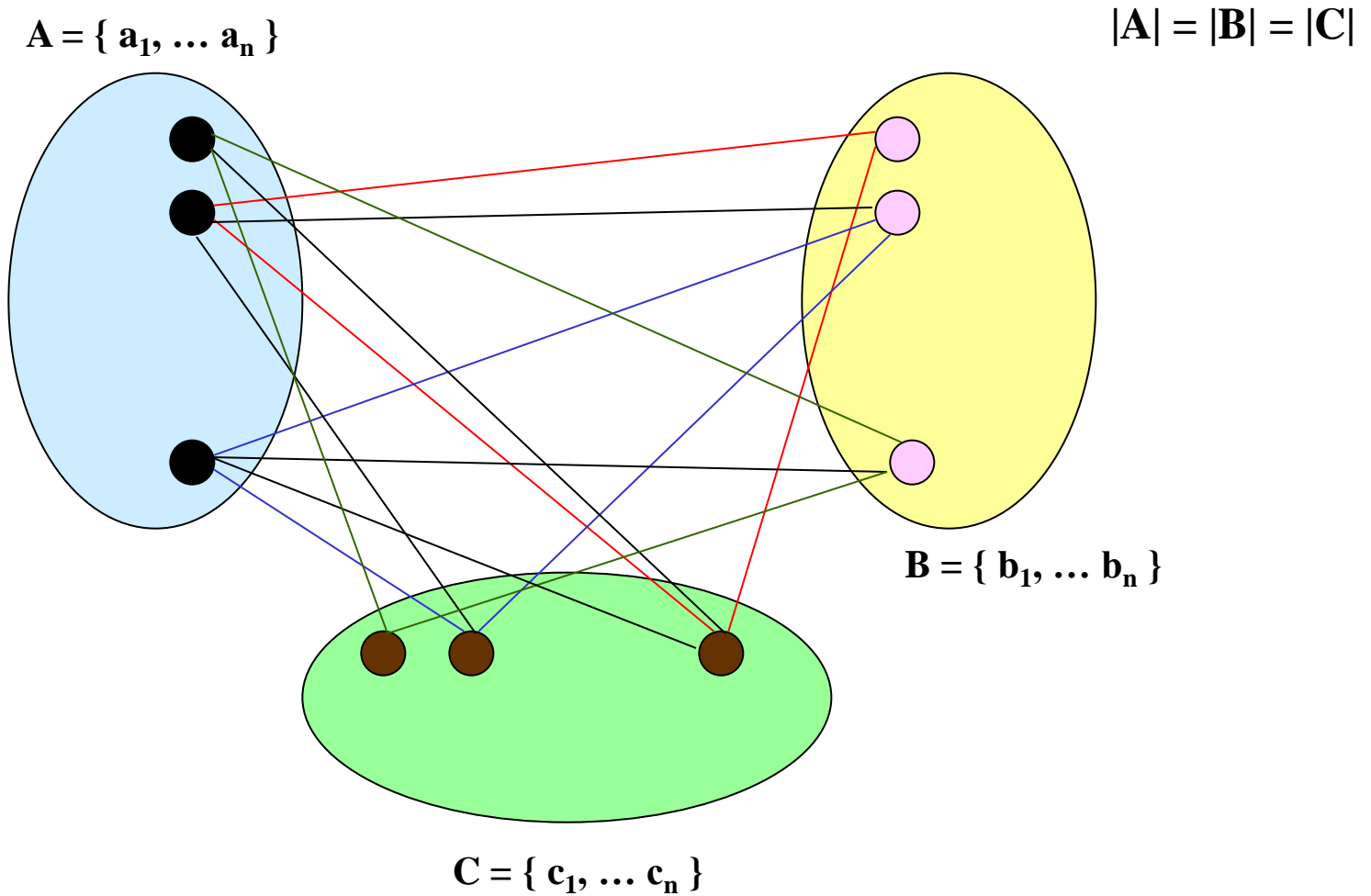
...

# Satisfiability Problem

## BASIC 6 NP-COMPLETE PROBLEMS AND DIAGRAM



# 3-matching problem



**PROBLEM: Exist covering by vertex triples (without intersection) OR not**

## Partitioning problem

Set of elements  $A = \{ 1, \dots, i, \dots, n \}$   
“weights” of elements  $\{ b_1, \dots, b_i, \dots, b_n \}$

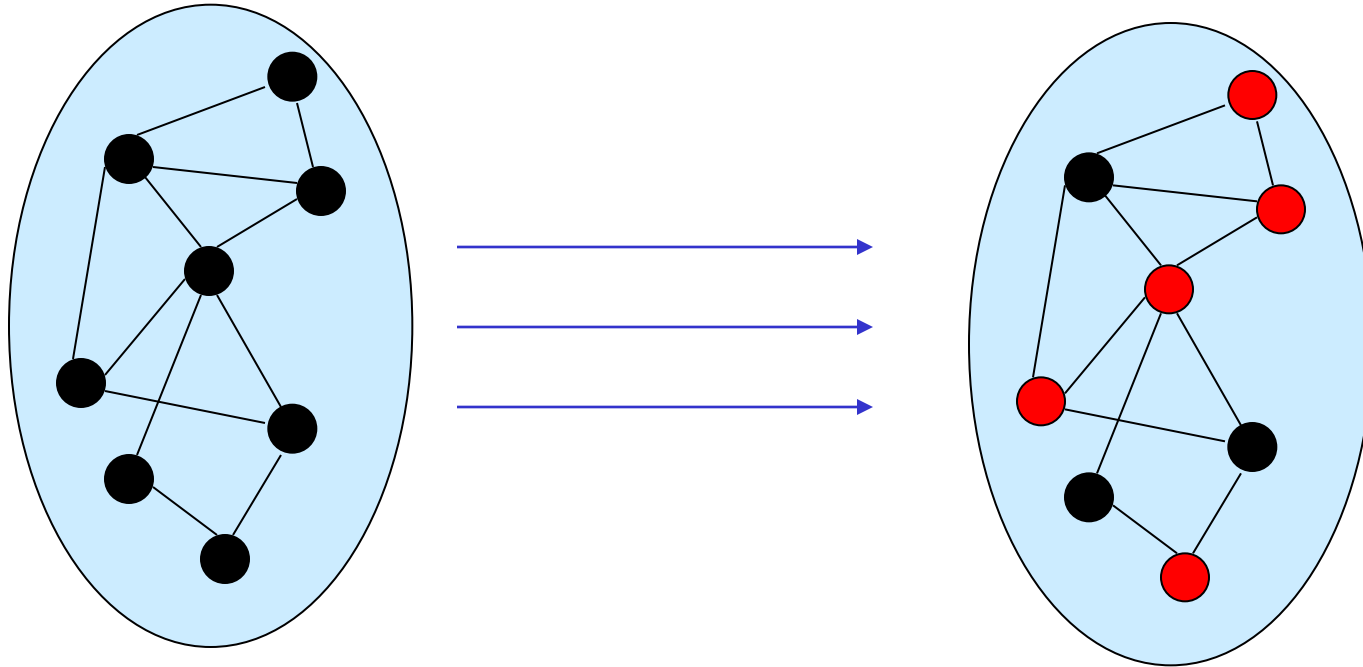
**PROBLEM:** Exist  $A' \subset A$  that

$$\sum_{i \in A'} b_i = \sum_{j \in A \setminus A'} b_j$$

**OR not**

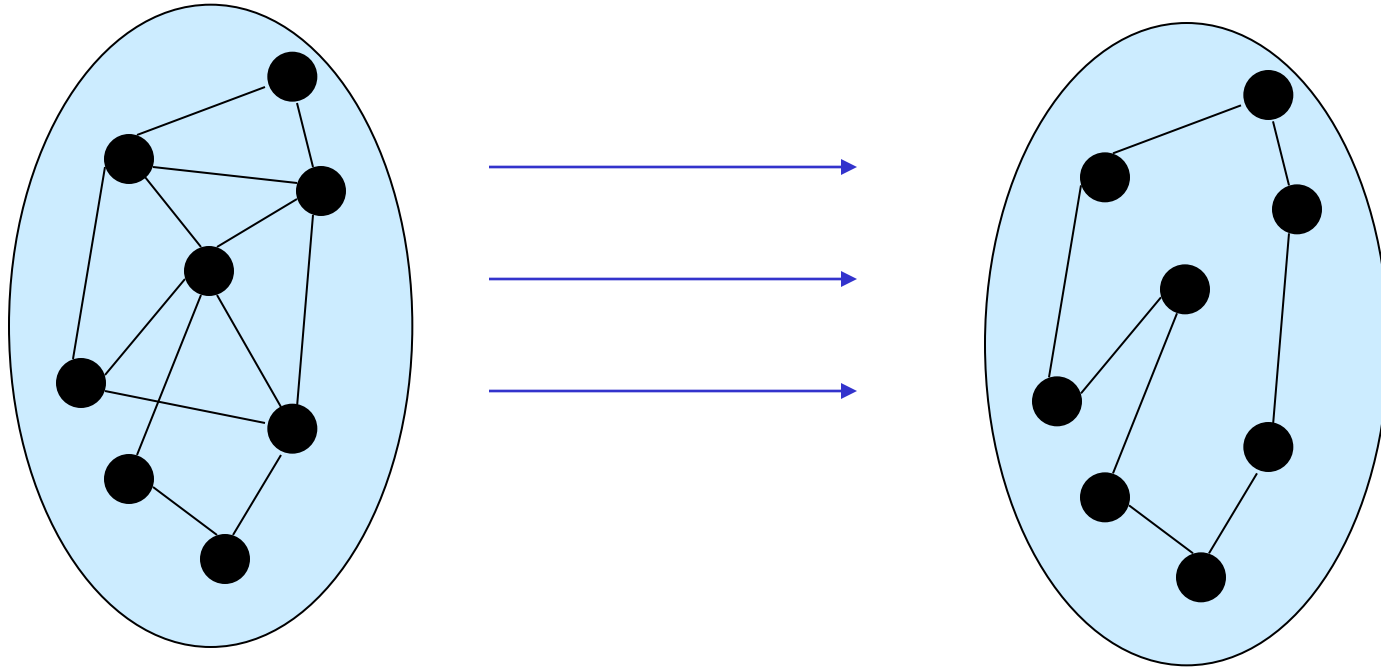
## Vertex covering problem

Vertex set  $A = \{ a_1, \dots, a_n \}$ , edge set  $E = \{ e_1, \dots, e_k \}$ , graph  $G = (A, E)$



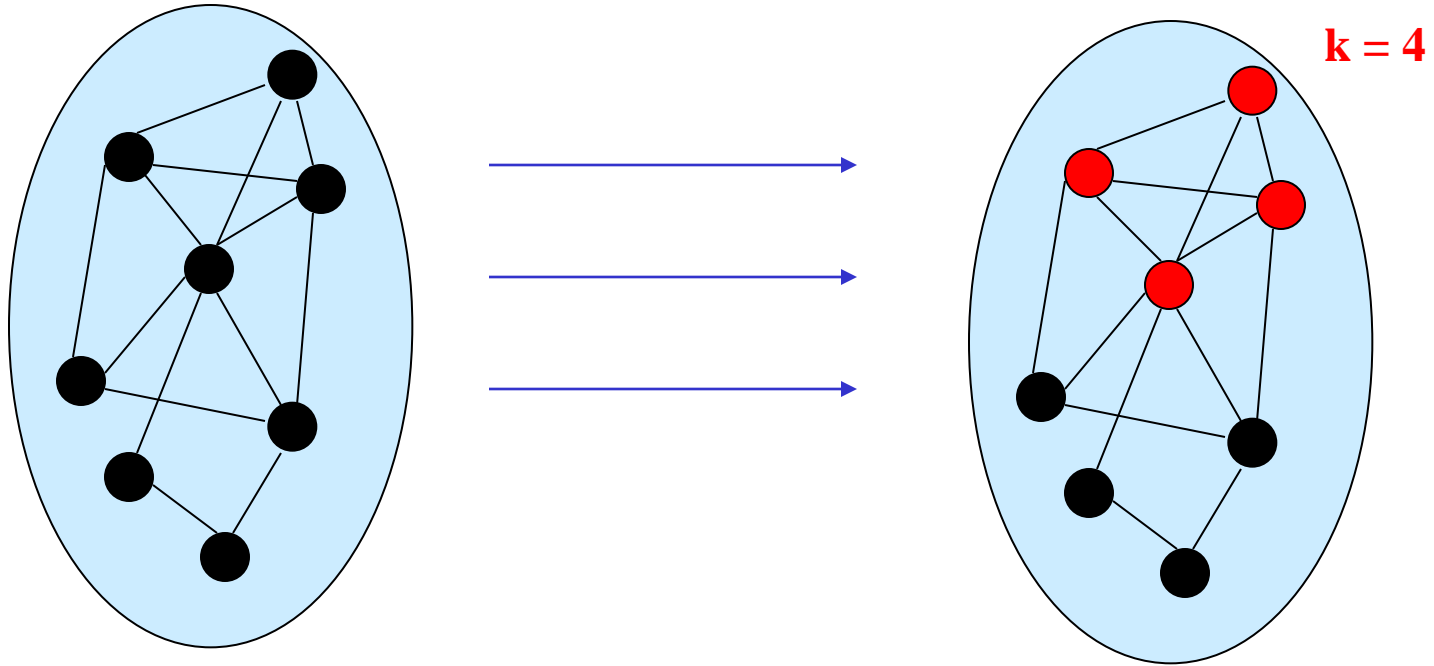
**PROBLEM:** find vertex covering ( $A' \subseteq A$ ) that covers  $\forall e \in E$

## Hamiltonian cycle problem



**PROBLEM: Exist Hamiltonian cycle OR not**

## Clique problem



**PROBLEM: Exist subgraph as clique with vertex number  $k$  OR not**

**BASIC (& CLOSE) COMBINATORIAL MODELS:**

**1.Graph coloring**

**2.Assignment / Allocation / Matching problems**

**3.Marriage problems (e.g., stable marriage problem)**

**4.Scheduling problems**

**5.Cyclic scheduling problems**

**6.Maximum clique problem**

**7.Combinatorial design (e.g., Latin square)**

**Etc.**



## **ALGORITHMS AND ALGORITHM SCHEMES:**

### **1.ENUMERATIVE METHODS**

### **2.AI METHODS:**

**expert systems (e.g., production-based systems)**

**knowledge-based systems**

**neural networks**

### **3.CONSTRAINED PROGRAMMING**

### **4.METAHEURISTICS (various local optimization techniques):**

**ant colony optimization,**

**iterated local search**

**simulated annealing**

**Tabu search**

**genetic algorithms**

### **5.COMBINATORIAL DESIGN**

### **6.EVOLUTIONARY COMPUTATION**

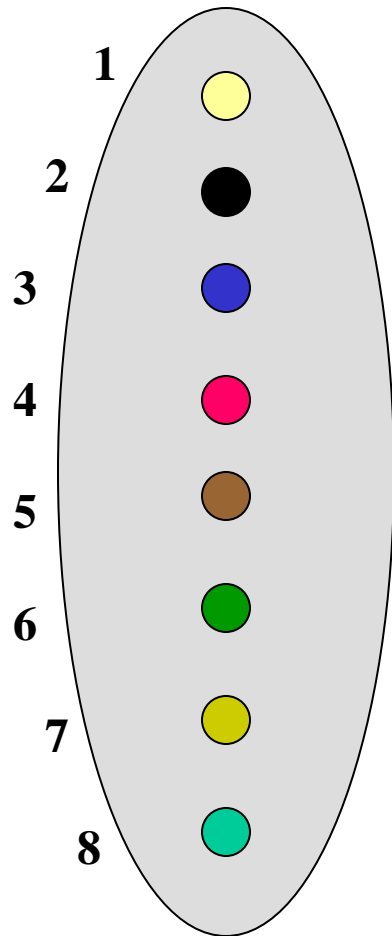
**(e.g., multi-objective evolutionary optimization)**

### **7.HYBRID SOLVING SCHEMES**

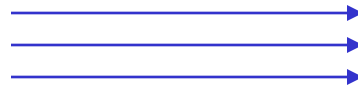


**Allocation (assignment, matching, location):**

Set of elements  
(e.g., personnel, facilities)



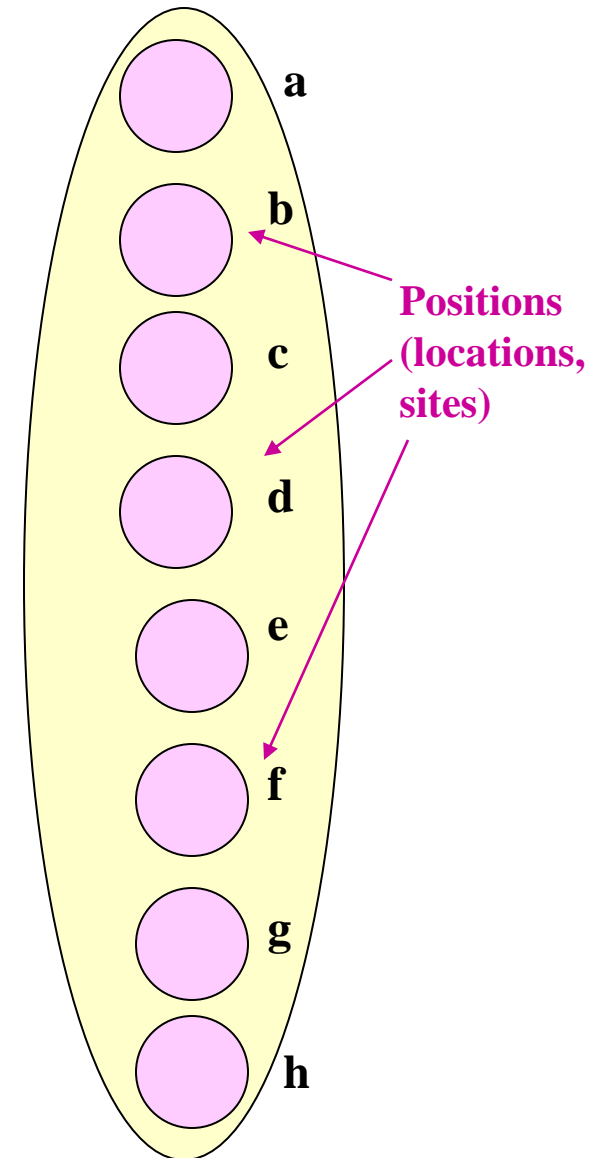
MAPPING



matrix of weights  $c_{ij}$

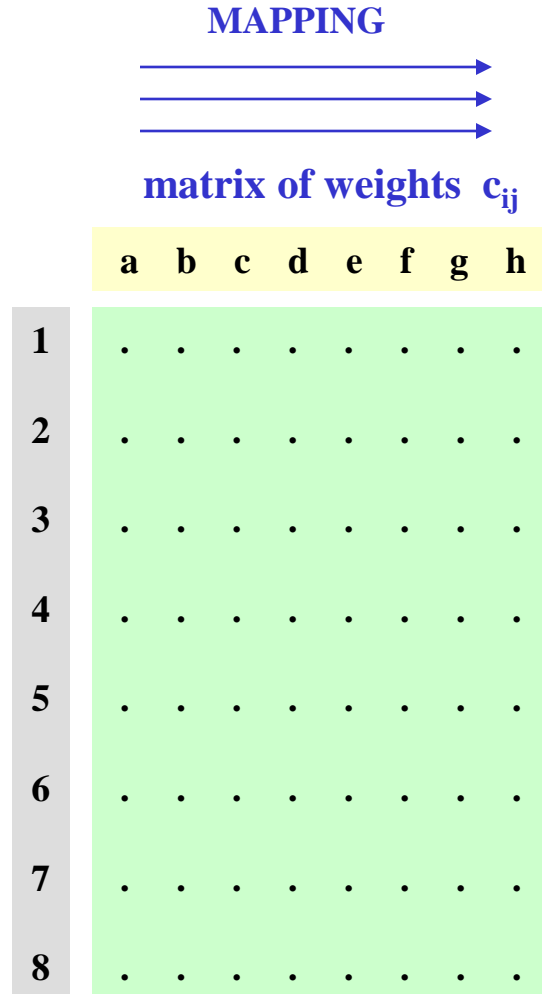
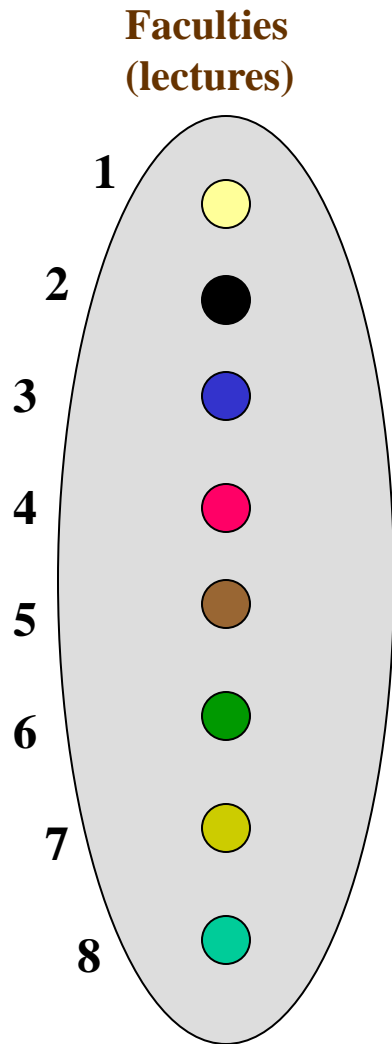
	a	b	c	d	e	f	g	h
1	.	.	.	.	.	.	.	.
2	.	.	.	.	.	.	.	.
3	.	.	.	.	.	.	.	.
4	.	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.
6	.	.	.	.	.	.	.	.
7	.	.	.	.	.	.	.	.
8	.	.	.	.	.	.	.	.

**BIPARTITE GRAPH**

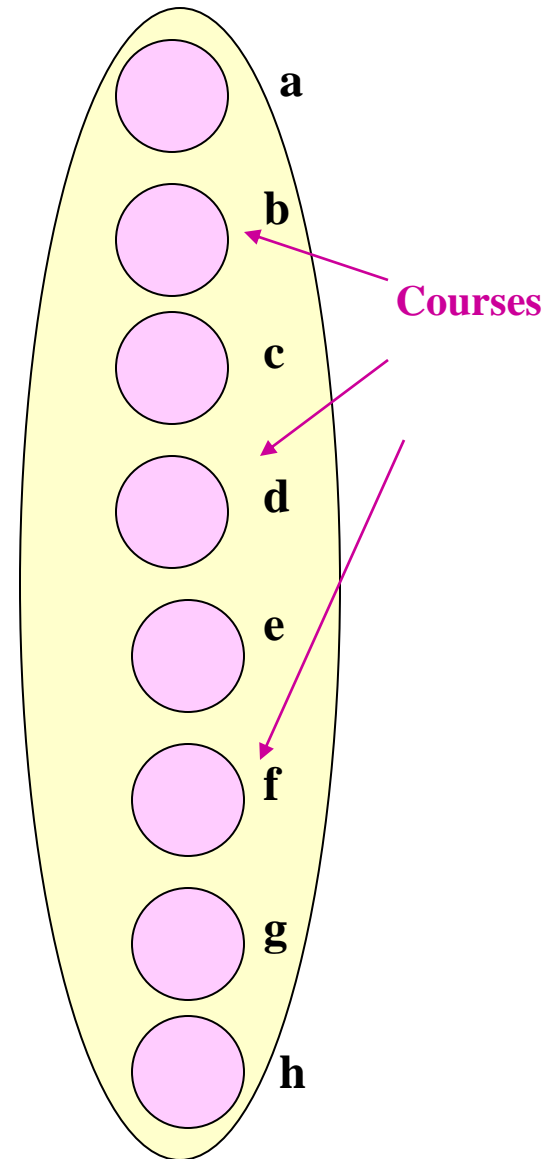


Allocation problem (from lecture 30)

Allocation (assignment, matching, location):



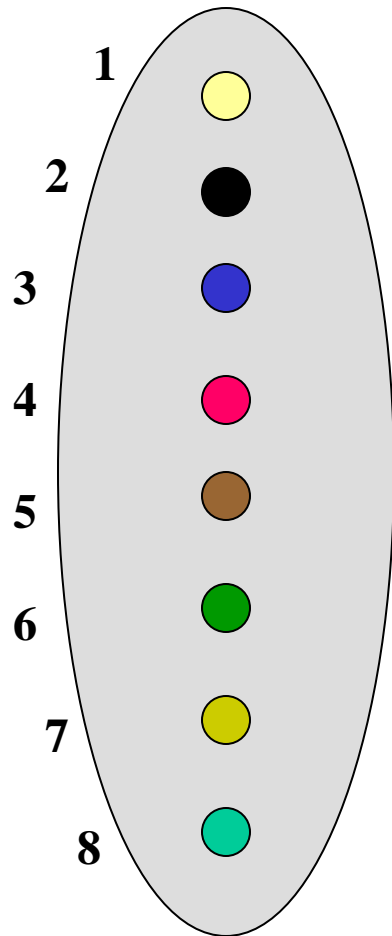
**BIPARTITE GRAPH**



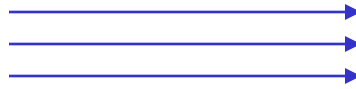
Allocation problem (from lecture 30)

Allocation (assignment, matching, location):

Student groups



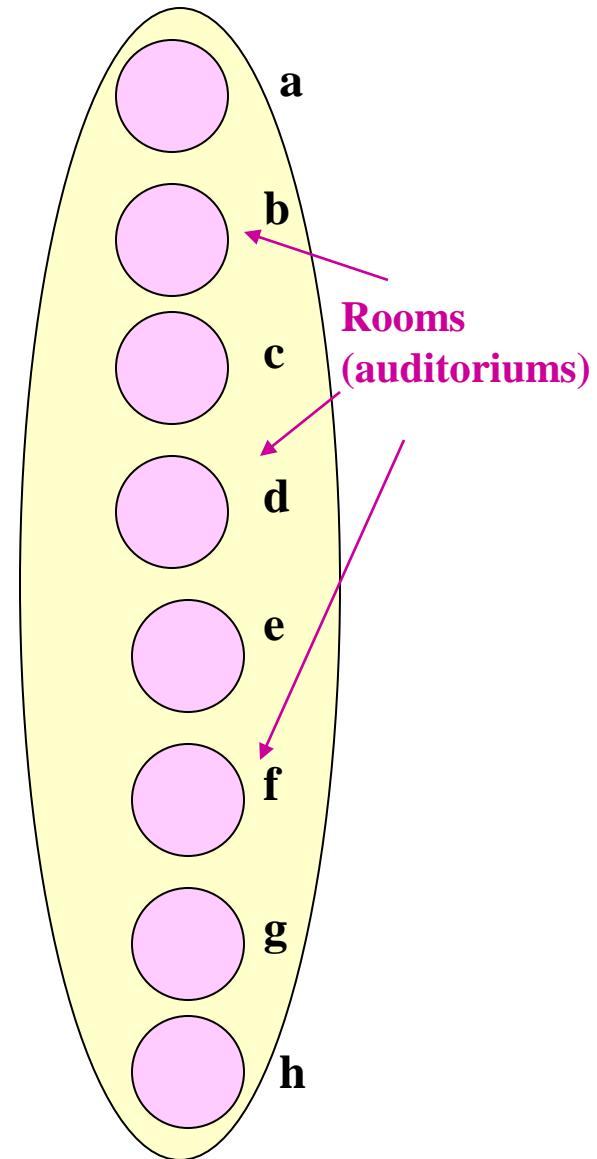
MAPPING



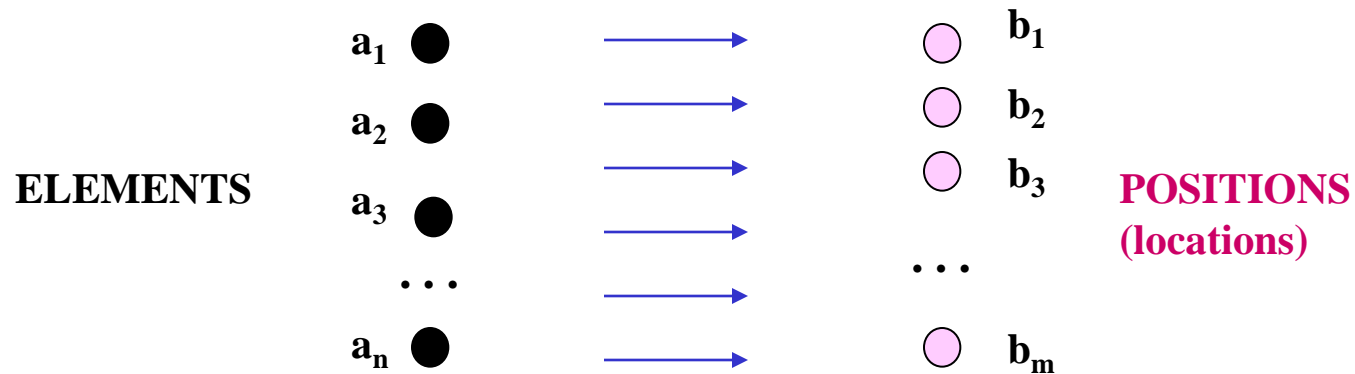
matrix of weights  $c_{ij}$

	a	b	c	d	e	f	g	h
1	.	.	.	.	.	.	.	.
2	.	.	.	.	.	.	.	.
3	.	.	.	.	.	.	.	.
4	.	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.
6	.	.	.	.	.	.	.	.
7	.	.	.	.	.	.	.	.
8	.	.	.	.	.	.	.	.

BIPARTITE GRAPH



## Assignment problem AP (from lecture 30)



### ANOTHER FORMULATION (algebraic):

Set of elements (e.g., personnel, facilities, tasks):  $A = \{ a_1, \dots, a_i, \dots, a_n \}$

Set of positions (e.g., locations, processors)  $B = \{ b_1, \dots, b_j, \dots, b_m \}$

(now let  $n = m$ )

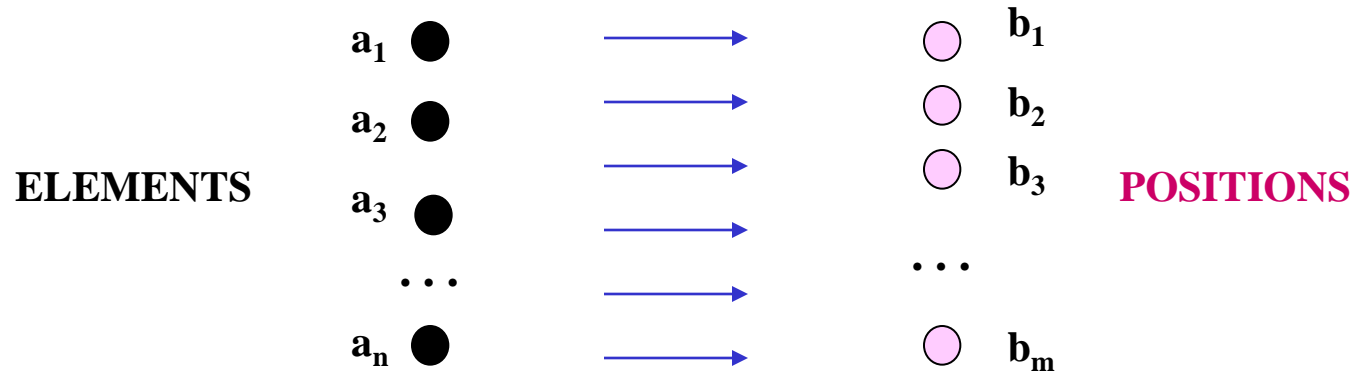
Effectiveness of pair  $a_i$  and  $b_j$  is:  $c(a_i, b_j)$

$x_{ij} = 1$  if  $a_i$  is located into position  $b_j$  and 0 otherwise ( $x_{ij} \in \{ 0, 1 \}$ )

The problem is:  $\max \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$

s.t.  $\sum_{i=1}^n x_{ij} = 1 \quad \forall j$   
 $\sum_{j=1}^n x_{ij} = 1 \quad \forall i$

## Quadratic Assignment Problem QAP (from lecture 30)



matrix of weights (“flow”)  $c_{ij}$

	$b_1$	$\dots$	$b_j$	$\dots$	$b_n$
$a_1$	.		.		.
$\dots$	.		.		.
$a_i$	.		$c_{ij}$		.
$\dots$	.		.		.
$a_n$	.		.		.

matrix of distance  $d_{ij}$

	$b_1$	$\dots$	$b_j$	$\dots$	$b_n$
$b_1$	.		.		.
$\dots$	.		.		.
$b_i$	.		$d_{ij}$		.
$\dots$	.		.		.
$b_n$	.		.		.

## A Basic (“flow”) Mathematical Formulation of the Quadratic Assignment Problem

An assignment of elements  $\{ i \mid i \in \{1, \dots, n\} \}$  to positions (locations)  $\{ p(i) \}$ , where  $p$  is permutation of numbers  $\{1, \dots, n\}$ , a set of all possible permutations is  $\Pi = \{ p \}$ .

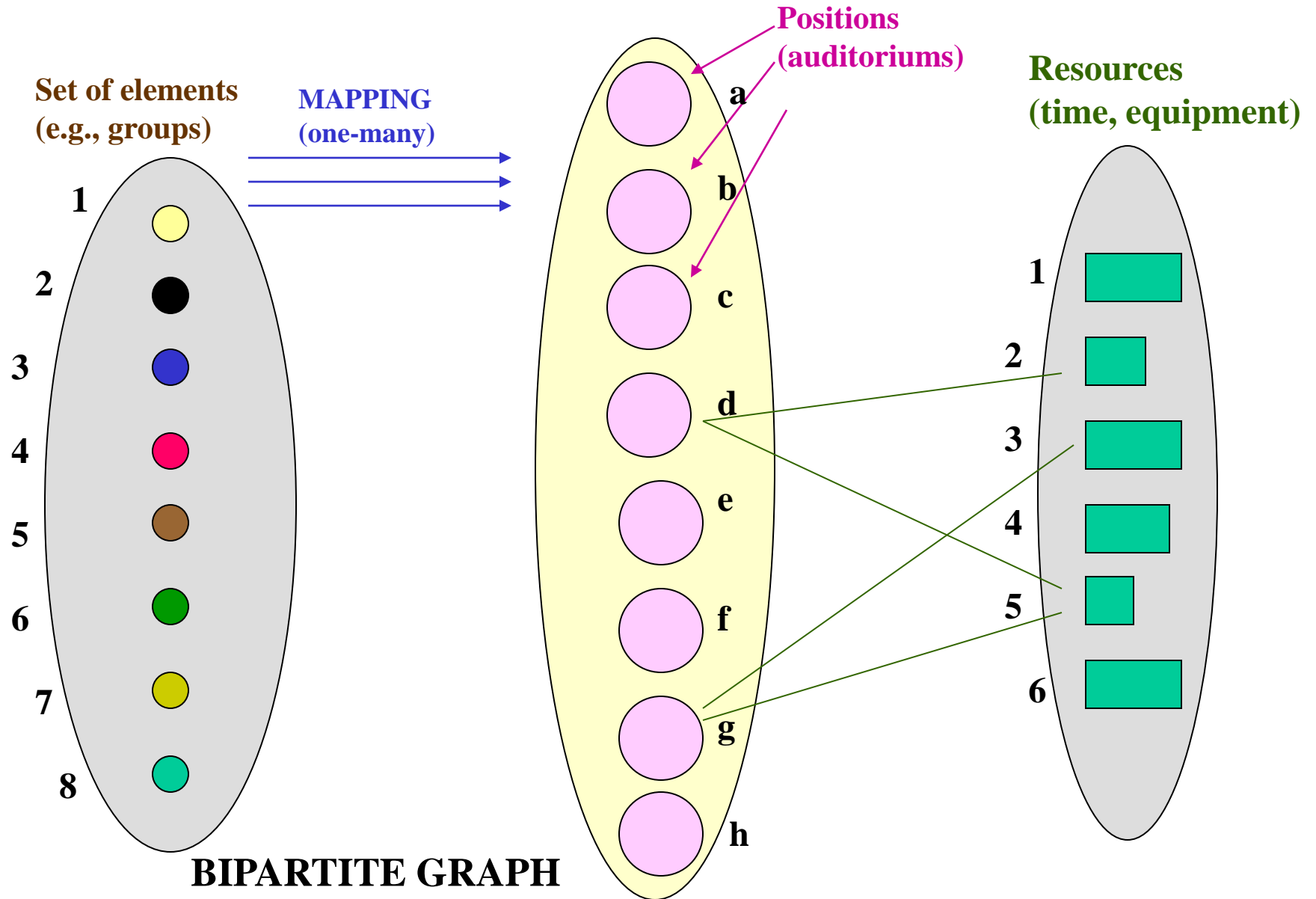
Let us consider two  $n$  by  $n$  matrices:

- (i) a “flow” (or “utility”) matrix  $C$  whose  $(i, j)$  element represents the flow between elements (e.g., facilities)  $i$  and  $j$ , and
- (ii) a distance matrix  $D$  whose  $(i, j)$  ( $p(i), p(j)$ ) element represents the distance between locations  $p(i)$  and  $p(j)$ .

With these definitions, the QAP can be written as

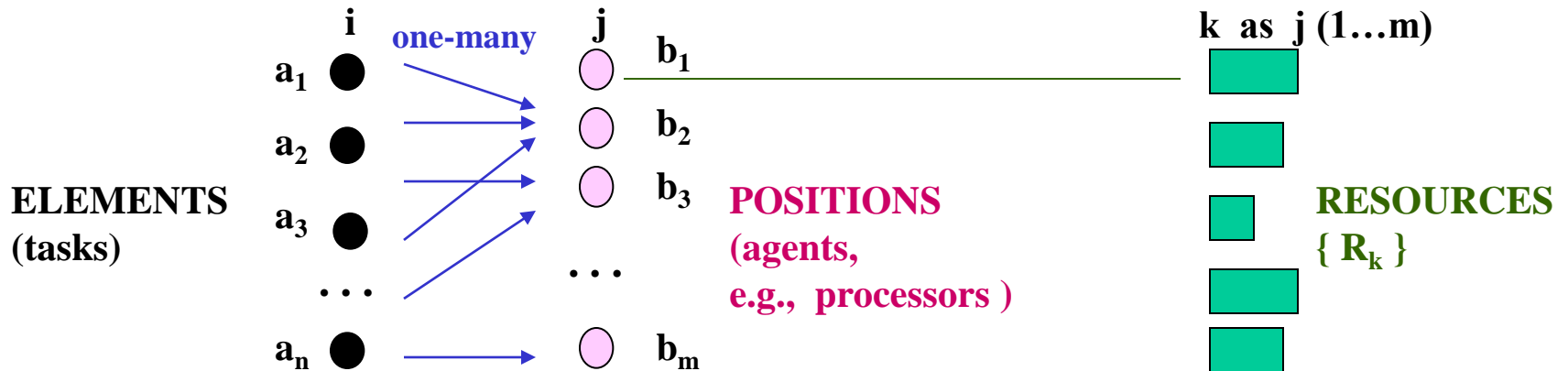
$$\max_{p \in \Pi} \sum_{i=1}^n \sum_{j=1}^n c_{ij} d_{p(i)p(j)}$$

Generalized Assignment Problem GAP (from lecture 30)





## Generalized Assignment Problem GAP (from lecture 30)



### FORMULATION (algebraic):

Set of elements (e.g., personnel, facilities, tasks):  $A = \{ a_1, \dots, a_i, \dots, a_n \}$

Set of positions (e.g., locations, processors)  $B = \{ b_1, \dots, b_j, \dots, b_m \}$

(now let  $n = m$ )

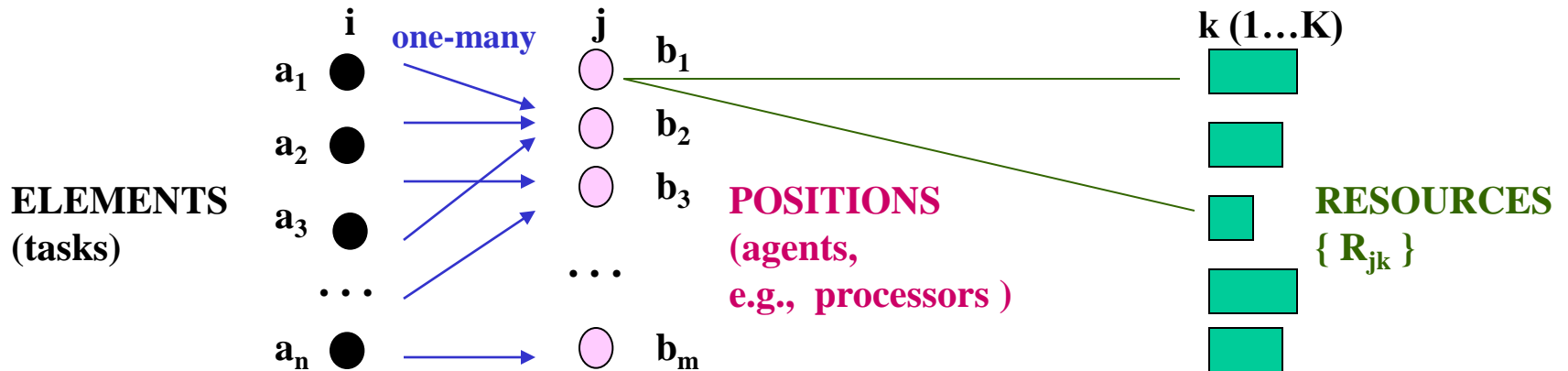
Effectiveness of pair  $a_i$  and  $b_j$  is:  $c(a_i, b_j)$

$x_{ij} = 1$  if  $a_i$  is located into position  $b_j$  and 0 otherwise ( $x_{ij} \in \{0, 1\}$ )

The problem 1 is:  $\max \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$

s.t.  $\sum_{i=1}^n r_{ik} x_{ij} \leq R_j \quad \forall j$  ( $R_k$  is resource of agent  $k$ )  
 $\sum_{j=1}^n x_{ij} = 1 \quad \forall i$

## Generalized Assignment Problem GAP (multiple common resources) (from lecture 30)



### FORMULATION (algebraic):

Set of elements (e.g., personnel, facilities, tasks):  $A = \{ a_1, \dots, a_i, \dots, a_n \}$

Set of positions (e.g., locations, processors)  $B = \{ b_1, \dots, b_j, \dots, b_m \}$

(now let  $n = m$ )

Effectiveness of pair  $a_i$  and  $b_j$  is:  $c(a_i, b_j)$

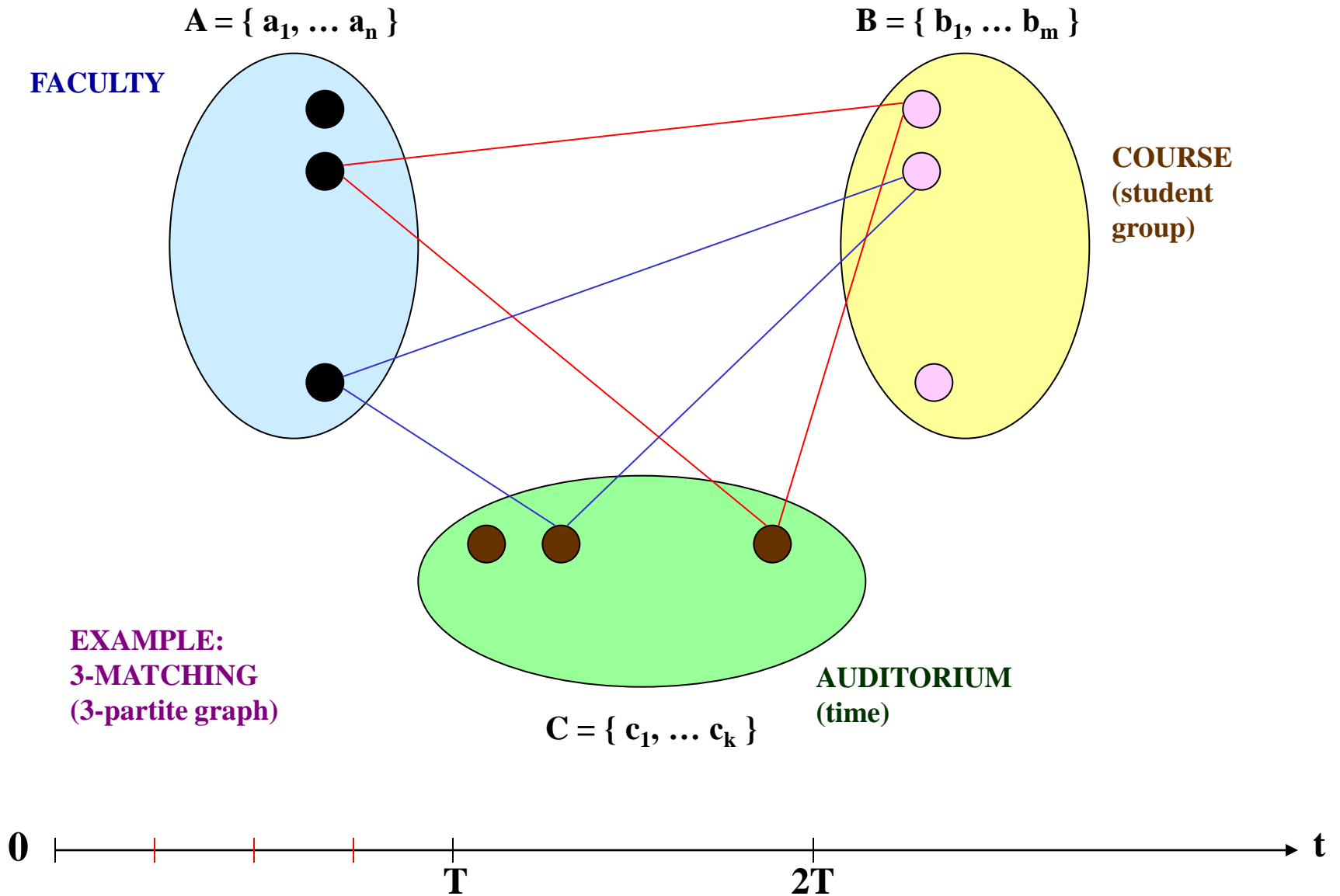
$x_{ij} = 1$  if  $a_i$  is located into position  $b_j$  and 0 otherwise ( $x_{ij} \in \{0,1\}$ )

The problem 2 is:  $\max \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$

s.t. 
$$\sum_{j=1}^m \sum_{i=1}^n r_{ik} x_{ij} \leq R_k \quad \forall k \text{ (common } K \text{ resources)}$$

$$\sum_{j=1}^m x_{ij} = 1 \quad \forall i$$

Timetabling problem: PLUS time (cycles)



**CONSTRAINTS (examples):**

**FOR LECTURERS:**

Lecturer 2 can teach only on Monday and Friday (**lecturers - time**)

Lecture 11 must be after Lecturer 12 (**over lecturers**)

Lecturers 5 and 7 can teach only in auditoriums 9 or 10 (**lecturers-auditoriums**)

**FOR STUDENT GROUPS:**

Group 1 needs auditorium 5 on Monday morning (**groups - time**)

Groups 7, 8, and 9 must have the same Lecturer 1 (**the same course**)

(**groups - lecturers**) \* (**groups - groups**)

Group 4 prefers Lecturer 10 (**groups - lecturers**)

Group 5 needs Lecturer 5 & Lecturer 8 as time neighbors (**groups - lecturers**)

**FOR AUDITORIUMS:**

Auditorium 1 is maintained on Wednesday (**closed**) (**auditoriums - time**)

Auditorium 4 corresponds only for groups 5, 7, 8, and 9 (**auditoriums - groups**)

**TYPES OF CONSTRAINTS (by elements):**

**Lecturers – lecturers**

**Lecturers - groups**

**Lecturers-auditoriums**

**Lecturers - time**

**Groups - groups**

**Groups – auditoriums**

**Groups - time**

**Auditoriums – auditoriums**

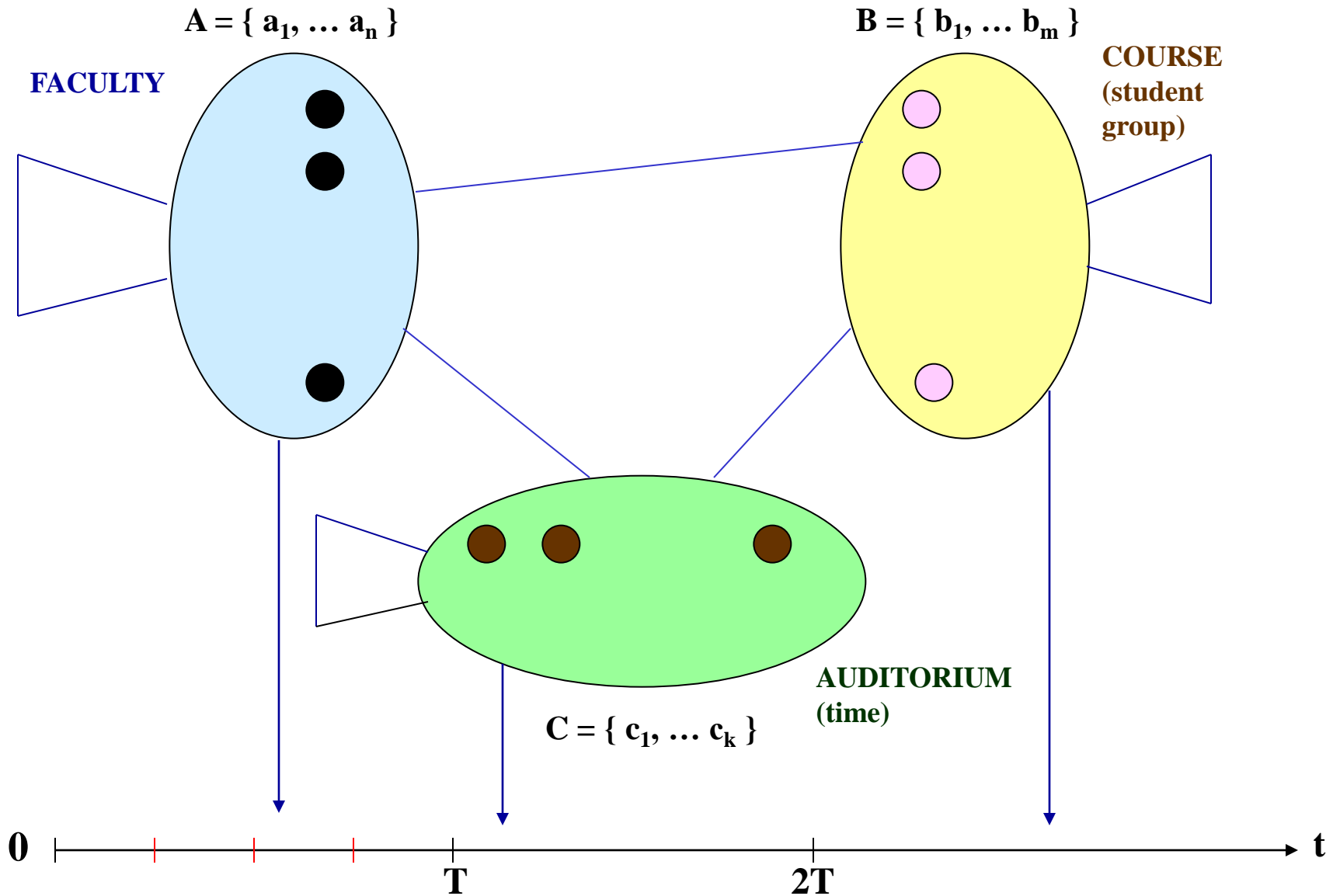
**Auditoriums - time**

**TYPES OF CONSTRAINTS (by kind):**

**Logical (e.g., binary relations)**

**Quantitative (e.g., resource constraints)**

Timetabling problem: illustration for constraints



Prospective application domains for timetabling problems

**1.SCHEDULING IN COMMUNICATION SYSTEMS  
AS COMMUNICATION TIMETABLING**

**2.SCHEDULING IN MONITORIG SYSTEMS  
AS MONITORING TIMETBLING**

**G.L. Nemhauser, M.A. Trick, Scheduling a major college basketball conference. Operations Research, 46(1), 1-8, 1998.**

**FORMULATION, DATA & CONSTRAINS:**

- 1. Basic objects: teams, slots (weekday slots, weekend slots),  
each team plays twice in a week, 8 home slots, 8 away slots**
- 2. Constraints for teams and slots: home slots, away slots, chain: home-away-etc.**
- 3. Patterns and constraints:  $\leq 2$  away games,  $\leq 2$  home games**
- 4. Team pairing constraints: candidates of team pair (from previous schedule)**

**SOLVING SCHEME:**

**STEP 1. A pattern is a string of H (home), A (away), and B (bye).**

**Examples: HAA, HBW**

**To find a set of pattern, example: HHA, AHA, HAH, AAH (teams a, b, c, d)**

**(enumeration & integer programming)**

**STEP 2. To assign games to the patterns. This is timetabling**

**(integer programming)**

**STEP 3. To assign teams to the patterns. This with patterns gives schedule**

**(quadratic assignment problem)**

**COMPUTING: 24 hours**